Aggregation Factors and Aggregation Bias in Consumer Demand.

by

Massimo Baldini

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INTRODUCTION

The aim of this paper\(^1\) is to examine, using a time series of individual cross sections, the problem of aggregation over households in the context of the empirical study of consumer demand for goods.

More than 40 years by now have passed since Gorman showed that working only with aggregate data permits recoverability of micro parameters only under very stringent behavioural assumptions, but the attention devoted to the issue of aggregation by past and contemporaneous researchers has often been inadequate; many studies explicitly rely on the hypothesis of the "representative agent", which simply amounts to completely ignoring any form of heterogeneity, and is now increasingly criticized (Kirman (92)), while on the other hand the growing production of micro data is encouraging empirical studies which do not need any restrictive assumption about aggregation.

While, as will be shown, it is easy to verify that the lack of consideration for differences in individual preferences leads to biased parameter estimates and possibly to the rejection of the implications of the theory of choice, at the end the importance of this problem can be evaluated only by examining what happens to the empirical results with and without a proper consideration of the sources of aggregation bias; Hicks (55), for example, suggested that even if micro differences are pervasive, in the aggregation process they tend to neutralize each other, so that an empirical analysis which uses only macro data may not be too misleading.

A distinctive advantage of the availability of a time series of micro data consists in the possibility of comparing the results obtained by estimating a demand function first under the hypothesis of the representative agent, and then in the case of a consistent aggregation over consumers of the individual demand relationships. In the first case the aggregate series are obtained by summing the same micro data and are considered to be the outcome of the decisions of a single agent (a big Robinson Crusoe), while in the second case each household contributes with its own demographic attributes to the definition of the relevant regressors.

The failure to take account of the aggregation problem can have effects not only on the biasedness of the estimates, but also on a more general bad definition of the model, so that the traditional misspecification tests could detect the presence of others specification problems (serial correlation, heteroscedasticity, etc.) even when these problems are actually absent, if the tests turn out to be affected by the aggregation procedure followed.

I shall divide the exposition into three main sections: section 1 is a survey of aggregation theory, section 2 contains an empirical analysis of the presence and relevance of aggregation

\(^1\) I would like to express my gratitude to my supervisor during the period spent in Ucl, Prof. Richard Blundell, for his guidance, and to Dr. James Banks of IFS for providing the data.
bias, and section 3 tests whether, as suggested by some researchers, the omission of consistent aggregation can lead to problems of (spurious) dynamic misspecification.

1) A BRIEF SURVEY OF AGGREGATION THEORY

1.1) Theory

Economic theories are often specified at the individual or micro level, trying to describe the behaviour of a single agent (a firm, a consumer, a worker, etc.), but empirical tests of these theories are generally carried out on aggregate data, usually the only available.

The essence of the aggregation problem consists in verifying under what conditions it is legitimate to use aggregate data for estimating the parameters of the theoretical model, so as to obtain a consistent description of individual behaviour.

In many studies this problem is completely neglected, under the implicit assumption that the economy is composed of a multitude of identical agents, so that macro data can be assumed to be the outcome of the decision of a single, representative agent: this is for example the approach followed in a great number of studies on the Euler equation in consumption, starting from the work by Hall (78) 2. Deaton and Muellbauer (80b) call this the exact aggregation approach; the conditions under which this approach turns out to be valid are, however, very restrictive.

Gorman (53), in the context of consumer theory, showed that aggregate (average) demand for a commodity depends only on prices and average income only if the marginal propensities to consume are the same for all agents, irrespectively of personal income or other socio-demographic characteristics: this is the unique case where a mean-preserving redistribution of purchasing power among agents with different level of income would leave aggregate consumption unchanged. This in turn implies that individual Engel curves must be linear with a common slope, corresponding to quasi-homothetic preferences.

Formally, if $q_i^h$ is expenditure on good $i$ by family $h$, $m^h$ is its income, $z^h$ is a vector of demographic variables (number of children, age of head, race, type of residence, etc.) and $p$ is the price vector, then the demand function at the individual level is $q_i^h = g_i^h(m^h, p, z^h)$; if we want that the aggregate model may be expressed as $\bar{q}_i = g_i(p, \bar{m})$, where the bars indicate population averages, then individual functions must take the form $q_i^h = a_i(p, z^h) + b_i(p)m^h$, where only the coefficient $a_i$ is allowed to vary across different households (so a certain degree of demographic heterogeneity is allowed even in this simple setting), but $b_i(p)$ is the same for all agents3.

2In what follows, however, I will deal only with static models of consumer demand, thus all aggregation problems and possible solutions suggested for intertemporal models (finite lives, differences in preferences across generations, cohort analysis, etc.) do not appear in this exposition.

3$a_i(p, z^h)$ and $b_i(p)$ are linearly homogeneous functions of prices.
The basic problem of the theory of exact linear aggregation is the virtually universal rejection of linear and parallel Engel curves in applied studies (Working(43), Leser(63)), because agents with different income level or demographic position will typically have different marginal propensities to spend, so that a more complex relationship between micro and macro functions is required.

A first attempt towards generality consists in keeping a linear form for the individual and aggregate demand functions, while allowing for non linear Engel curves. The Pigl class of demand systems proposed by Muellbauer (75), to which the Almost Ideal model belongs, replaces the average expenditure with a representative expenditure level, computed by taking into account the distribution of both income levels and demographic attributes. If "exact non linear aggregation" holds, the average budget share can be assumed to derive from the behaviour of a single representative agent, whose outlay is no longer the average expenditure, but a certain point in the range of individual outlays.

When this expenditure level does not depend on prices, we have the case of price independent generalized linearity, with the representative cost function given by
\[ c(\alpha, p) = \left[ a(p)^\alpha (1-u_0) + b(p)^\alpha u_0 \right]^{1/\alpha} \]

If \( \alpha \) tends to zero, this cost function takes a logarithmic form, and the Almost Ideal model is obtained. The restrictive conditions required for the consistent estimation of the A.I. model on aggregate data only are shortly indicated in the last part of this section.

Stoker (93) emphasizes three advantages of linear aggregation models:
1) It is immediate to obtain the aggregate equation starting from the individual relationship, provided of course that we know the characteristics of each agent.
2) Linearity is only "intrinsic", in the sense that each function of the individual attributes can be non linear, even if they are connected additively, so a sufficient degree of generality is guaranteed.
3) If only the aggregate model is available (because we know only the joint distribution of income and attributes across the population, not the characteristics possessed by each single agent) it is nevertheless possible to fully recover the individual level model.

When the individual-level model is non linear, or when interactions between demographic variables and real expenditure are allowed, exact aggregation theory doesn't apply, and the basic problem becomes that of verifying when we can recover the micro-parameters from the aggregate relationship. Recoverability is not guaranteed, it depends on the specific form of non linearity, and on the form of the density function of the joint distribution of income and demographic variables (cfr. Stoker(84)).

Even if the individual model is linear, we cannot recover all the individual parameters from the aggregate model if aggregate-level data show insufficient variation of demographic variables. The general solution consists in the integration of macro with individual data in estimation. If
estimation is carried out only at the aggregate level, misleading conclusions (biased parameter values) are likely to be obtained.

A brief synthesis of the more relevant problems incurred when dealing with the topic of aggregation can be provided using a simple model:

if the theoretical model estimated on aggregate data is \( \bar{q} = b_1 \bar{x} + e \), omitting the constant (and the temporal index) for simplicity\(^4\), and if the micro model is \( q_h = \beta_1 x_h \), then \( E(\hat{b}_1) = \beta_1 \), but if the true individual relationship is \( q_h = \beta_1 x_h + \beta_2 (x_h)^2 \), then the correct aggregate model is given by \( \bar{q} = b_1 \bar{x} + b_2 \bar{x}^2 + u \), so that if we keep estimating the previous macro expression, we have that

\[
\hat{b}_1 = (\bar{x}^T x)^{-1} x^T \bar{q} = (\bar{x}^T x)^{-1} x^T (\beta_1 \bar{x} + \beta_2 \bar{x}^2 + u)
\]

so that \( E(\hat{b}_1) = \beta_1 + (\bar{x}^T x)^{-1} x^T x \beta_2 = \beta_1 + \beta_2 \frac{S_{xx}^2}{S_{xx}} \)

where the term multiplying \( \beta_2 \) is the slope in the regression of \( \bar{x}^2 \) on \( \bar{x} \) over time; only in the case of \( \beta_2 = 0 \), i.e. of linearity of the micro formula, is \( \beta_1 \) estimated unbiased\(^5\).

If the aggregate regression tries to take into account the non-linearity of the behavioural relationship, the best way in which it could be expressed is simply \( \bar{q} = b_1 \bar{x} + b_2 (\bar{x})^2 + e \), but the correctly aggregated model would be \( q = b_1 x + b_2 x^2 + e \); since the square of the mean is different from the mean of the squared income terms (the difference is the variance of the income distribution), \( \hat{b}_2 \) is biased. Its bias depends on the variance of the distribution of income, and this shows that non-linearities in the micro relations produce distributional biases in the aggregate equation. The same problem arises if the micro equation presents logarithmic terms in income.

Finally, if the individual expression relates \( q_h \) not only to income but also to demographic characteristics, this could have two different effects on the aggregation problem:

a) No interactions between income and demographics, so that for example \( q_h = \beta_1 x_h + \beta_2 x_h^2 + \gamma D_h \), where \( D_h \) is a dummy variable equal to 1 if, for example, the head of the household is a white collar worker, or if children are present; in this case the parameter \( \gamma \) is fully recoverable from the aggregate (averaged) equation provided that data on the proportions of households in different demographic groups are available, which is a fairly easily satisfied assumption.

\(^4\)The bars indicate averages over households.

\(^5\)\( \hat{b}_1 \) is unbiased also when in the sample \( S_{xx}^2/S_{xx} \) turns out to be zero, but \( \beta_1 \) is not the household's marginal propensity to consume if \( \beta_2 \) is different from zero, so the knowledge of \( \beta_1 \) alone would be useless.
b) In the more realistic case where the presence of a certain demographic characteristic has, on the demand for a good, an influence which depends also on the level of income, aggregate data alone are not able to guarantee recoverability, because measures of the joint distribution of demographic variables and income are necessary, and these measures can be obtained only from individual-level data.

In the empirical model I will consider in the two other sections, both quadratic terms in income and interactions between income and demographic variables are present, so that a simple (linear) treatment of aggregation is prevented.

1.2) Empirical tests of aggregation bias

Before considering the ways followed in current research to take account of the aggregation problem, it is interesting to consider some studies aimed at identifying the importance of distributional considerations in aggregate data. The general scheme of these works consists in the introduction of distributional measures in a standard macro equation, in order to test whether they have a significant effect.

Blinder (75) didn't find any significant effect of income distribution on an aggregate consumption function, but other researchers have attributed this result to an insufficient degree of variability over time of the distributional variable considered.

Stoker (86) considers whether the inclusion of variables indicating the proportion of households in various income ranges in the Linear Expenditure System of consumer demand implies any difference in the estimated parameters. The author estimates a system of 4 groups of goods with and without the distributional statistics; in both cases a version with an AR(1) error term is estimated as well. If we consider the standard system, the introduction of an AR(1) error implies big changes in the parameter values, and the autoregressive coefficient is strongly significant; this can be interpreted as a sign of dynamic misspecification.

On the other hand, the system with distributional factors doesn't show significant differences in estimated coefficients between the static and the dynamic version, and the autoregressive coefficient is not significantly different from zero. Moreover, estimates of the system with proportions are very similar to those of the dynamic LES without proportions. A likelihood ratio test leads to a strong rejection of the basic LES in favour of both the dynamic model without proportions and the static model with proportions. So we have two statistically equivalent models, and aggregate data alone cannot distinguish between them; if the issue of individual heterogeneity is neglected by appealing to the idea of representative agent, it is easy to conclude that the "correct" model is a dynamic one, but this paper shows that a completely different explanation for the same data is possible. Another important finding of this study is that the values of the parameter estimates are very different in the two models with and without
proportions, so inconsistent estimates are obtained if no attention is paid to the aggregation problem.

Even if the linear expenditure system is very restrictive, Buse (92) has shown that the same methodology applied on a different demand system, the Quadratic Expenditure System, and on a different data set leads to the same conclusion.

Another aspect of the study by Stoker which is worth mentioning is the not precise estimate of the proportion parameters, due to the little variation over time of distributional measures of demographic characteristics; he concludes that for this reason aggregate data, even integrated by proportions, are unlikely to measure precisely individual demand patterns, and that "more full micro-macro modelling" is necessary if we want to separately recover distributional and behavioural effects.

Blundell, Pashardes and Weber (93), using a demand system with non linear terms in income and interactions between demographic elements and income, show that estimation on aggregate data allows to recover individual parameters only if certain "aggregation factors" are constant over time. To give a simplified example, and anticipating a bit of notation presented more extensively in the next section, we may represent the budget share of good i for household h only as a function of log income, i.e. \( w_{ih}^h = \beta_i \log x_i^h \); then the aggregate budget share is \( w_i = \sum_h \mu_i^h w_{ih}^h = \beta_i \sum_h \mu_i^h \log x_i^h \), an expression which is not computable with aggregate data only. Still, we can write \( w_i = \beta_i \left( \frac{\sum_h \mu_i^h \log x_i^h}{\log x_i} \right) \log x_i = \beta_i \pi_{oi} \log x_i \), where \( \pi_{oi} \) is average real expenditure; if the aggregation factor \( \pi_{oi} \) is constant over time (and close to one), then \( \beta_i \) can be recovered using only macro data, otherwise there will be signs of parameter instability over the period.

The authors show that the aggregation factors are not constant in their sample (FES data from 1970 to 1984), and then compare the estimates obtained from a micro-level model with the estimates from an aggregate equation integrated with some distributional variables (the linear and quadratic entropy terms presented later) and trend and seasonal terms; as can be expected from the non constancy of the aggregation factors, the aggregate model leads to biased results, but interestingly its forecasting ability is not lower than that of the micro reference model, perhaps as a consequence of the low variability of the aggregation factors over the post-sample period.

\[ \mu_i^h = \frac{m_i^h}{\sum_h m_i^h}, \quad m_i^h \text{ is nominal expenditure and } x_i^h \text{ is real expenditure.} \]
1.3) Three empirical approaches to aggregation

There are three general ways to build econometric models which allow recoverability of behavioural parameters: the estimation on aggregate data alone, the joint use of macro and micro data sources, and the use of a panel (or, more commonly, a time series of cross section data) of individual observations.

The first of the three methods succinctly indicated is of course based on the strong and unrealistic hypothesis of the representative agent, if only averaged aggregate terms are considered in the estimated macro model. As already said, recoverability in this context is possible only if all Engel curves are linear and parallel. The concept of generalized linearity introduced by Muellbauer (75) allows a more general form for the Engel curve, provided that aggregate budget shares depend also on the entropy index $\sum_h x^h_i \log \frac{x^h_i}{\sum_h x^h_i} , $ obtained by noting that the aggregate budget share is a weighted average of the individual budget shares, with weights given by $\frac{x^h_i}{\sum_h x^h_i} .$

The "representative expenditure level" is given by $\frac{H_t}{Z_t} \bar{x}_t , $ where $-\log Z_t$ is equal to our entropy measure and $H_t$ is the number of households in period $t$.

Since the time series of the entropy term is difficult to obtain, Deaton and Muellbauer(80a) used the logarithm of the average income in place of the representative expenditure; this replacement is legitimate if the index of relative entropy $\sum_h x^h_i \log \frac{x^h_i}{\bar{x}_t} $ is constant over time\(^7\).

This model is no longer based on the representative agent approach, but it allows recoverability only if three very unrealistic assumptions are met: if relative entropy is constant (i.e., changes in income involve equiproportionally all households) or uncorrelated with $\bar{x}$ and $p$, if the only difference across households is in the level of income, and if $H_t = Z_t , $ i.e. if the expenditure shares are identical, because otherwise $\bar{x}$ is lower than the representative expenditure\(^8\).

A more general approach to heterogeneity among individuals uses both macro and micro data, the first to measure the impact of prices, the second to infer the importance of demographic and

\(^7\)Because $\log H_t - \log Z_t = \log H_t + \sum_h x^h_i \log \frac{x^h_i}{\sum_h x^h_i} = \sum_h x^h_i \log \frac{x^h_i}{x_t} , $\n\(^8\)Z reaches the maximum value of H in the case of perfect equality in income distribution.
income effects. In this framework agents are allowed to be different not only in income but also in socio-demographic attributes.

One of the first demand systems reflecting this approach is the translog model of Jorgenson, Lau and Stoker (80,82), where aggregate expenditure shares depend on prices, on the distribution of total expenditure over agents (the entropy index), and on the joint distribution of income and demographic variables \( \frac{\sum h x^h D^h}{\sum h x^h} \). They use a single cross section of individual data to estimate the parameters of distributive variables, and a time series of annual aggregate data to estimate the intercept and the price coefficients. The distributional terms imply, in the share-income space, a translation of the fitting curve.

Even if this model could be estimated with aggregate data only, the limited variability over time of the proportion terms suggests that the parameters associated to them are very unlikely to be precisely estimated on macro-data, so that individual-level information is required. Moreover, the pooling of micro and micro sources makes this approach particularly appealing for all the countries or fields of research for which sufficiently detailed and long time series of cross sections are still lacking.

A third possibility consists in the exclusive use of a panel of individual observations, allowing the researcher to estimate all parameters directly at the micro level; in this case, of course, no aggregation bias problem can arise. The already cited study by Blundell, Pashardes and Weber (93) (BPW henceforth) applies the Almost ideal model (with quadratic terms in the logarithm of income) to a large sample of households, and constitutes the starting point for my empirical analysis.
2) INDIVIDUAL BEHAVIOUR AND AGGREGATION BIAS

2.1) The two models

In this section I will follow closely BPW in outlining a model representing individual preferences, as a framework for the analysis of the aggregation problem.

We can denote with $q_{it}^h$ the expenditure on good $i$ in period $t$ by family $h$, and make it dependent not only on the vector of prices and on income, but also on a vector of demographic variables, so that the marshallian demand is given by the function $q_{it}^h = f_i (p, m_t^h, z_t^h)$, where $m_t^h$ is non durable expenditure in period $t$, and $z_t^h$ is a vector of conditioning variables, including both durable goods and socio-demographic variables, as well as other goods whose demand is not particularly flexible, like for example tobacco and housing. This formulation corresponds to a two-stage budgeting framework, where the household in each period chooses first how to divide total expenditure between two broad groups weakly separable in preferences (non durables and others), and then allocates $m_t^h$ among non durable commodities.

BPW consider a demand system of 7 groups of goods, while this research is limited to the analysis of two goods, food and alcohol, since the purpose is not an exhaustive representation of consumer preferences, but the assessment of aggregation bias problems, for which a single equation approach may be sufficient.

The functional form chosen is the quadratic extension of the Almost Ideal model, where the budget share depends also on the square of the logarithm of real expenditure: with the availability of household data, it is easy to make preferences depend also on household's characteristics, so that the budget share for good $i$ takes the form

$$(1) \quad w_{it} = \alpha_i^h + \sum_j \gamma_j \log p_j + \beta_i^h \log x_{it}^h + \lambda_i^h (\log x_{it}^h)^2$$

where $x_{it}^h$ is real expenditure at time $t$ (nominal expenditure $m_t^h$ deflated using the retail price index), and the parameters $\alpha_i^h$, $\beta_i^h$, and $\lambda_i^h$ depend also on individual characteristics:

$$(2) \quad \alpha_i^h = \alpha_0 + \sum_k \alpha_{ik} z_{it}^h + \sum_k \delta_k T_k$$

where $T_k$ are seasonal dummies and a time trend, and

$$(3) \quad \beta_i^h = \beta_i + \beta_i^D D_i$$
$$\lambda_i^h = \lambda_i + \lambda_i^D D_i$$

where $D_i$ is a dummy associated to the presence of a certain demographic characteristic in the household.

The consistently aggregated relationship is expressible as

$$w_{it} = \frac{P_i \sum_k q_{it}^h}{\sum_i m_t^h} = \frac{\sum_i w_{it}^h m_t^h}{\sum_i m_t^h} = \Sigma_h \mu_i^h w_{it}^h$$

where $\mu_i^h = m_t^h / \Sigma_i m_t^h$
so that $w_{it}$ is the weighted sum of individual shares.

By applying this weighted sum to our functional form, we obtain:

$$w_{it} = \alpha_0 + \sum_{h} \alpha_h \mu_{it}^{h} x_{it}^{h} + \sum_{j} \gamma_j \log p_j + \beta_i \sum_{h} \mu_{it}^{h} \log x_t^{h}$$

$$+ \beta_i^2 \sum_{h} \mu_{it}^{h} D_k^i \log x_t^{h} + \lambda_i \sum_{h} \mu_{it}^{h} (\log x_t^{h})^2$$

$$+ \lambda_i^2 \sum_{h} \mu_{it}^{h} (\log x_t^{h})^2$$

(4)

Since the model is non-linear and income parameters can vary with demographic variables, this formulation should not suffer from lack of generality as in the exact linear aggregation case.

The term $\sum_{h} \mu_{it}^{h} \log x_t^{h}$ can be rewritten as $\sum_{h} \mu_{it}^{h} \log(x_t^{h} x_t^{h}) = \log \bar{x}_t + \sum_{h} \mu_{it}^{h} \log \frac{x_t^{h}}{x_t}$

where $\bar{x}_t = \sum_{h} x_t^{h}/H_t = $ average total real expenditure, and

$$\sum_{h} \mu_{it}^{h} \log \frac{x_t^{h}}{x_t} = \sum_{h} \frac{x_t^{h}}{x_t} \log \frac{x_t^{h}}{x_t} + \log H_t = - \log Z_t + \log H_t$$

(5) $\sum_{h} \mu_{it}^{h} \log \frac{x_t^{h}}{x_t}$

where $Z_t$ is the entropy index of relative expenditure inequality; the term $\sum_{h} \mu_{it}^{h} \log \frac{x_t^{h}}{x_t}$ reaches a minimum value of 0 in the case of perfect distributional equity (where $Z=H$).

Equation (4), with $\log \bar{x}_t$ and the entropy term replacing $\sum_{h} \mu_{it}^{h} \log x_t^{h}$, represents the aggregate expression that takes correctly into account the distributional factors, in the form of the entropy index, of the terms in $\alpha_{it}$, and of the interactions among individual characteristics and the logarithm of income (and its square). The estimation of such expression on aggregate data, using all the weighted sums, should yield unbiased estimates of the individual preference parameters.

The alternative model, estimated under the maintained hypothesis of the representative agent, is simply:

$$w_{it} = \alpha_0 + \sum_{j} \gamma_j \log p_j + \beta_i \log \bar{x}_t + \sum_{h} \delta_k T_{it} + \lambda_i (\log \bar{x}_t)^2$$

(6)

Clearly in (6) the parameter estimates should be biased, due simply to the omission of many relevant regressors, if these variables are indeed correlated with income and its square or with prices.

The availability of a time series of individual data allows the estimation of both equations (4) and (6), and the comparison between the estimates. If the estimates differ significantly, then aggregation bias is present, and the use of simple aggregate data should be avoided for theoretical or policy analysis purposes.

BPW in their study do not estimate equations (4) and (6), but make a comparison between price and income elasticities obtained from a fully microeconomic model and from an aggregate model that is a "compromise" between our two expressions, with aggregate data plus some distributional factors that should be available working only at the macro level. They find a clear evidence of aggregation bias, particularly in the case of income elasticities.
In this work I estimate both (4) and (6), and then compare the resulting coefficient estimates.

2.2) The data

The data set used for this study is obtained from the British Family Expenditure Survey and covers 25 years, from 1968 to 1992. The sample selection has led to consider (for computational reasons) nearly 44000 households, whose head is less than 60 years old, resident in the areas of Greater London, South East and South West of England. Real variables are measured in 1987 pounds, and 1987 is also the basis for the price series. Average data are obtained by dividing the total by the number of households interviewed in each period.

It is important to underline the length of the period: during an interval of 25 years the structure of English population has considerably changed, as well as the income distribution; thus, it should be possible to estimate with a good degree of precision the coefficients associated to the distributive variables.

As already indicated I consider only two categories of goods, food and alcohol, but among the regressors there are the prices of two other groups of goods, clothing and fuel, for which the complete time series were available.

Atkinson and Micklewright (83) have shown that alcohol is the only category of good for which under reporting in the FES survey is a relevant problem; a related problem for data on alcohol is the great number of zero responses, which can be due to two indistinguishable reasons: either the household never buys the good, or it buys it at discrete intervals, but not during the two weeks of the survey.

Fig.1 and Fig.2 show the behaviour of food and alcohol shares during the period, and fig 3 reports the series of the entropy index7.

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7The entropy measure reported in the figure is given by \( \frac{\sum h x_i^h \log x_i^h}{\sum h x_i^h} \log H_t = \frac{\log H_t - \log Z_t}{\log H_t} \); since \( Z=H \) with perfect equality and \( Z=1 \) with perfect inequality, this measure, taking the value of 0 with perfect equality and of 1 with perfect inequality, doesn't depend on the numerosity of the sample.
Fig. 4 describes one of the variables representing the effect of interactions between demographic characteristics and income (DK is the dummy for the presence of children). The series, as many others not reported, is clearly neither constant nor smoothly decreasing, so that the idea that the socio demographic attributes of the population change slowly over time, and thus their effects can be captured by a simple time trend, cannot be supported.
2.3) Estimation of the aggregate equations

The macro series are obtained by quarterly aggregating the FES data, so that the sample is made up of 100 observations; each quarter grouping nearly 430 observations, the value of the entropy term should be fairly reliable. As Fig.3 shows, our entropy measure is increasing over the period, suggesting an increase in the inequality in income distribution. Fig.1 and 2 suggest that seasonal factors are relevant, so among the regressors three seasonal dummies and a time trend are introduced. To avoid the outliers problem, observations with total expenditure (including durables) greater than the mean plus three times the standard deviation have been dropped.

As is well known, the distribution of income or total expenditure is not normal, so all observations with total expenditure lower than the mean have been kept in the sample.

As BPW point out, working with aggregate data greatly reduces the number of degrees of freedom available for estimation, so only a small number of demographic variables, with their interactions with income terms, could be introduced in the consistently aggregate model.

The \( z \) variables considered are: number of adults (ADUL), number of females (FEM) and number of rooms (ROOMS) in the household, while the dummy variables interacting with the income terms are the following: presence of children (DK), married couple (DMC), working wife (DWW), house owned (or mortgaged) (DOW).

On the basis of the plot of the share on log \( x \) (not reported), quadratic terms do not appear to be relevant for food, so that they have been introduced only in the alcohol equation (see also Blundell, Banks and Lewbel (94)).

As far as the estimation method is concerned, BPW note that a Wu-Hausman test for the exogeneity of the logarithm of income terms leads to the conclusion that endogeneity is not likely in the macro equation, then use ordinary least squares for the macro model (and GMM for the micro one); being unable to compute the exogeneity test for lack of a sufficient number of instruments, I follow BPW in using OLS in all subsequent estimations.

Table 1 reports the coefficient estimates for equations (4) and (6) in the case of the budget share for food.
Tab.1. Food share regression

<table>
<thead>
<tr>
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<th>MODEL (6)</th>
<th>MODEL (4)</th>
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<tbody>
<tr>
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<td>s.e.</td>
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<td>-.05212</td>
<td>.019455</td>
</tr>
<tr>
<td>log((\bar{x}_i))</td>
<td>-.10435</td>
<td>.013345</td>
</tr>
<tr>
<td>entr</td>
<td></td>
<td></td>
</tr>
<tr>
<td>adul</td>
<td>.03777</td>
<td>.013635</td>
</tr>
<tr>
<td>fem</td>
<td>.03135</td>
<td>.022171</td>
</tr>
<tr>
<td>rooms</td>
<td>-.00604</td>
<td>.005757</td>
</tr>
<tr>
<td>dk*lx</td>
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<td>.003646</td>
</tr>
<tr>
<td>dmc*lx</td>
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<td></td>
</tr>
<tr>
<td>dww*lx</td>
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</tr>
<tr>
<td>dow*lx</td>
<td></td>
<td></td>
</tr>
<tr>
<td>s1</td>
<td>.00203</td>
<td>.001898</td>
</tr>
<tr>
<td>s2</td>
<td>-.00074</td>
<td>.001578</td>
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<tr>
<td>s3</td>
<td>-.00017</td>
<td>.000276</td>
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<tr>
<td>quart</td>
<td>.77694</td>
<td>.064424</td>
</tr>
<tr>
<td>const</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

R\(^2\) = 0.9709  \quad \text{adj.R}\(^2\) = 0.9679  \\
R\(^2\) = 0.9792  \quad \text{adj.R}\(^2\) = 0.9749  

If we look at the graph of the temporal evolution of this share, it appears to be dominated by a trend, but once we insert it among a set of regressors, it turns out to be not significant, clearly because income has a dominant effect.

The two models can be compared using different, but ultimately equivalent, tests; for example, we can simply check whether the individual estimates are significantly different in the two equations: considering only the estimates which are significantly different from zero, the t-test of the difference lies in the rejection region only in the case of log income, while the coefficients of pfuel, s1 and the constant are fairly similar.

The coefficient of \(\log(\bar{x}_i)\) changes by 35.6%, while the coefficients of pfuel by 5%, of the constant by 2.6% and of s1 by 23%.

Another possibility, noting that equation (6) is nested in the other, consists in an F test of the hypothesis that the subset of the coefficients of the demographic variables and their interactions with income are jointly zero; the RSS is 0.00244 in the restricted (eq.(6)) model and 0.001746 in the unrestricted, so with 8 and 82 degrees of freedom the value of the F test is 4.074, while the critical
value is 1.99 at 5% (2.8 at 1% significance level); the distributional factors, therefore, matter, as can be also seen by noting that four out of eight of them are individually significant, and their omission has an effect mainly on the income coefficient; this point has already been underlined in the work by BPW, where none of the differences in the price elasticities between a micro and an aggregate model with some entropy terms is significantly different from zero, while the budget elasticities are much more influenced by the choice of the functional form.

It should be noted that the regression equation doesn't impose the constraint that the coefficients of log($x_i$) and ENTR must be equal, as is required eq.(4); in the case of food, an F test of the hypothesis that their difference is zero leads to accept the Null ($F(1,82) = 0.52$, P-value = 0.47), and in the alcohol case the value of the test is $F(1,77) = 0.17$ (P-value=0.68), so that the null hypothesis is still acceptable.

Another possible test is a LR test: twice the difference between the log likelihoods is 33.8, while the critical value of the chi-square distribution with 8 degrees of freedom at the 5% significance level is 15.5 (20.1 at 1%), so model (6) is clearly rejected by the other.

Finally, the two models can be compared by considering the values taken by the budget and own-price elasticities. The budget elasticity computed at the average share and at the average values of the households' characteristics is 0.5652 for model (6) and 0.4589 for model (4) (19% change from (6) to (4)), while the uncompensated own price elasticity is -0.799 for eq.(6) and -0.869 for eq.(4), with a change of 8.8%.

Table 2 provides the same estimates as the previous one, for the alcohol budget share.
### Tab.2. Alcohol share regression

<table>
<thead>
<tr>
<th>MODEL (6)</th>
<th>MODEL (4)</th>
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<tr>
<td>palc</td>
<td>-0.04434</td>
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<tr>
<td>pclcloth</td>
<td>-0.00208</td>
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<tr>
<td>pfuel</td>
<td>0.02927</td>
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<tr>
<td>log (\bar{x})</td>
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</tr>
<tr>
<td>entr</td>
<td>0.49064</td>
</tr>
<tr>
<td>((\log \bar{x})^2)</td>
<td>-0.08299</td>
</tr>
<tr>
<td>adul</td>
<td></td>
</tr>
<tr>
<td>fem</td>
<td></td>
</tr>
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<td>dk*lx</td>
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<td>dk*lxsq</td>
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<td>dww*lxsq</td>
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</tr>
<tr>
<td>dow*lxsq</td>
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<tr>
<td>(\sum_{h} \mu_i^*(\log x_i)^2)</td>
<td>-2.3224</td>
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<tr>
<td>R^2 = 0.7026</td>
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<tr>
<td>adj.R^2 = 0.6692</td>
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</tr>
<tr>
<td>R^2 = 0.7834</td>
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</tr>
<tr>
<td>adj.R^2 = 0.7215</td>
<td></td>
</tr>
</tbody>
</table>

The dynamic evolution of this share appears to be dominated by seasonal and price effects.

While the coefficients associated to the prices of fuel and alcohol (the only two significant) are very similar in the two regressions, the estimate of the logarithm of average income in model (6) is nearly twice as the other.

If we consider also the quadratic terms in income, in order to nest model (6) within the other it is necessary to make another substitution in the regressors; following the change described in section 2.1), \(\sum_{h} \mu_i^*(\log x_i)^2\) can be replaced by the three terms.
\[ \sum_i \alpha_i^2 \left( \frac{\log \frac{x_i^b}{x_i}}{x_i} \right)^2 + (\log \frac{\bar{x}}{x})^2 + 2 \log \frac{\bar{x}}{x} \sum_i \alpha_i^2 \log \frac{x_i^b}{x_i}, \] where now a quadratic entropy measure is present as well. After the substitution we could compute an F test as before.

An alternative, simpler procedure consists in applying a non nested test, for example the Davidson-McKinnon test (J test): if we define the two hypotheses as

H₀: (6) is true
H₁: (4) is true

we can regress the budget share on the regressors of (6) plus the estimated budget share from equation (4); if this last variable has an explanatory power on its own, then (6) cannot be the correct model; the estimate associated to \( \hat{\omega} \) is 0.9922 with a t statistic of 5.279, so H₀ is strongly rejected in favour of H₁.

Reversing the roles of the two hypotheses, a regression of the share on the explanatory variables of model (4) together with the predicted values from (6) gives an estimate of 0.33 with a t test of 1.19, so that we cannot reject H₁ using H₀ as alternative hypothesis. The conclusion is that model (4) is acceptable, model (6) is not.

It is possible to check the results of the J test using an F test on a general model which nests both models (4) and (6): in this comprehensive regression equation, the t test of \( (\log \frac{\bar{x}}{x})^2 \) is 0.279, while the F test of the joint significance of the 13 variables belonging only to equation (4) is 2.19, with a P-value of 0.018 so that, at the 5% significance level, we can accept model (4) instead of (6).

Particularly significant is the comparison between the elasticities: the budget elasticity is 1.314 for model (6) and 2.2453 for the consistently aggregated model (70.87% change), while the uncompensated own price elasticities are respectively -1.9855 and -1.7953 (variation of 9.58% only).

Only a few of the demographic factors are significantly different from zero, but the J test leads to accept model (4); this could imply that the low level of individual significance is due to the introduction of too many regressors, and that perhaps a more parsimonious model is preferable (we are estimating 23 parameters with 100 observations).

It is interesting to note that the orders of magnitude of these elasticities are fairly similar to the values obtained by BPW, who report average values for the budget and own price elasticities for alcohol of 1.88 and -1.55 respectively. Actually, the food elasticities are even more similar: 0.501 for budget elast. (compared with 0.4589 for model (4)) and -0.514 for the own price elast. (-0.869 for eq.(4)).

A common result of these estimations is that problems arise particularly for the income parameters; as repeatedly said, this is very likely to be due to the presence of a significant degree of correlation between income and one or more of the omitted variables in the representative agent model; an informal verification of this hypothesis can be obtained by regressing the logarithm of average income (and its square) on the demographic factors. The results, reported in the appendix,

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8In this way \((\log \frac{\bar{x}}{x})^2\) can be introduced also in the consistently aggregated model.
show that income is significantly correlated with most of these terms, particularly with the dummy indicating if the wife works, and with the entropy term. This could be a useful indication for the estimation of a macro model with only a few aggregation terms.

Finally, an interesting question is whether the demographic factors can improve the forecasting ability of the macro equation. I reestimated the two models excluding the last four quarters, and computed the 95% confidence intervals around the forecasted values. The continuous lines are the actual values, the dashed lines represent the fitted values and the out of sample forecasts. In the forecasting period, the points indicate the borders of the confidence intervals. The equations show a satisfactory fit, but the food equations tend to overpredict the actual values, which are at the limit of the band. In order to make clearer the comparison between the two equations, the last two figures illustrate the forecast errors; while eq.(4) performs better in the food case, the opposite is true for the alcohol equation (even if the fitted values of (4) are still acceptable), so no definite conclusion can be drawn as far as the relative forecasting ability of the two models is concerned. As a temporary conclusion, we could say that aggregation is more relevant for the study of the elasticities than for the forecast of aggregate quantities.
Fig. 8

Alcohol: equation (6)

Fig. 9

Food: forecast errors

Alcohol: forecast errors

---
eq(4)  ---  eq(6)
---
eq(4)  ---  eq(6)
2.4) Comparison with the micro estimates

Appendix 2 reports the results of a micro regression of the two budget shares on the same price, income and demographic variables considered in the consistently aggregated model. Comparing the two sets of results, important discrepancies emerge; for example, in the alcohol equation the coefficient associated with log income is of a completely different degree of magnitude, and sometimes there is a reversion in the signs. On the other hand, some similarities are present as well: in the food equation the income coefficient is quite similar to our aggregate results, the more significant price effect comes again from fuel, and the seasonal factors confirm their decisive importance for alcohol consumption.

It is difficult to express a judgement on the comparison between the two sets of estimates, since various problems are surely present. For example, the estimation method used in the micro regression is still OLS, due to a lack of a proper set of instrumental variables, which is probably inadequate for more than one reason. First of all, the presence of many zero expenditures for alcohol (nearly 25% of the households) implies that the OLS estimates are very likely to be biased, and this could help to explain why the estimated coefficient for log income is so different from the macro results only in the alcohol case. A correct treatment of this problem would require the use of IV techniques. If the demographic variables included do not give a full account of individual heterogeneity, the error terms are heteroscedastic. Finally, if the household's error term contains an individual factor, the coefficient estimates from a simple regression on the levels are biased: for example, if \( w^h = \sum \beta_j x_{h,j} + u_h + \eta_h \), the individual factor \( \eta_h \) is correlated with the regressors \( x \), so that the estimates of the vector \( \beta \) are biased. On the other hand, if we suppose that the individual effects are distributed with zero mean in homogeneous subgroups of observations, by working with the means of all the variables the individual factors should disappear, and the estimates of \( \beta \) should be consistent; this is one of the main reasons behind the increasing diffusion of studies considering cohort data, typically constructed for instance by averaging on the households with different ranges of age of the head. In general, working with aggregated data should decrease the impact of the individual effects. An interesting verification could be a set of estimates on more homogeneous groups of households; some attempts with the alcohol equation didn't provide significantly different results.
3) AGGREGATION BIAS AND DYNAMIC MISSPECIFICATION

3.1) Aims of this section

In section 1.2) I have already described the principal findings of the work by Stoker (86): in brief, estimation of a Linear Expenditure System of consumer demand shows clear signs of dynamic misspecification (AR(1) residuals), but the introduction of simple distributional variables is enough to make the autoregressive estimate not significantly different from zero.

Stoker concludes suggesting that an insufficient attention to the aggregation problem could have lead researchers to overestimate the importance of dynamic misspecification in demand analysis.

The basic intuition behind these results is simple: the aggregate demographic variables (for example the proportions of households in various income classes) tend to change slowly over time, so they should be highly autocorrelated; if we don't insert them in the macro equation, they are part of the error term, and make it autocorrelated as well. It follows that, if the temporal dependence of the disturbance is due only to this omission, an explicit consideration of the demographic factors should eliminate the autocorrelation. If, on the other hand, the residual is still autocorrelated, then the temporal dependence does not derive from this problem.

The purpose of this part of the paper is to analyze the relationship between aggregation and dynamics, following two complementary ways: first of all, it is possible simply to check for the differences in the dynamic structure of our two basic equations, and second, we can replicate the test of Stoker using a properly computed set of distributional factors from the micro data.

3.2) Tests of dynamic structure

Table 3 reports some tests and estimates aimed at analyzing the dynamic structure of the two alternative equations.

<table>
<thead>
<tr>
<th></th>
<th>DW</th>
<th>TR²(1)</th>
<th>c.v.(1)</th>
<th>t₀ 1</th>
<th>r(t)</th>
<th>t₄ 4</th>
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<tbody>
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<td>1) FOOD</td>
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<tr>
<td>EQ.(6)</td>
<td>1.49</td>
<td>5.029</td>
<td>2.078</td>
<td>0.226</td>
<td>0.33</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(2.25)</td>
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<td>EQ.(4)</td>
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<td>10.51</td>
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<td>(3.43)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) ALC</td>
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<tr>
<td>EQ.(6)</td>
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<td>3.11</td>
<td>1.679</td>
<td>0.19</td>
<td>-0.158</td>
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<tr>
<td></td>
<td></td>
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<td></td>
<td>(1.95)</td>
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<tr>
<td>EQ.(4)</td>
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<td>3.564</td>
<td>1.673</td>
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<td></td>
<td>(2.45)</td>
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</table>

DW is the Durbin-Watson test, always falling in the inconclusive region, so I proceeded to compute the Breusch-Godfrey test for first and fourth order autocorrelation in the error term. TR²(1) is the value of this test for I order autocorrelation, to be compared with the critical value of
the chi-square distribution with one degree of freedom, c.v. (1) (5% significance level). R²(1) is the R² of the auxiliary regression \( \hat{u}_t = \beta X_t + \gamma \hat{u}_{t-1} + v_t \), where T=99, \( \hat{u}_t \) is the OLS residual of the original equation and \( X_t \) is the set of all the regressors in the original equation.

\( t_{u_{t-1}} \) is the value of the t test for the parameter of \( \hat{u}_{t-1} \), and \( r \) is the estimated AR(1) coefficient of the original equation (with t in brackets).

In the case of the food equation, both models accept the presence of first-order autocorrelation, with \( r \) significantly different from zero. For the alcohol equation, there is a slight sign of AR(1) dynamics only in the consistently aggregated model.

On the whole, the two demand equations show a very similar behaviour; despite the use of quarterly data, there is no indication of fourth-order autocorrelation, but more important is the lack of significant differences between the "representative agent" model and the correctly aggregated equation: the introduction of the weighted averages of the demographic variables doesn't imply that the autoregressive term becomes insignificant in the food equation, on the contrary it is more distant from zero according to all indicators for both goods.

Thus, as can be clearly seen particularly in the food equation, there is no sign that the omission of distributive variables can account for dynamic misspecification, so in this case a specific modelling of dynamics seems to be required.

3.3) The Stoker-Buse model

In his study, Stoker uses cell proportion data, i.e. time series of the proportions of households in given categories. In particular, Stoker deals with expenditure categories, but nothing would prevent us from using other groupings, for example by number of children, or by age or working activity of the head.

If \( P_{jt} \) denotes the proportion of agents (households) in group \( j \) at time \( t \) (\( j=1,...,N \)), average budget share of good \( i \) is given by \( w_{it} = \sum_j E_t(w_{ij})P_{jt} \), and average expenditure by

\[ E_t(x) = \sum_j E_t(x|j)P_{jt}, \]

where \( E_t(w_{ij}) \) and \( E_t(x|j) \) are the within cell \( j \) averages of period \( t \).

If we write \( w_{jt} = E_t(w_{ij}) \) and \( x_{jt} = E_t(x|j) \), then in vector form \( w_{it} = W_{it}^T P_t \) and \( E_t(x) = X_t^T P_t \).

Making the assumption (easily testable using our data set) that the within-cell averages are constant over time, then \( w_{it} = W_{it}^T P_t \) and \( E_t(x) = X_t^T P_t \).

Orthogonally decomposing \( W_t \) (and suppressing the index \( i \) for simplicity), we can write

\[ W = RW + (I-R)W \]

where \( R = X_c(X_c^TX_c)^{-1}X_c^T \) and \( X_c = (1,X) \) (1 is the unit vector), so that

\[ w_t = W_t^T P_t = W_t^T R P_t + W_t^T (I-R) P_t \]

\[ \begin{align*}
  &= W_t^T X_c (X_c^TX_c)^{-1} X_c^T P_t + W_t^T P_t \\
  &= W_t^T X_c (X_c^TX_c)^{-1} X_c^T P_t + W_t^T P_t \\
  &= W_t^T X_c (X_c^TX_c)^{-1} X_c^T P_t + W_t^T P_t \\
  &= W_t^T X_c (X_c^TX_c)^{-1} X_c^T P_t + W_t^T P_t \\
\end{align*} \]

\[ 9W_{it} = (w_{i1t},...,w_{iNt})^T \quad \text{and} \quad X_t = (x_{i1t},...,x_{iNt})^T \]
where \( \tilde{W} \) is the vector of the residuals of the regression of \( W \) on \( X_c \), given that I-R is the familiar "residual maker" of linear regression.

If the micro equation is linear, then \( W^TX_c(X_c'X_c)^{-1}X_c^T \) is equal to \( \alpha + \beta E_i(x_n) \), so that \( w_i = \alpha + \beta E_i(x_n) + \tilde{W}^T P_i + u_i \) and aggregate (average) budget share is decomposed in a linear component of the regressors and a term which should not be significant in the individual relation is linear. A test of micro-linearity is given by an F test of the \( \tilde{W} \) parameters of the aggregate equation, but alternatively Stoker suggests to collect N-2 of the proportions in a vector \( P^* \) and to run the regression \( w_i = \alpha^* + \beta E_i(x_n) + \tilde{W}^* P_i^* + v_i \) which corresponds to joining two cell means and taking the residuals as deviations from the line joining these two points. If \( \tilde{W} \) is not significantly different from zero, there are no distributional effects in the aggregation process; otherwise, the test shows that using aggregate data alone we can obtain only biased estimates.

Buse(92) generalizes this test, starting from a quadratic micro relation. since, as already said, micro linearity is usually rejected in empirical studies, the test of Stoker would lead to reject the null even when distributional effects are absent, since in Stoker the presence of distributional effects coincides with micro non linearity.

Starting from a non linear but quadratic individual equation, the macro relation is

\[
w_i = \alpha + \beta E_i(x) + \gamma E_i(x^2) + \tilde{W}^T P_i
\]

\[
= \alpha + \beta E_i(x) + \gamma (E_i(x))^2 + \gamma \text{var}(x) + \tilde{W}^T P_i
\]

In this case we break the link between non linearity and aggregation bias: if the estimates of \( \tilde{W} \) are not significant, distributional factors do not matter even if the micro-relation is not linear\(^{10}\).

Unlike the cited authors, we can compute the vectors of income proportions and the variance directly on micro-data, and avoid the approximation of considering the distribution of income in place of the distribution of expenditure.

I have computed the time series of the proportions for 6 expenditure classes: \( P1= <100, P2=100-150, P3= 150-200, P4= 200-300, P5= 300-400, P6= >400 \) (in 1987 pounds), and in the regressions P3 and P4 have been omitted, thus following the approach suggested by Stoker.

As for the variance of expenditure, I didn't insert it in the final estimation since, dealing only with aggregate data, this information should not be known.

The *dynamic model considered is, as usual, a static model with first order autocorrelation in the disturbances.

Table 4 reports the estimates for the two goods in the four cases of:

a) static model without proportions
b) dynamic model without proportions

---

\(^{10}\) From section 1, we know that with non linearity in the micro relation, only a macro equation which takes account of distributional factors is correctly specified, but under scrutiny here is the statistical significance of the distributional variables, not their presence.
c) static model with proportions  
d) dynamic model with proportions

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<th>ALCOHOL</th>
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<td>b</td>
<td>c</td>
<td>d</td>
</tr>
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<td>(2.05)</td>
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<td>s2</td>
<td>0.002</td>
<td>0.002</td>
<td>0.0017</td>
<td>0.0014</td>
</tr>
<tr>
<td></td>
<td>(1.18)</td>
<td>(1.10)</td>
<td>(0.968)</td>
<td>(0.813)</td>
</tr>
<tr>
<td>s3</td>
<td>-0.0007</td>
<td>-0.0008</td>
<td>-0.0008</td>
<td>-0.00084</td>
</tr>
<tr>
<td></td>
<td>(-0.47)</td>
<td>(-0.58)</td>
<td>(-0.478)</td>
<td>(-0.592)</td>
</tr>
<tr>
<td>const</td>
<td>0.776</td>
<td>0.79</td>
<td>1.517</td>
<td>1.45</td>
</tr>
<tr>
<td></td>
<td>(12.06)</td>
<td>(11.93)</td>
<td>(5.24)</td>
<td>(5.47)</td>
</tr>
<tr>
<td>r</td>
<td>0.226</td>
<td>0.229</td>
<td>0.175</td>
<td>0.245</td>
</tr>
<tr>
<td></td>
<td>(2.25)</td>
<td>(2.31)</td>
<td>(1.686)</td>
<td>(2.34)</td>
</tr>
<tr>
<td>P1</td>
<td>-0.118</td>
<td>-0.107</td>
<td>-0.033</td>
<td>-0.034</td>
</tr>
<tr>
<td></td>
<td>(-2.35)</td>
<td>(-2.29)</td>
<td>(-0.96)</td>
<td>(-1.06)</td>
</tr>
<tr>
<td>P2</td>
<td>-0.10</td>
<td>-0.096</td>
<td>-0.024</td>
<td>-0.018</td>
</tr>
<tr>
<td></td>
<td>(-2.62)</td>
<td>(-2.79)</td>
<td>(-0.93)</td>
<td>(-1.06)</td>
</tr>
<tr>
<td>P5</td>
<td>0.10</td>
<td>0.087</td>
<td>-0.023</td>
<td>-0.035</td>
</tr>
<tr>
<td></td>
<td>(1.89)</td>
<td>(1.74)</td>
<td>(-0.64)</td>
<td>(-0.099)</td>
</tr>
<tr>
<td>P6</td>
<td>0.18</td>
<td>0.166</td>
<td>-0.015</td>
<td>-0.035</td>
</tr>
<tr>
<td></td>
<td>(1.89)</td>
<td>(1.81)</td>
<td>(-0.215)</td>
<td>(-0.52)</td>
</tr>
</tbody>
</table>

The two groups of estimates (for food and alcohol) show some interesting regularities; first of all, we have a confirmation of the different impact of the introduction of demographic variables on the estimates related to other regressors: estimates of price coefficients and seasonal effects are nearly
insensitive to their introduction, while the coefficients of income and its square (and the constant) are greatly affected. In the case of food, the coefficient of \( \log x \) more than doubles, while for alcohol both income coefficients are more than halved (in absolute value).

The second point to note is that the demographic terms do not imply any significant change in the dynamic structure of the data; in fact, in the alcohol case the degree of first order autocorrelation is clearly increased by the presence of the proportion terms.

The coefficients associated to these terms are nearly all significantly different from zero for food, and all insignificant for alcohol; this could be due perhaps to some misspecification in this equation, but what is relevant is that even with these poor estimates the parameters of prices and income terms are affected in the same way as in the food equation.

A likelihood ratio test is used to compare these models, and the results are reported in Table 5. At the 1% significance level no test is significant, so only at 5% level it is possible to reject the static model, but the last row shows that the static model with proportions is not statistically equivalent to an AR(1) model with proportions, and this should at least suggest two independent roles for dynamics and aggregation.

**Tab.5. Likelihood Ratio Tests**

<table>
<thead>
<tr>
<th></th>
<th>df</th>
<th>c.v.1%</th>
<th>c.v.5%</th>
<th>food L.R.</th>
<th>alcohol L.R.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a / b</td>
<td>1</td>
<td>6.63</td>
<td>3.84</td>
<td>5</td>
<td>2.9</td>
</tr>
<tr>
<td>a / c</td>
<td>4</td>
<td>13.3</td>
<td>9.49</td>
<td>8.76</td>
<td>4.12</td>
</tr>
<tr>
<td>b / d</td>
<td>4</td>
<td>13.3</td>
<td>9.49</td>
<td>3.76</td>
<td>6.24</td>
</tr>
<tr>
<td>c / d</td>
<td>1</td>
<td>6.63</td>
<td>3.84</td>
<td>5.22</td>
<td>5.02</td>
</tr>
</tbody>
</table>

In the following table I report first the results of single-equation estimations of each of the demographic factors on itself and all the others lagged one period, and then the results of simple autoregressions; the aim is to verify the temporal structure of these terms: if they are not strongly autocorrelated, then our previous results become more understandable, in the sense that the explicit consideration of the proportion terms should not significantly change the autocorrelation structure of the residuals.
Tab. 6. Temporal correlations of the proportion terms

<table>
<thead>
<tr>
<th></th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
<th>P6</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1(-1)</td>
<td>0.195</td>
<td>0.29</td>
<td>0.09</td>
<td>-0.07</td>
<td>-0.27</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.96)</td>
<td>(0.78)</td>
<td>(0.45)</td>
<td>(-0.29)</td>
<td>(-1.36)</td>
<td></td>
</tr>
<tr>
<td>P2(-1)</td>
<td>0.048</td>
<td>0.89</td>
<td>0.41</td>
<td>-0.40</td>
<td>-0.44</td>
<td>-0.25</td>
</tr>
<tr>
<td></td>
<td>(0.33)</td>
<td>(3.24)</td>
<td>(2.61)</td>
<td>(-1.88)</td>
<td>(-3.18)</td>
<td>(-2.55)</td>
</tr>
<tr>
<td>P3(-1)</td>
<td>-0.031</td>
<td>0.91</td>
<td>0.14</td>
<td>-0.17</td>
<td>-0.42</td>
<td>-0.17</td>
</tr>
<tr>
<td></td>
<td>(-0.18)</td>
<td>(2.79)</td>
<td>(0.72)</td>
<td>(-0.67)</td>
<td>(-2.51)</td>
<td>(-1.86)</td>
</tr>
<tr>
<td>P4(-1)</td>
<td>0.051</td>
<td>0.45</td>
<td>0.17</td>
<td>-0.04</td>
<td>-0.23</td>
<td>-0.14</td>
</tr>
<tr>
<td></td>
<td>(0.27)</td>
<td>(1.27)</td>
<td>(0.85)</td>
<td>(-0.15)</td>
<td>(-1.30)</td>
<td>(-1.66)</td>
</tr>
<tr>
<td>P5(-1)</td>
<td>0.017</td>
<td>0.33</td>
<td>0.23</td>
<td>-0.02</td>
<td>-0.32</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.71)</td>
<td>(0.85)</td>
<td>(-0.05)</td>
<td>(-1.35)</td>
<td>(0.15)</td>
</tr>
<tr>
<td>P6(-1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(1.64)</td>
</tr>
<tr>
<td>CONST</td>
<td>0.106</td>
<td>-0.36</td>
<td>0.02</td>
<td>0.40</td>
<td>0.41</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>(0.67)</td>
<td>(-1.19)</td>
<td>(0.10)</td>
<td>(1.71)</td>
<td>(2.67)</td>
<td>(2.49)</td>
</tr>
<tr>
<td>F</td>
<td>0.85</td>
<td>28.62</td>
<td>10.48</td>
<td>13.46</td>
<td>17.4</td>
<td>31.4</td>
</tr>
<tr>
<td>(P val.)</td>
<td>(0.52)</td>
<td>(0)</td>
<td>(0)</td>
<td>(0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.044</td>
<td>0.606</td>
<td>0.36</td>
<td>0.42</td>
<td>0.46</td>
<td>0.63</td>
</tr>
<tr>
<td>Pi(-1)</td>
<td>0.18</td>
<td>0.73</td>
<td>0.28</td>
<td>0.53</td>
<td>0.57</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td>(1.95)</td>
<td>(10.5)</td>
<td>(2.88)</td>
<td>(6.32)</td>
<td>(6.91)</td>
<td>(11.44)</td>
</tr>
<tr>
<td>CONST</td>
<td>0.13</td>
<td>0.07</td>
<td>0.17</td>
<td>0.11</td>
<td>0.031</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>(8.58)</td>
<td>(3.69)</td>
<td>(7.17)</td>
<td>(5.63)</td>
<td>(4.91)</td>
<td>(3.21)</td>
</tr>
</tbody>
</table>

(t stats in brackets)

Since the proportion terms sum to one, at least one of them had to be excluded from each regression. Once we take into account the whole structure of interrelations among these terms (a sort of Var model), the degree of first-order autocorrelation is not, as expected, particularly big.
CONCLUSIONS

In this paper I have investigated the importance of aggregation problems in the estimation of consumers' demand for goods. The availability of a time series of individual-level data allows to compare the results obtainable from a correctly aggregated model and from a model which simply ignores the aggregation problem, based instead on the hypothesis of the representative agent.

I think that the main findings can be summarized as follows:

a) Aggregation surely matters: if the micro relation is not linear, as in my case (QUAIDS), the estimates from an aggregate model that doesn't take into account distributional factors are biased, and the consistently aggregated model has been shown to outperform the "representative agent" equation. A micro regression leaves open the problems of a proper estimation procedure at the individual level and of the comparison between a properly aggregated model and a micro one. These results confirm the findings of many empirical works, which agree in the belief that the use of simple macro models is very likely to lead to misleading or completely wrong conclusions. A possible objection is that in many cases long panels are yet unavailable, so that if one wants to do some research the recourse to simple aggregate relationships is unavoidable but, as BPW have shown, in this case too it is advisable to supplement the macro data with distributive variables, even if not particularly sophisticated, usually obtainable from other sources.

b) Estimates of income effects are more influenced by individual heterogeneity than price or seasonal variables; this can imply that if access is limited to aggregate data, inferences for policy purposes based on macro regressions should be limited to the effects of changes in relative prices (for example, only indirect and not direct taxation).

c) Contrary to the findings of Stoker (86) and Buse (92), I cannot conclude that the presence of dynamic misspecification appears as a consequence of neglecting aggregation problems, because the two main models considered in this study (the correctly aggregated and the simple macro equation) basically present the same dynamic structure. It follows that aggregation and dynamic structure seem to be separate problems, that must be tackled using distinct procedures.
APPENDIX 1: CORRELATION AMONG INCOME AND OTHER REGRESSORS.

<table>
<thead>
<tr>
<th>DEP. VAR.: $\log \bar{x}_t$</th>
<th>DEP. VAR.: $(\log \bar{x}_t)^2$</th>
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</thead>
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<tr>
<td></td>
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</tr>
<tr>
<td>entr</td>
<td>.92511</td>
</tr>
<tr>
<td>adulmi</td>
<td>.26334</td>
</tr>
<tr>
<td>femmi</td>
<td>-.19082</td>
</tr>
<tr>
<td>roomsmi</td>
<td>.06152</td>
</tr>
<tr>
<td>dkml1x</td>
<td>.02984</td>
</tr>
<tr>
<td>dmcm1lx</td>
<td>.05107</td>
</tr>
<tr>
<td>dwmml1x</td>
<td>.07069</td>
</tr>
<tr>
<td>dowmml1x</td>
<td>.03533</td>
</tr>
<tr>
<td>lpfood</td>
<td>-.57248</td>
</tr>
<tr>
<td>lpalc</td>
<td>-.20643</td>
</tr>
<tr>
<td>lpcloth</td>
<td>.50155</td>
</tr>
<tr>
<td>lpfuel</td>
<td>.18211</td>
</tr>
<tr>
<td>quart</td>
<td>.0070</td>
</tr>
<tr>
<td>s1</td>
<td>-.05859</td>
</tr>
<tr>
<td>s2</td>
<td>-.02487</td>
</tr>
<tr>
<td>s3</td>
<td>-.03615</td>
</tr>
<tr>
<td>dkml1xsx</td>
<td></td>
</tr>
<tr>
<td>dmcm1lxsx</td>
<td></td>
</tr>
<tr>
<td>dowmml1xsx</td>
<td></td>
</tr>
<tr>
<td>dwmml1xsx</td>
<td></td>
</tr>
</tbody>
</table>

R-square = 0.9441  \quad \text{Adj R-square} = 0.9333  \quad \text{F}(16, 83) = 87.54

R-square = 0.9930  \quad \text{Adj R-square} = 0.9913  \quad \text{F}(20, 79) = 563.79
### APPENDIX 2

#### MICRO ESTIMATIONS

<table>
<thead>
<tr>
<th>Food share regression</th>
<th>Alcohol share regression</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>coef.</strong></td>
<td><strong>s.e.</strong></td>
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<tr>
<td>pfood</td>
<td>0.04293</td>
</tr>
<tr>
<td>palc</td>
<td>0.01097</td>
</tr>
<tr>
<td>pcloth</td>
<td>-0.034</td>
</tr>
<tr>
<td>pfuel</td>
<td>-0.06773</td>
</tr>
<tr>
<td>log x</td>
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</tr>
<tr>
<td>(log x)^2</td>
<td>-0.005871</td>
</tr>
<tr>
<td>adul</td>
<td>0.03493</td>
</tr>
<tr>
<td>fem</td>
<td>-0.00471</td>
</tr>
<tr>
<td>rooms</td>
<td>0.002838</td>
</tr>
<tr>
<td>dk*lx</td>
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</tr>
<tr>
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</tr>
<tr>
<td>dww*lx</td>
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</tr>
<tr>
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<tr>
<td>dk*lxsq</td>
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</tr>
<tr>
<td>dmc*lxsq</td>
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</tr>
<tr>
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</tr>
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</tr>
<tr>
<td>const</td>
<td>0.62798</td>
</tr>
</tbody>
</table>

R² = 0.3461  
adj. R² = 0.3459

R² = 0.1240  
adj. R² = 0.1236
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