Using a VECM to characterise the relative importance of permanent and transitory components

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Summary

This paper uses cointegrated two-variable autoregressions (VECM) with the identifying assumptions proposed by Blanchard and Quah (1989) to characterise the transitory components in U.S total disposable income and consumption of nondurables and services. The results indicate that: all the variance of consumption change is explained by the trend, while the variance of the cyclical component accounts for an estimated 64 percent of the total variance of income change, consumption is the trend in income, but is not a random walk, excess sensitivity and excess smoothness in consumption change equation are found. While multivariate trend-cycle decomposition isolates large cyclical component in income, the persistence is high. This outcome suggests that the persistence of total income is not a useful indicator of the size of trend in the series confirming two well-known results: (i) the random walk specification for the trend, common in the univariate analysis, is biased towards establishing the permanent component as important, (ii) the importance of the cyclical component depends on the information set and is higher when multivariate estimate is performed.

J.E.L. Classification: C32, E32
Keywords: business fluctuations, multivariate trend-cycle decomposition, permanent income hypothesis.

1. Introduction

A large part of macroeconomic time series can be modelled as non-stationary stochastic processes with at least one unit root. In particular, there is general agreement that the real variables are integrated process of order one and have a difference stationary representation. An I(1) process can be written in the sum of a I(1) component (permanent component) and an I(0) component (transitory component). We simply call these components trend and cycle.

During the '80s, following the influential work of Nelson and Plosser (1982), a voluminous literature has estimated the magnitude of the stochastic trend of a time series in a univariate framework. A partial list includes Harvey (1985), Watson (1986), Clark (1987), Campbell and Mankiw (1987), Cochrane (1988). These studies have, in general, found no mean-reversion, especially in real GNP. This does not provide an empirical support for the macroeconomic theory designed to produce and understand transitory
fluctuations. However, the univariate results crucially depend on assumptions about the
two components required to identify the model: correlation versus orthogonality and the
random walk specification of the trend (Watson 1986, Christiano and Eichenbaum 1990,
Lippi and Reichlin 1992, Quah 1992). Recent literature has proposed a multivariate
analysis of economic fluctuations: a partial list includes Evans (1989), Blanchard and
Quah (1989), King et al. (1991), Blanchard (1993), Cochrane (1994), Evans and
Reichlin (1994), Lippi and Reichlin (1994). The multivariate trend-cycle decomposition
is not necessarily linked to the a-priori restrictions on the dynamics of the two
components, in particular to the restrictive hypothesis that the trend is a random walk. In
general, these studies have found a cyclical component, characterising the economic
fluctuations, more important than the univariate estimates.

This paper intends to illustrate the trend-cycle decomposition in the context of
cointegrated two-variable autoregressions of U.S total disposable income and
consumption of non-durables and services using the identification of Blanchard and Quah
(1989). Why the choice of consumption-income VECM? The permanent income
hypothesis under rational expectation, PIH, suggests to us that consumption and total
income are cointegrated and that consumption change is a good forecaster of the trend in
total income. Our results are the following: i) consumption "nearly" represents the trend
in total disposable income, but is not a random walk and exhibits both excess sensitivity
and excess smoothness; ii) almost all the variance of consumption change is due to trend,
while the 64 percent of the total variance of income change is explained by the cycle. The
results in (i) are in line with recent theoretical and empirical literature on consumption
(Deaton 1987, Campbell and Mankiw 1989, Pischke 1991, Goodfriend 1992, Forni and
Lippi 1994). Moreover, the result of non random walk can also explain why the
univariate persistence measures are not useful to describe the permanent component in
the series: total income shows significant persistence in its innovations, but nevertheless
has its fluctuations dominated by cyclical components. The result in (ii) is in line with
Cochrane (1994). He uses an approximation of the PIH in consumption and total income level to log consumption and log GNP and uses a different identifying assumption: the conventional c first orthogonalisation. The uniformity of results excepting the (i), with respect to the Cochrane's paper, suggests the robustness of the multivariate trend-cycle decomposition vis-a'-vis the use of a different data set and a different identifying assumption\(^1\).

The paper is organised as follows: section 2 briefly describes the identifying assumptions in order to obtain the trend-cycle decomposition from the cointegrated two-variable autoregressions; section 3 presents the estimate of the VECM, the variance decomposition and the impulse response functions of consumption and total income. It also contains a comparison between the importance of the trend in income obtained by bivariate estimate and that suggested by the univariate persistence measure; section 4 concludes.

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\(^1\)As far as this latter point is concerned, see Evans and Reichlin (1994), Lippi and Reichlin (1994).
2. The model

In this section we show the identifying assumptions to characterise the trend-cycle representation of a VECM in consumption and total income. Full identification is reached by imposing restrictions on the matrix of the long run multipliers and assuming that the disturbances of the two components are orthogonal. More precisely, we make the following assumptions: 1) one shock is temporary and one is permanent; 2) the variance-covariance matrix of the shocks is the identity matrix. This procedure has been used by Blanchard e Quah (1989), King et al. (1991).

We suppose that the two variables are I(1) following VECM representation (reduced form)

\[ \Gamma(L) \Delta Z_t = -\alpha \beta' Z_{t-1} + \epsilon_t, \]  

(1)

where \( Z_t \) is a (2x1) vector of the two variables, \( \Gamma(L) = \begin{bmatrix} \Gamma_{11}(L) & \Gamma_{12}(L) \\ \Gamma_{21}(L) & \Gamma_{22}(L) \end{bmatrix} \) is a (2x2) polynomial matrix with the normalisation \( \Gamma(0) = I \), \( \alpha \), \( e \), \( \beta' \) are respectively the (2x1) vector of the adjustment coefficients to the long run equilibrium and the (1x2) cointegrating vector that characterise this equilibrium, \( \epsilon_t \) is the (2x1) vector of the disturbances such that \( E(\epsilon_t) = 0 \) and \( E(\epsilon_t, \epsilon_t') = \Sigma_\epsilon \), \( p \) is the truncation order of the VECM. We ignore, for simplicity, the constant and the dummies variables. Assumptions (1) and (2) above imply the existence and uniqueness of the following trend-cycle representation (structural model)

\[
\begin{pmatrix}
\Delta Z_{1t} \\
\Delta Z_{2t}
\end{pmatrix} = \begin{pmatrix}
E_{11}(L)u_{1t} + E_{12}(L)u_{2t} \\
E_{21}(L)u_{1t} + E_{22}(L)u_{2t}
\end{pmatrix},
\]  

(2)
where \( E_{11}(L) \), \( E_{12}(L) \), \( E_{21}(L) \), \( E_{22}(L) \) are rational functions in \( L \), the variance-covariance matrix of the shocks is \( \Sigma_u = I \) and \( E_{11}(1) = E_{21}(1) = 0 \), so that \( u_1 \) is the transitory shock and \( u_2 \) the permanent shock. \( E_{11}(L)u_{1t} \) and \( E_{21}(L)u_{1t} \) are the first differences of the cycles, while \( E_{12}(L)u_{2t} \) and \( E_{22}(L)u_{2t} \) are the first differences of the trends of the two variables. Notice that there is a common trend, in the sense that the two permanent components are driven by the same shock, \( u_1 \), but, in general, these components have different impulses, so that the notion of common trend is not one of proportionality as in Stock and Watson (1988) definition, moreover, the trend is not necessarily a random walk. If the trend were a random walk then \( E_{12}(L) = E_{22}(L) = E_{12}(0) \). This outcome does not generally occur and the sole condition implied in the model is that \( E_{12}(1) = E_{22}(1) \neq 0 \). The sequence of steps to achieve the full identification of the bivariate model is contained in the Appendix.

The PIH under rational expectation implies that total income and consumption are \( I(1) \) and cointegrated (Campbell 1987), however the implied VECM has a very special form: since consumption change is a pure innovation (Hall 1978), all the coefficients in the consumption change equation are zero. It is total income, not consumption, that adjusts toward equilibrium, by means of the movements in capital income induced by savings. Moreover, since consumption change is proportional to the univariate labour income innovation times a constant (that depends on the market discount factor), it is thus possible to demonstrate that consumption change is the revision in the long run expectation of total income, that is the Beveridge and Nelson trend\(^2\) (Beveridge and Nelson 1981). This version of the PIH has generally been rejected

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\(^2\)Suppose that the labour income is \( I(1) \), so that the first difference has the following representation \( \Delta y_t = a(L)e_t \), where \( e_t \) is white noise and \( a(L) \) is a rational function. Under the hypotheses of quadratic utility function and that the market discount factor is equal to the individual discount factor, the consumption change is \( \Delta c_t = a(\beta)e_t \), where \( \beta = \frac{1}{1+r} \) is the market discount factor and \( r \) is the interest rate. Total income is also \( I(1) \), so that the first difference has the following representation: \( \Delta x_t = b(L)e_t \). It is possible show that \( \Delta c_t = a(\beta)e_t = b(1)e_t \), that is the Beveridge and Nelson trend in income.
by both macro and micro data: "excess sensitivity" and "excess smoothness" of consumption are generally found. These findings have stimulated a literature aimed at reconciling PIH to empirical evidence. For example, models that assume infinitely lived agents with aggregation effects arising from incomplete information and different income processes (Campbell and Mankiw 1989, Pischke 1991, Goodfriend 1992, Lippi and Forni 1994). The VECM in (1) is general in the sense that does not require the trend to be a random walk, as noted above, so that it is possible to test this implication of the PIH versus the recent PIH literature.

3. Empirical results

In this section the VECM (1) is estimated in order to characterise the trend-cycle decomposition, using U.S national income and product accounts (NIPA) quarterly data from 1947:1 al 1991:4. All data are seasonally adjusted at quarterly rates, taken in per-capita terms and expressed in thousands of 1987 U.S dollars. We analyse total disposable income (x) and consumption of nondurables and services (cn). To estimate the VECM it is necessary to verify that 1) the variables are I(1), 2) the variables are cointegrated.

x and cn are I(1) from the simple Dickey e Fuller tests (Table 1). Before applying the Johansen procedure (Johansen 1988) to test the cointegration it is necessary to determine the lag length (p truncation order) of the VAR that ensures approximately white noise normal residuals. Tables 2 and 3 suggest that a VAR(2) with four dummies (D50:1, D74:1, D87:2, D87:2) is sufficient to whiten the residuals vector. Many of the identified outliers, corrected by the dummies, seem to be associated with turning points

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3 A significant correlation between consumption change and past income change (and/or past consumption change), has been referred to in the literature as "excess sensitivity". Excess sensitivity has been tested employing both macro and micro data by many authors. A partial list is Flavin (1981), Hall and Mishkin (1982), Hayashi (1985), Campbell (1987), Campbell and Deaton (1989). The independence of consumption change from past information has (nearly) always been rejected by the data. The PIH predicts that consumption is more volatile than income when income is positively autocorrelated, as in the case of U.S data. This is paradoxical, since the early PIH was intended to explain precisely the fact that consumption changes are very small as compared with income changes (Deaton paradox). The empirical evidence that the variance of consumption changes is much less that of income changes is what is known as "excess smoothness"(Deaton 1987, Forni 1994).

4 The estimates are performed by using PcFlm and Matlab.
in the business cycle, particularly recessions, apart from the Korean War outlier, D50:1, that it is associated with an expansion. The positive outlier in income, D50:1, may also be due to one-time National Service Life Insurance dividend payment to World War II veterans. Balke and Fomby (1994) show that the outliers can explain a substantial proportion of the volatility of the series. Hence, it is necessary to take into these outliers in the estimate if the relative importance of trend and cycle are to be explained.

Table 1. Dickey-Fuller tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>DF</th>
<th>ADF(1)</th>
<th>ADF(2)</th>
<th>ADF(3)</th>
<th>ADF(4)</th>
<th>ADF(5)</th>
<th>ADF(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>With trend</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cnds</td>
<td>-2.77</td>
<td>-3.09</td>
<td>-2.72</td>
<td>-2.48</td>
<td>-2.62</td>
<td>-2.80</td>
<td>-2.69</td>
</tr>
<tr>
<td>ΔX</td>
<td>-12.18</td>
<td>-9.38</td>
<td>-6.98</td>
<td>-6.51</td>
<td>-6.41</td>
<td>-6.19</td>
<td>-4.95</td>
</tr>
<tr>
<td>ΔCnds</td>
<td>-9.79</td>
<td>-7.13</td>
<td>-5.14</td>
<td>-4.71</td>
<td>-4.77</td>
<td>-4.05</td>
<td>-3.43</td>
</tr>
<tr>
<td>Without trend</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>0.22</td>
<td>-0.049</td>
<td>0.81</td>
<td>-0.18</td>
<td>0.023</td>
<td>0.21</td>
<td>0.37</td>
</tr>
<tr>
<td>Cnds</td>
<td>1.12</td>
<td>0.81</td>
<td>0.43</td>
<td>-0.018</td>
<td>0.052</td>
<td>0.317</td>
<td>0.072</td>
</tr>
<tr>
<td>ΔX</td>
<td>-12.21</td>
<td>-9.39</td>
<td>-7.01</td>
<td>-6.52</td>
<td>-6.41</td>
<td>-6.16</td>
<td>-4.96</td>
</tr>
<tr>
<td>ΔCnds</td>
<td>-9.71</td>
<td>-7.11</td>
<td>-5.19</td>
<td>-4.75</td>
<td>-4.76</td>
<td>-4.11</td>
<td>-3.53</td>
</tr>
</tbody>
</table>

Note: Dickey - Fuller critical values: -2.87 (regression without trend), -3.43 (regression with trend).

Table 2. Mis-specification tests, VAR(2)

<table>
<thead>
<tr>
<th>Equation</th>
<th>Autoc- F(5,168)</th>
<th>Normality- $\chi^2$ (2)</th>
<th>ARCH- F(4, 165)</th>
<th>Heteros- F(8,164)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta X$</td>
<td>1.243 (0.290)</td>
<td>36.933 (0)*</td>
<td>1.362 (0.249)</td>
<td>1.53 (0.150)</td>
</tr>
<tr>
<td>$\Delta Cnds$</td>
<td>2.852 (0.016)*</td>
<td>6.020 (0.050)</td>
<td>0.859 (0.489)</td>
<td>0.573 (0.798)</td>
</tr>
</tbody>
</table>

Vector tests

<table>
<thead>
<tr>
<th>Vector</th>
<th>Autoc- F(20, 324)</th>
<th>Normality- $\chi^2$ (4)</th>
<th>Heterosc1- F(42,470)</th>
<th>Heterosc2- F(42,463)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.901 (0.011)*</td>
<td>46.109 (0)*</td>
<td>1.344 (0.129)</td>
<td>1.022 (0.436)</td>
</tr>
</tbody>
</table>

Table 3. Mis-specification tests after the inclusion of the dummies: D50:1, D74:1, D80:2, D87:2

<table>
<thead>
<tr>
<th>Equation</th>
<th>Autoc- F(5,164)</th>
<th>Normality- $\chi^2$ (2)</th>
<th>ARCH- F(4, 161)</th>
<th>Heteros-F(8,160)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta X$</td>
<td>1.435 (0.247)</td>
<td>4.909 (0.086)</td>
<td>1.402 (0.235)</td>
<td>2.514 (0.013)*</td>
</tr>
<tr>
<td>$\Delta Cnds$</td>
<td>1.508 (0.190)</td>
<td>2.303 (0.316)</td>
<td>0.147 (0.963)</td>
<td>0.665 (0.721)</td>
</tr>
</tbody>
</table>

Vector tests

<table>
<thead>
<tr>
<th>Vector</th>
<th>Autoc- F(20, 316)</th>
<th>Normality- $\chi^2$ (4)</th>
<th>Heterosc1- F(24,458)</th>
<th>Heterosc2- F(42,451)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.288 (0.183)</td>
<td>8.803 (0.066)</td>
<td>1.344 (0.084)</td>
<td>1.022 (0.157)</td>
</tr>
</tbody>
</table>

Note: Autoc: (LM test, F-version, Godfrey (1988)), $H_0$: no autocorrelation; Normality (see Doornick and Hansen (1994)): $H_0$: normality of residuals; ARCH: (LM test, F-version, Engle (1982)) $H_0$: no conditional autoregressive heteroscedasticity; Heterosc: White (1980), Heterosc1: (LM test, F-version, Kelejian (1982), Heterosc2: (similar to a functional form test), for all $H_0$: no heteroscedasticity. * indicates when $H_0$ is rejected, (probability value). See Doornick and Hendry (1994) for a discussion of these tests and for references.
Table 4 presents the likelihood ratio statistics for the determination of the rank of the long run matrix, it reports the maximum eigenvalue statistics, $\lambda_{\text{max}}$, calculated as $-T \log(1 - \lambda_i)$ for the eigenvalue of the $r_{th}$ cointegrating vector, and the trace statistics, calculated as $-T \sum_{i=r+1}^{p} \log(1 - \lambda_i)$. The 95% quantiles for testing the order $r$ from Osterwald-Lenum (1992) are also reported. Both tests lead to accept one cointegrating vector at 95%. The cointegrating vector, normalised with respect to income, is $\beta' = (1, -1.369)$. The vector of the associated adjustment coefficients is $\alpha' = (-0.224, 0.0298)$, a low coefficient indicates slow adjustment towards the estimated equilibrium state and a high coefficient indicates rapid adjustment. Having found one long run relationship, we may test restrictions on the $\alpha$ vector by the likelihood ratio test in order to test if the restriction is valid. The likelihood test is

$$\text{LRtest} = -T \sum_{i=1}^{r} \log \left( \frac{(1 - \lambda^R_i)}{(1 - \lambda^\text{UN}_i)} \right),$$

where $\lambda^R, \lambda^\text{UN}$ represent the characteristic roots from the restricted and unrestricted model respectively. Under the null hypothesis that the restriction is valid the test is asymptotically distributed as a $\chi^2(s)$, where $s$ is the number of restriction imposed. The test (weak exogeneity test) that $\alpha' = (-0.224, 0)$ yields an $LR = 1.51$, which, compared with $\chi^2(1) = 3.84$, enables us to accept the restriction. The associated cointegrating vector becomes $\beta' = (1, -1.373)$. The long run relationship can be represented either as $x_t = 1.37cnds_t + \epsilon_t$ or as $cnds_t = 0.74x_t + \epsilon_t$. Following Campbell (1987), total consumption is $c_t = 1.37cnds_t$, and the implied share of nondurables and services consumption in the total is 73 percent.\footnote{Campbell (1987) finds that the share is 71 percent.}
Table 4 Cointegration tests: testing the rank of the long run matrix

$\left( \lambda_1 = 0.1412, \lambda_2 = 0.0113 \right)$

<table>
<thead>
<tr>
<th>$H_0$</th>
<th>$H_1$</th>
<th>( \lambda_{\text{max}} )</th>
<th>agg-$\lambda_{\text{max}}$</th>
<th>95%</th>
<th>Trace</th>
<th>agg-trace</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r = 0$</td>
<td>$r = 1, r \geq 1$</td>
<td>27.1</td>
<td>26.4</td>
<td>14.1</td>
<td>29.12</td>
<td>28.47</td>
<td>15.4</td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>$r = 2, r \geq 2$</td>
<td>2.02</td>
<td>1.98</td>
<td>3.8</td>
<td>2.02</td>
<td>1.98</td>
<td>3.8</td>
</tr>
</tbody>
</table>

*Note:* Critical values in Osterwald-Lenum (1992); Adjusted statistics taking into account the degree of freedom, see Reimers (1992);

The PIH, as in Hall's formulation, suggests that, since the consumption change is a pure innovation, all the coefficients in the consumption equation must be zero. The weak exogeneity test presented above and the results reported in Table 5 imply that the Hall's model is only partially supported by our data. Income equation adjusts to the long run, while the consumption equation does not adjust, but the coefficients in the consumption equation are not all zero. The consumption change equation exhibits "excess sensitivity", on the border of significance, with respect to the coefficient of $\Delta x_{t-1}$ and a significant positive correlation between $\Delta c_t$ and $\Delta c_{t-1}$. This means that the independence of consumption change from past information, predicted by the theory, is rejected by our data. As consumption is not a random walk, we test if it still is the trend in income, that is if the cycle in consumption is not significant for predicting the volatility of the series.
Table 5 Short run dynamics

<table>
<thead>
<tr>
<th>Variables</th>
<th>Coefficients</th>
<th>Standard Errors</th>
<th>t-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta x_{t-1}$</td>
<td>0.12959*</td>
<td>0.0677</td>
<td>1.914</td>
</tr>
<tr>
<td>$\Delta c_{t-1}$</td>
<td>0.47943*</td>
<td>0.1175</td>
<td>4.080</td>
</tr>
<tr>
<td>ECM$_{t-1}$</td>
<td>-0.22571*</td>
<td>0.0506</td>
<td>-4.460</td>
</tr>
<tr>
<td>D50:1</td>
<td>0.08806*</td>
<td>0.0183</td>
<td>4.810</td>
</tr>
<tr>
<td>D74:1</td>
<td>-0.04493*</td>
<td>0.0190</td>
<td>-2.359</td>
</tr>
<tr>
<td>D80:2</td>
<td>-0.06833*</td>
<td>0.0506</td>
<td>-3.734</td>
</tr>
<tr>
<td>D87:2</td>
<td>0.08139*</td>
<td>0.0183</td>
<td>-4.443</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.02873*</td>
<td>0.0078</td>
<td>-3.683</td>
</tr>
</tbody>
</table>

Equation $\Delta c_t$

<table>
<thead>
<tr>
<th>Variables</th>
<th>Coefficients</th>
<th>Standard Errors</th>
<th>t-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta x_{t-1}$</td>
<td>0.07387</td>
<td>0.0438</td>
<td>1.683</td>
</tr>
<tr>
<td>$\Delta c_{t-1}$</td>
<td>0.23237*</td>
<td>0.0761</td>
<td>3.050</td>
</tr>
<tr>
<td>ECM$_{t-1}$</td>
<td>0.03364</td>
<td>0.0328</td>
<td>1.025</td>
</tr>
<tr>
<td>D50:1</td>
<td>0.00967</td>
<td>0.0118</td>
<td>0.816</td>
</tr>
<tr>
<td>D74:1</td>
<td>-0.03278*</td>
<td>0.0123</td>
<td>-2.655</td>
</tr>
<tr>
<td>D80:2</td>
<td>-0.04064*</td>
<td>0.0118</td>
<td>-3.426</td>
</tr>
<tr>
<td>D87:2</td>
<td>0.00755</td>
<td>0.0118</td>
<td>0.636</td>
</tr>
<tr>
<td>Constant</td>
<td>0.01303*</td>
<td>0.0050</td>
<td>2.557</td>
</tr>
</tbody>
</table>

* significance at 5% level

3.1 Variance decomposition

Having obtained the VECM estimate, we can decompose the variables in trend-cycle in a multivariate context following the identification procedure presented in section 2. The results, reported in Table 6, are: 1) the variance of the trend in consumption explains 93 percent of total volatility, while the variance of the cycle only explains remaining 7 percent; 2) the variance of the trend in income explains 36 percent of the total volatility, while the cycle remaining 64 percent; 3) the first order autocorrelation

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coefficient in the trend in income is 0.51, while in consumption is 0.34, this means that the trends are not random walks; 4) given that the relative importance of the cycle in consumption is low, the consumption is "nearly" the trend in income, but not the Beveridge and Nelson trend as predicted by Hall's model and as found by Cochrane (1994). Figure A shows the total income series and the estimated stochastic trend.

**Table 6 Variance decomposition and some persistence measures**

<table>
<thead>
<tr>
<th>Series</th>
<th>$\sigma^2_{\Delta \text{trend}}$</th>
<th>$\sigma^2_{\Delta \text{cycle}}$</th>
<th>$\sigma^2_{\Delta \text{trend}}/\sigma^2_{\Delta \text{series}}$</th>
<th>$\sigma^2_{\Delta \text{cycle}}/\sigma^2_{\Delta \text{series}}$</th>
<th>$V_i^{\text{PH}} = \frac{\sigma^2_{\Delta c}}{\sigma^2_{\Delta x}}$</th>
<th>$V_i = \frac{b(1)^r\sigma^2_{\Delta c}}{\sigma^2_{\Delta x}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta x$</td>
<td>0.1880</td>
<td>0.3321</td>
<td>0.36</td>
<td>0.64</td>
<td>0.32</td>
<td>0.60</td>
</tr>
<tr>
<td>$\Delta c$</td>
<td>0.1546</td>
<td>0.0122</td>
<td>0.93</td>
<td>0.07</td>
<td>------</td>
<td>2.08</td>
</tr>
</tbody>
</table>

Note: $\sigma^2_{\Delta \text{trend}}, \sigma^2_{\Delta \text{cycle}}$: trend and cycle variances;

$\frac{\sigma^2_{\Delta \text{trend}}}{\sigma^2_{\Delta \text{series}}}, \frac{\sigma^2_{\Delta \text{cycle}}}{\sigma^2_{\Delta \text{series}}}$: respectively the contributions of the trend and cycle variances to the volatility of the series;

$V_i^{\text{PH}} = \frac{\sigma^2_{\Delta c}}{\sigma^2_{\Delta x}}$: persistence measure (Cochrane) of total income implied by the PIH;

$V_i = \frac{b(1)^r\sigma^2_{\Delta c}}{\sigma^2_{\Delta x}}$: univariate persistence measures (Cochrane) implied by the multivariate estimate.
Figure A - - - - - x, _______trend

Figure B shows the impulse response functions of consumption and income to a one standard deviation shock. More precisely, these responses describe the dynamic effects of permanent and transitory shocks. The vertical axis denotes the consumption and income responses, while the horizontal axis denotes time in quarters. Income has a hump-shaped response to the permanent shock, while the same shock has a smaller impact on consumption. The transitory shock has no effect on consumption at any horizon, while the effect of this shock on income is large and decays after 15-18 quarters.
3.2 Some remarks about the persistence measures

Given the results, regarding the relative importance of trend and cycle in the series, obtained in section 3.1, some remarks about the utility of the univariate measures of persistence are necessary. Under the hypothesis of random walk of the trend, the higher the persistence, the further the series from the traditional representation in which the shocks have only transitory effects (trend stationary representation) - in other words the bigger is the size of the permanent component.

Table 7 reports non parametric estimates of Cochrane's measure of persistence, while Table 8 reports the parametric estimates of the Beveridge and Nelson persistence, when first difference is modeled as various low-order ARMA processes. The non-
parametric persistence of total income ranges from 0.91 to 1.08, while the parametric one ranges from 1.059 to 1.210. These estimates seem to indicate that trend in income is very important in accounting for the total volatility. However, $x$ shows significant persistence in its innovation, but nevertheless its fluctuations are dominated by transitory disturbances (as indicated by the VAR estimate). Hence, since the trend in income is not a random walk, but a positive autocorrelated process, the information coming from a high persistence is not useful in establishing the importance of the permanent component in the series.

Table 6 shows the univariate persistence implied by the PIH, $V_1^{PH}$, and the univariate persistence implied by the multivariate model $V_2$. The $V_1^{PH}$ on the hypothesis of cointegration between total income and consumption and that consumption is the Beveridge e Nelson trend in income $^7$ is

\[ V_1^{PH} = \frac{\text{var} \Delta c}{\text{var} \Delta x}, \]

this expression tells us that either consumption is smooth or total income is persistent (Tables 7 and 8), we cannot have both together. The excess of smoothness implied by the result: $V_1^{PH} = 0.32$ and generally found in the data has been used as an argument against the PIH (see note 2). If the consumption was a random walk $V_1^{PH}$ and $V_2$ should be the same. This is not verified because $V_2 = 0.60$. This is further evidence that the aggregate consumption is not a random walk. This fact can explain the excess smoothness result$^8$.

$^7$See Forni (1994).
$^8$For example, Forni and Lippi (1994) have shown that following for heterogeneity and incomplete information the PIH can be partially reconciled with evidence. This model produces non-randomwalkness of aggregate consumption, excess smoothness, excess sensitivity, cointegration between consumption and total income and the implied error correction model is general, as (1) in section 2, i.e. lagged consumption and lagged income appear in the consumption equation as well as in the income equation.
Table 7 Non-parametric estimates of persistence of total income: $\Delta X$

<table>
<thead>
<tr>
<th>Window Size</th>
<th>Bartlett</th>
<th>Tukey</th>
<th>Parzen</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.98</td>
<td>1.005</td>
<td>1.088</td>
</tr>
<tr>
<td></td>
<td>(0.26)</td>
<td>(0.29)</td>
<td>(0.26)</td>
</tr>
<tr>
<td>20</td>
<td>0.93</td>
<td>0.93</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>(0.35)</td>
<td>(0.38)</td>
<td>(0.33)</td>
</tr>
<tr>
<td>30</td>
<td>0.96</td>
<td>0.91</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>(0.45)</td>
<td>(0.46)</td>
<td>(0.39)</td>
</tr>
<tr>
<td>40</td>
<td>0.97</td>
<td>0.94</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>(0.53)</td>
<td>(0.54)</td>
<td>(0.45)</td>
</tr>
<tr>
<td>50</td>
<td>0.94</td>
<td>0.94</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>(0.57)</td>
<td>(0.60)</td>
<td>(0.51)</td>
</tr>
<tr>
<td>60</td>
<td>0.93</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>(0.62)</td>
<td>(0.66)</td>
<td>(0.56)</td>
</tr>
</tbody>
</table>

*Note:* Cochrane (1988) persistence estimates: ratio of the variance of trend changes to the variance of income changes or, equivalently, ratio of the spectral density of total income evaluated at zero frequency to the variance of income changes, $\nu = \frac{\text{b}(1)^2 \sigma_x^2}{\sigma_{\Delta x}^2} = \frac{S(0)}{\sigma_{\Delta x}^2}$. Standard errors in brackets.
Table 8. Beveridge and Nelson persistence of $\Delta X$

<table>
<thead>
<tr>
<th>ARMA(p, q)</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
<th>$\phi_3$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>b(1)</th>
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<tr>
<td>(0,1)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.110</td>
<td>-</td>
<td>-</td>
<td>1.111 [0.082]</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>(1.347)</td>
<td></td>
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</tr>
<tr>
<td>(0,2)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.119</td>
<td>-0.802</td>
<td>-</td>
<td>1.039 [0.106]</td>
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<tr>
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<td>(1.563)</td>
<td>(-1.00)</td>
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<tr>
<td>(0,3)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.127</td>
<td>-0.044</td>
<td>0.127</td>
<td>1.210 [0.166]</td>
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<tr>
<td></td>
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<td></td>
<td>(1.697)</td>
<td>(-0.538)</td>
<td>(1.453)</td>
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<tr>
<td>(1,0)</td>
<td>0.093</td>
<td>-</td>
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<td>-</td>
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<td>1.103 [0.090]</td>
</tr>
<tr>
<td></td>
<td>(1.259)</td>
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<td></td>
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<tr>
<td>(1,1)</td>
<td>-0.615</td>
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<td>-</td>
<td>0.769</td>
<td>-</td>
<td>-</td>
<td>1.095 [0.041]</td>
</tr>
<tr>
<td></td>
<td>(-3.487)</td>
<td></td>
<td></td>
<td>(4.898)</td>
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<tr>
<td>(1,2)</td>
<td>-0.577</td>
<td>-</td>
<td>-</td>
<td>0.717</td>
<td>-0.028</td>
<td>-</td>
<td>1.070 [0.087]</td>
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<tr>
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<td>(-2.601)</td>
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<td></td>
<td>(3.080)</td>
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<td>(1,3)</td>
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<td>-</td>
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<td>-0.014</td>
<td>0.054</td>
<td>1.117 [0.126]</td>
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<tr>
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<td></td>
<td>(2.046)</td>
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<td>(0.528)</td>
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<td>-</td>
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<td>(3.658)</td>
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<tr>
<td>(2,2)</td>
<td>-0.172</td>
<td>0.321</td>
<td>-</td>
<td>0.519</td>
<td>-0.394</td>
<td>-</td>
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<td>(-0.287)</td>
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<td>(0.528)</td>
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<tr>
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<td>0.224</td>
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<td>0.044</td>
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<td>(-0.507)</td>
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<td>1.158 [0.165]</td>
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<tr>
<td></td>
<td>(1.624)</td>
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<td>(1.394)</td>
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<tr>
<td>(3,1)</td>
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<td>0.089</td>
<td>0.345</td>
<td>-</td>
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<td></td>
<td>(-0.648)</td>
<td>(-0.653)</td>
<td>(1.127)</td>
<td>(1.060)</td>
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<tr>
<td>(3,2)</td>
<td>-0.081</td>
<td>-0.529</td>
<td>0.171</td>
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<td>0.495</td>
<td>-</td>
<td>1.177 [0.131]</td>
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<td>(-2.054)</td>
<td>(2.219)</td>
<td>(0.865)</td>
<td>(1.768)</td>
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<td>(3,3)</td>
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<td>-0.492</td>
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<td>0.177</td>
<td>0.725</td>
<td>1.166 [0.065]</td>
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<tr>
<td></td>
<td>(1.894)</td>
<td>(-1.555)</td>
<td>(-3.235)</td>
<td>(-1.438)</td>
<td>(1.153)</td>
<td>(5.124)</td>
<td></td>
</tr>
</tbody>
</table>

Note: Beveridge and Nelson decomposition of total income: $\Delta x_t = b(1)e_t + (1-L) \left[ \frac{b(L)-b(1)}{(1-L)} \right]$, where $b(1)e_t$ represents the trend change and $b(1)$ the persistence of total income. (t-ratio), [ standard errors ].

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4. Conclusions

In this work the VECM in consumption of nondurables and services and total disposable income is utilised to obtain the trend-cycle decomposition and to derive the dynamic effects of transitory and permanent shocks on the series.

The main results are: the variance of the first permanent component explains 93 percent of all the variance of the series, while the permanent component in income explains only 34 percent of total volatility. Given that relative importance of cycle in consumption is low, the consumption is "nearly" the trend in income, but not the Beveridge and Nelson trend as predicted by the PIH, as in Hall's formulation (Hall 1978) and as found in the related empirical work by Cochrane (1994). In other words, our estimates show that consumption is not a random walk and exhibits "excess sensitivity" and "excess smoothness". Income has a hump-shaped response to permanent shock, while the same shock has a smaller impact on consumption. The transitory shock has no effect on consumption at all horizon, while the effect on income is large and decays only after 15-18 quarters.

The multivariate procedure isolates a large cyclical component in total income, whereas the univariate representation shows little transitory component - in other words income shows high persistence in its innovations, but its fluctuations are nevertheless dominated by the cyclical component. This difference is in line with two well-known results: i) the univariate persistence measure is not informative about the size of the permanent component when the random walk hypothesis for the trend is removed or in other words the random walk specification is biased towards establishing the trend as important, ii) the relative importance of the cyclical component seem to depend on the information set and is necessarily higher when multivariate estimate is performed.
Acknowledgements

I would like to thank Mario Forni, Marco Lippi and Marcello D'Amato for helpful discussions and comments. All errors are mine alone.
References


Appendix

This Appendix presents a discussion about the identification. More precisely, it contains the sequence of stages for achieving the trend-cycle representation (structural model), from the VECM representation (reduced form). If the variables are I(1) and the first difference is a stationary ARMA, so that we have the VECM representation (1) of section 2

\[
\Gamma(L)\Delta Z_t = -\alpha\beta'Z_{t-1} + \varepsilon_t, \tag{A.1}
\]

where \(Z_t\) is a (2x1) vector of the two variables, \(\Gamma(L)\) is a (2x2) polynomial matrix with the normalisation \(\Gamma(0) = I\), \(\alpha\) e \(\beta'\) are respectively the (2x1) vector of the adjustment coefficients to the long run equilibrium and the (1x2) cointegrating vector that characterise this equilibrium, \(\varepsilon_t\) is the (2x1) vector of the disturbances (or shocks) such that \(E(\varepsilon_t) = 0\), and \(E(\varepsilon_t, \varepsilon_t') = \Sigma_{\varepsilon}\). \(p\) is the truncation order of the VECM. We ignore, for simplicity, the constant and the dummies variables. If model (A.1) exists (i.e cointegration exists), the following VAR(p) in the level of the variables also exists

\[
A(L)Z_t = \varepsilon_t, \tag{A.2}
\]

where

\[
A(L) = \Gamma(L)(1-L) + \alpha\beta' L = I - A_1 L - ... - A_p L^p
\]

and

\[
A(1) = I - A_1 - ... - A_p = \alpha\beta' \text{ with } A(0) = I.
\]

Multiply the LHS and the RHS of the (A.2) by the adjoint of \(A\), \(A^*(L)\), to obtain

\[
A^*(L)A(L)Z_t = A^*(L)\varepsilon_t, \tag{A.3}
\]
where \(A^*(L)A(L)\) is the determinant of \(A(L)\). Define \(d(L) = \frac{\det A(L)}{(1-L)}\) to exclude the presence of the unit root from the determinant of \(A(L)\) and rewrite the (A.3) as follows

\[
d(L)(1-L)Z_t = A^*(L)\varepsilon_t, \tag{A.4}
\]

\(d(L)\) can be inverted, because now it does not contain a unit root. The invertibility of \(d(L)\) enables the (A.4) to be rewritten as the Wold representation:

\[
\Delta Z_t = C(L)\varepsilon_t, \tag{A.5}
\]

where \(C(L) = \frac{A^*(L)}{d(L)}\). The variance-covariance matrix \(\Sigma_e\) is a non-singular, symmetric and definite positive matrix; hence we can write it as \(\Sigma_e = PA\Lambda P'\), where \(P\) is the orthonormal matrix of eigenvectors, such that \(PP'=I\), and \(\Lambda\) is the diagonal matrix of the eigenvalues. We can rewrite the variance-covariance matrix as \(\Sigma_e = P\sqrt{\Lambda}\sqrt{\Lambda}P' = RR'\), where \(R = P\sqrt{\Lambda}\). The (A.5) is now

\[
\Delta Z_t = C(L)RR^{-1}\varepsilon_t,
\]
or

\[
\Delta Z_t = D(L)\eta_t, \tag{A.6}
\]

where \(D(L) = C(L)R\) and \(\eta_t = R^{-1}\varepsilon_t\). The disturbances are now orthonormal, that is \(\Sigma_{\eta} = I\). The reason for this is contained in the following passages

\[
E(\eta_t, \eta_t') = \Sigma_{\eta} = E(R^{-1}\varepsilon_t\varepsilon_t' R^{-1}) = R^{-1}\Sigma_e R^{-1} = R^{-1}RR'R^{-1} = I
\]
If we consider only that $\Sigma_\eta = I$, the (A.6) is not a unique representation. It is possible to obtain another representation via the Cholesky orthogonalisation (Sims 1980), moreover it is always possible multiply the (A.6) by an orthonormal matrix $O$ to obtain an equivalent representation:

$$\Delta Z_t = D(L)OO^{-1}\eta_t,$$  \hspace{1cm} (A.7)

where $O$ is the orthonormal matrix. More generally

$$\Delta Z_t = E(L)u_t,$$  \hspace{1cm} (A.8)

where $E(L) = D(L)O$, $u_t = O^{-1}\eta_t$ and $E(0) \neq 1$. Using $O$, it is possible to preserve the orthogonalisation of the variance-covariance matrix, $\Sigma_\eta$, so that $\Sigma_u = I$. We must choose one of the $\infty^1$ orthonormal matrix to achieve the full identification. This is possible by imposing a restriction on $E(L)^9$. The cointegration suggests this restriction, given that it implies the existence of one permanent shock. Assume that $u_{1t}$ is the transitory shock, so that long run response of $Z_{1t}$ to this shock is $E_{11}(1) = 0$. This condition also implies that $E_{21}(1) = 0$, given that $E(1)$ is singular owing to cointegration. We have fully identified the bivariate model and we obtained the trend-cycle representation (2) in section 2

$$
\begin{pmatrix}
\Delta Z_{1t} \\
\Delta Z_{2t}
\end{pmatrix}
= 
\begin{pmatrix}
E_{11}(L)u_{1t} + E_{12}(L)u_{2t} \\
E_{21}(L)u_{1t} + E_{22}(L)u_{2t}
\end{pmatrix}
$$  \hspace{1cm} (A.9)

---

9Given that $E(L) = D(L)O$, imposing $E_{11}(1) = 0$, we implicitly choose one orthonormal matrix through $o_{1t} = \frac{D_{12}(1)}{D_{11}(1)}o_{2t}$, where $o_{1t}$ is one element of $O$. 

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