Co-movements and convergence in international output.
A Dynamic Principal Components Analysis.

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Ottobre 1995

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A Dynamic Principal Components Analysis

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Abstract: This paper utilises a dynamic principal components analysis and Johansen tests to analyse co-movements and convergence in real outputs of the main OECD economies on a sample period from 1960 to 1992. We reject convergence, but find evidence for common stochastic trends and long run co-movements in international output more important than the business cycle ones.

Key words: dynamic principal components method, Johansen procedure, stochastic convergence.

JEL classification numbers: C14, C32, O40, O57

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1. Introduction

Bernard and Durlauf (1991, 1995) propose a definition of convergence entirely referred to the time series properties of outputs rather than cross-section one. According to their definition, a necessary condition for stochastic convergence in (logged) real per capita outputs, towards a unique steady-state equilibrium, requires that a permanent shock to one country is related with a permanent shock to other countries, in other words it requires the same stochastic trend (or unit root) in outputs. A large class of real business cycle models, stochastic versions of the standard neoclassical growth model, interprets the presence of unit root in national output and a common unit root in international output movements as due to permanent technological shocks, or in other words the stochastic trend (or common stochastic trend) in output (international outputs) is representative of the random walk technology process. We refer to this as the purely technological interpretation of unit root in outputs and will adopt this representation as a benchmark for testing convergence. However, this outcome not necessarily implies convergence in international outputs. While the presence of one common (stochastic) trend (n-1 cointegrating vectors) gives relatively weak restrictions on technology

*We wish to thank Mario Forni for helpful suggestions. We are also grateful to Mario Forni and Lucrezia Reichlin for generously providing their software. The usual disclaimers apply.


2 See, for example, King et. al. (1991).
movements requiring just some links between national production functions, so that the permanent shocks partially migrate, convergence requires that permanent (technological) shocks fully migrate from one country to another. In other words, convergence requires that each country of the group under examination has identical long run trends, while common trends or cointegration allow for proportionality of the stochastic elements. Obviously, under the null of \( n \) common stochastic trends (no-cointegration) the innovations do not exhibit linear transmission mechanism in the long run and the source of fluctuations is idiosyncratic at domestic level and not transferred from a country to another. This analysis has been considered as a test for the implication of the thesis, derived in the context of optimal growth models, that if all countries share technology and preferences, then output levels will converge over time. If this is not the case idiosyncratic microeconomic factors are important to explain the growth.

Testing for the presence of just one common persistent part explaining most of the total variance of international output movements of 21 OECD economies is equivalent to testing for the necessary condition for convergence and "purely" technological interpretation of unit root. In this paper we use a dynamic principal component analysis to count up the number of common components in particular to long run economic fluctuations. This method allows to test for long run co-movements and convergence using large cross-section of countries simultaneously over a long time period and permits to check for co-movements across outputs at the cycle frequencies. For comparison, we also use cointegration procedure proposed by Johansen (1991) that enables us to test directly the convergence hypothesis for dimensionally not too large cross section of countries.

Our analysis leads to two basic conclusions. First, we find little evidence of convergence across 21 OECD economies and smaller samples of countries (ten European countries, six original European Community countries and countries of the G7 and G5 group) using both multivariate methods. However, we find strong evidence of a small number of common stochastic elements in international output. We isolate two or three common principal components to long run economic fluctuations explaining at zero frequency at least 0.95 of total variance of outputs. Second, at zero frequency the high weight of the first common component (at least 0.80 of total variance) prevents output levels to diverge too much in the long run, while at the business cycle frequencies, there is not evidence of important co-movements.

Our work is related, among others, to papers by Bernard and Durlauf (1995), Campbell and Mankiw (1989), Cogley (1990), Quah (1990), Pagano (1993), who explore convergence in a time series prospective. All these papers find persistence and divergence in outputs of the OECD economies. In particular, Bernard and Durlauf, Cogley and Pagano stress that the no-convergence result is a substantial outcome for many countries, but, as in our case, there is strong evidence of common stochastic elements in long run economic movements. Our analysis differs from these papers in some aspects. We use a different data set and a different econometric technique: a dynamic principal component analysis over all the frequencies, which seem appropriate to study not only the convergence but also long run and short run co-movements; moreover this methods permits to establish the weight of the common shocks to the variables at different frequencies.

The plan of the paper is as follow. Section 2 contains a brief description of the dynamic principal component analysis. Section 3 presents the empirical results. Section 4 concludes.

\footnote{More formally, convergence among \( n \) economies requires \((n-1)\) cointegrating vectors of the form \((1, -1)\) and identical deterministic components.}
2. Dynamic principal components approach and Johansen procedure

In this section, we present the dynamic principal component analysis and Johansen procedure used in this paper to identify the number of common shocks in international output and to test for cointegration and convergence.

Let \( Y_t \) denote the \((n \times 1)\) vector of individual output levels (log real per capita GDP for \( n \) economies). Assume that the individual elements of the output vector are integrated of order one. It is then natural to write a multivariate Wold representation of outputs as

\[ \Delta Y_t = \mu + B(L)\zeta_t, \]

where \( B(L) \) is a \((n \times n)\) polynomial matrix and \( \zeta_t \) is an \((n \times 1)\) vector of white noises and \( \mu \) is an \((n \times 1)\) vector of constant terms. The spectral density of \( \Delta Y_t \) is

\[ f_{\Delta Y}(e^{-\omega}) = B(e^{-\omega})\sum \sigma \sigma^T, \]

where \( \sigma \) indicates the frequency and \( \sum \sigma \sigma^T \) the variance-covariance matrix of innovations. The rank of the spectral density matrix is smaller equal to the dimension of \( \zeta_t \). Test of the number of common shocks requires to compute the number of principal components of \( f_{\Delta Y}(e^{-\omega}) \), that explain the most of the variance of \( \Delta Y_t \) at each frequency. We can ask how many principal components explain at least the 0.95% of total variance of \( \Delta Y_t \). If \( p \) components are sufficient we conclude that there are \( p \) common elements in the vector of international outputs.

It is possible to decompose \( f_{\Delta Y}(e^{-\omega}) \) in the following way:

\[ f_{\Delta Y}(\omega) = P(\omega)D(\omega)P(\omega)^T, \]

where \( D(\omega) \) is a diagonal matrix with \( r \) non-zero elements on the principal diagonal (the eigenvalues): \([\lambda_1(\omega), \ldots, \lambda_r(\omega)]\) and \( \text{rank}(D(\omega)) = \text{rank}(f(\omega)) \). Ordering the eigenvalues from the largest to the smaller it is possible to compute the variance ratio between the variance of the \( p \) principal components and the variance of all the components (\( n \)):

\[ \sum \lambda_i(\omega) \Delta \omega \sum \lambda_i(\omega)\Delta \omega, \quad p = 1, \ldots, n. \]

At zero frequency, the number of principal components (equal to the rank of the spectral density matrix at zero frequency) gives indication for the number of common permanent components or common stochastic trends in international output.

If idiosyncratic elements dominate for every country, then we would expect to find \( n \) common permanent parts (or equivalently \( n \) common trends) for \( n \) countries, if countries converge we expect to find the necessary condition for convergence respected, that is one common permanent component (or equivalently one common trend) that explains most of the total variance of \( \Delta Y_t \).

For Johansen tests for cointegration (Johansen 1991), we impose additional structure on the output series. Assume that representation (1) can be rewritten as the following vector autoregressive representation

\[ \Gamma(L)\Delta Y_t = \mu + \Pi Y_{t-1} + \varepsilon_t, \]

where the \( \Pi \) matrix represents long run relations across output levels and the polynomial matrix \( \Gamma(L) \) represents the short run impact of shocks on the system. Tests for cointegration concerns the rank, \( r \), of the long run matrix \( \Pi \). If \( r = 0 \), we have no-cointegration and \( n \) idiosyncratic trends in international output. If \( \Pi \) is a full rank matrix, that is \( r = n \), we have \( n \) cointegrating vectors and no common trends, finally if \( \Pi \) is a reduced rank matrix, \( r < n \), we have \( r \) cointegrating vectors and \((n - r)\) common trends.

Convergence, as defined by Bernard and Durlauf (1991, 1995), requires identical common long run trends in international output. Representation (2) under the hypothesis of \((n - 1)\) cointegration vectors of the form \((1, -1)\) implies both identical stochastic trend and identical deterministic component in international output that is convergence.

\[ \text{See} \] Brillinger (1981) and Phillips and Ouliaris (1988). For application of principal components methods involving the analysis at zero frequency, see Bernard and Durlauf (1991, 1995) and at any frequencies see Forni and Reichlin 1995.

\[ \text{Cointegrating vectors can be thought of as representing constraints that an economic system imposes on the movements of the outputs in the long run within the n-dimensional space. More cointegrating vectors there are, the more stable the system. In a system with no common trends, so it is stationary, the vector} \]
3. Empirical results

The annual data used relate to log per capita real output (in US $ at 1985 price levels) of 21 OECD economies, 1960-1992, and are taken from OECD Annual National Account: Main Aggregates. Results (not reported) from Dickey- Fuller tests, with autoregressive corrections of order 0 through 4 in regressions with and without trend, suggest that the 21 series of real outputs utilized in this paper are reasonably I(1) with drift in levels. In testing for co-movements and convergence, we use separate groupings of economies: all 21 countries together, the main ten European countries, the original six European Community countries, the G5 and G7 group of main industrialised countries.

Starting with the case of 21 economies, Figure 1 reports the ratios:

\[
\frac{\lambda_1(\omega)}{\sum_{i=1}^{21} \lambda_i(\omega)} \frac{\lambda_2(\omega)}{\sum_{i=1}^{21} \lambda_i(\omega)} \ldots \frac{\lambda_n(\omega)}{\sum_{i=1}^{21} \lambda_i(\omega)} = 1 \text{ at each frequency } \omega, \text{ for } r - n = 21.
\]

The spectra have been estimated using Bartlett's window with lag window size equal to ten. The variance of \( \Delta Y_{11} \), explained by the first principal component at zero frequency, where it is possible to capture permanent components in the series, is slightly inferior to 0.80 of total variance. It suggests that there is one very important common shock presumably a technological shock. However, to capture at least 0.95 of total variance of \( \Delta Y_{11} \) at zero frequency are necessary at least three components. This means that there are at least three common permanents shocks (or common stochastic trends) that explain the long run international output movements.

of outputs never wavers too far from its steady state equilibrium value. If there is one common trend and \((n-1)\) cointegrating vectors, there are only \((n-1)\) directions where the variance is finite and one in which it is infinite. In other words the system converges to a unique long run equilibrium. The fewer the number of cointegrating vectors, the less constrained is the long run relationship across outputs. If there are no cointegrating vectors, the outputs are free to wander anywhere, they are unbounded.

The countries are: Austria (AUT), Belgium (BEL), Canada (CAN), Denmark (DNK), Finland (FIN), France (FRA), Western Germany (DEU), Greece (GRC), Iceland (ISL), Ireland (IRL), Italy (ITA), Japan (JPN), Luxembourg (LUX), Netherlands (NLD), Norway (NOR), Portugal (PRT), Spain (ESP), Sweden (SWE), Switzerland (CHE), United Kingdom (GBR), United States (USA).


This outcome suggests that the cointegration of groups of economies is more probable than cointegration two by two (which is equivalent to testing the presence of just one common trend) and that the necessary condition to have global convergence in all the OECD economies does not hold. However, there are some economies that are linked by common long run movements, suggesting the possibility to have convergence clubs.

We move now to test this hypothesis.

Figures 2, 3 and 4 summarise the dynamic principal components analysis for the sub-aggregate of ten European countries (10 EC), six original European Community countries (6 ECC) and G7 group. In each of these cases, two components at zero frequency are able to capture 0.95 of the total variance suggesting that there are at least two common trends, representing presumably a technological component and a non technological component in GDP. In particular, the first component (always at the zero frequency) is able to capture 0.85 of total variance of the 6 ECC outputs and nearly 0.90 of the G7 outputs suggesting weak evidence about the interpretation of persistence (or unit root) in output as technology based and preventing outputs to diverge too much.

More interesting is the case of G5, see Figure 5, where the first principal component capture more than 0.90 of total variance of \( \Delta Y_{11} \), suggesting that it is possible to find convergence for this group of economies. Relying on this evidence we will perform a direct test of convergence using Johansen tests.

Focusing on the first principal component, that tracks the importance of short run and long run co-movements in international output, we see, in all the figures, that there
is a peak at zero frequency explaining most of the total variance (at least 0.80 percent). This outcome suggests that outputs comove more in the long run than at the business cycle frequencies.

% of explained variance of $\Delta Y_{\text{EC.}j}$

Figure 1 Variance of $\Delta Y_{\text{EC.}j}$ explained by 21 principal components at different frequencies

% of explained variance of $\Delta Y_{\text{EC.}j}$

Figure 2 Variance of $\Delta Y_{\text{EC.}j}$ explained by 10 principal components at different frequencies

% of explained variance of $\Delta Y_{\text{EC.}j}$

Figure 3 Variance of $\Delta Y_{\text{EC.}j}$ explained by 6 principal components at different frequencies
In Tables 1 and 2 we present the results from the Johansen trace and maximum eigenvalue statistics on convergence and common trends for dimensionally not too large cross section of countries: the 6 ECC, the G5 and G7 group. The lag length of the models has been chosen using standard diagnostic testing procedure and information criterion tests, when necessary we insert dummies to correct the normality of residuals\textsuperscript{16}.

The convergence hypothesis fails in all samples of countries. Having failed to find convergence, or an identical long run trend, we turn to the test for the number of common trends. In all the cases, we reject the null hypothesis that there are four or more distinct trends. Trace statistics suggest a number of common trends lower than the number revealed by maximum eigenvalue tests: we can not reject the presence of two common trends in the G5 group of countries and one in the six ECC and in the G7 group. These results match the previous outcomes derived using dynamical principal component method.

Table 1 Multivariate Test for Convergence

<table>
<thead>
<tr>
<th>Countries</th>
<th>Tests for Convergence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Six ECC+</td>
<td>$\chi^2(5) = 53.77^*$</td>
</tr>
<tr>
<td>G5++</td>
<td>$\chi^2(4) = 30.35^*$</td>
</tr>
<tr>
<td>G7++</td>
<td>$\chi^2(6) = 47.29^*$</td>
</tr>
</tbody>
</table>

Notes: VAR lag length: $n = 3$, \textsuperscript{++} = 2 plus two dummies D74, D82.
Test for Convergence distributed as a $\chi^2(n-1)$ where $n$ is the number of countries.
\* rejected at 5% critical value.

\textsuperscript{16}Diagnostic tests for autocorrelation, normality and heteroscedasticity and information criterion tests have been performed using PC-FIML program. For references and a discussion of these tests, see Doornik and Hendry (1994).
Table 2 Multivariate Tests for Cointegration or Common Stochastic Trends

<table>
<thead>
<tr>
<th>Trends</th>
<th>Max Eig</th>
<th>Trace</th>
<th>Max Eig</th>
<th>Trace</th>
<th>Max Eig</th>
<th>Trace</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 6</td>
<td>88.44</td>
<td>226</td>
<td>47.93</td>
<td>137.6</td>
<td>53.49</td>
<td></td>
</tr>
<tr>
<td>&gt; 5</td>
<td>81.48</td>
<td>182.9</td>
<td>47.93</td>
<td>137.6</td>
<td>53.49</td>
<td></td>
</tr>
<tr>
<td>&gt; 4</td>
<td>40.43</td>
<td>101.4</td>
<td>42.08</td>
<td>106.7</td>
<td>36.17</td>
<td>89.66</td>
</tr>
<tr>
<td>&gt; 3</td>
<td>28.27*</td>
<td>61</td>
<td>30.4*</td>
<td>64.67</td>
<td>24.93*</td>
<td>53.49</td>
</tr>
<tr>
<td>&gt; 2</td>
<td>19.29</td>
<td>32.73</td>
<td>20.83</td>
<td>34.27*</td>
<td>16.9</td>
<td>28.56</td>
</tr>
<tr>
<td>&gt; 1</td>
<td>12</td>
<td>13.44*</td>
<td>13.38</td>
<td>13.44</td>
<td>11.61</td>
<td>11.67*</td>
</tr>
<tr>
<td>&gt; 0</td>
<td>1.44</td>
<td>1.44</td>
<td>0.056</td>
<td>0.056</td>
<td>0.056</td>
<td>0.056</td>
</tr>
</tbody>
</table>

Notes: See notes in Table 1

Max Eig: maximum eigenvalue test for cointegration = $T \ln(1 - \lambda_m)$

Trace: trace test for cointegration = $\sum \ln(1 - \lambda_i)$

Critical Values in Osterwald-Lenum (1992). * Rejected at 5%

4. Conclusion

We find little empirical evidence of convergence in OECD outputs. There are at least three common components at zero frequency (three common trends) across the 21 OECD economies and at least two common permanent components across the 10 EC, 6 ECC and G7 group. These outcomes reject the purely technological interpretation of unit root, even if the first common permanent component captures a large part of total variance in international output fluctuations at zero frequency and probably a so large effect may be roughly interpreted as not fully technological transmission, that is permanent shocks only partially migrate across countries. In the case of the G5 group, there is some evidence about the interpretation of the unit root in output technology based, because the first permanent component is able to capture more than the 0.90 of total variance. The economic growth in advanced industrialised economies cannot be reduced exclusively to idiosyncratic country-specific factors. A relatively small set of common components interact with individual economic characteristics to determine growth rates and, given the high weight of the first permanent shock, the long run dynamics prevent output levels from diverging by too much, while at the cycle frequencies there is little evidence of important co-movements. This latter outcome suggests that idiosyncratic transitory shocks are important and stabilisation policy response is a foundational problem for designing effective monetary policy institutions who may want to take account of this problem.

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