Is an Aggregate Error Correction Model Representative of Disaggregate Behaviours? 
An example

by

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Abstract: This paper tests for the existence of cointegration between employment, real wage and output both for the main sectors of Italian economy (agriculture, industry, services and public administration) and for the aggregate series. The Johansen (1991a) estimation procedure is utilised on the period from 1950 to 1990. The tested macro cointegration is strong, while the sectoral cointegration is weaker owing to the non-uniformity of the cointegration tests. This could be due to the presence of I(1) non-common components in the disaggregated series that make the sectoral cointegration difficult, but nearly cancel out when the aggregation process is performed. Moreover, the macro dynamics and the micro one are very different, in particular, the aggregation induces a more dynamically complex macro relation. Therefore, even though an Error Correction Mechanism is implicit in the aggregate result, such a dynamic model has not an empirical microbackground and it does not seem to be the outcome of maximisation under adjustment costs, as in the customary interpretation of the aggregate ECM's.

1. Introduction

In the last fifteen years the Italian economy, like the majority of the European countries, has experienced a steady increase in both employment and unemployment. In this economic context there has been a proliferation of estimates of labour-demand functions both in a pure neoclassical specification in the relative prices (Jenkinson 1986, Lucifora 1987, Symons 1981, Symons and Layard 1984) and in a mixed specification in which output becomes a key determinant of the movement of employment (Beckerman and Jenkinson 1986, Briscoe and Wilson 1991, Chiarini and Placidi 1991, Layard and Nickell 1978, Modigliani, Rossi and Padoa-Schioppa 1986, Parigi and Urga 1993, Prosperetti and Urga 1989, Smith and Hagan 1993).

*University of Modena, Department of Economics, Via Berengario 51, 41100 Modena, Tel.059-417826, Fax.059-417948, E-Mail: Pistoressi@Merlino.Unimo.It. My thanks are due to Jeremy Smith, my supervisor at the Department of Economics of the Warwick University, for constructive comments on the earlier draft. This revised version of the paper has benefited from the suggestions by Marcello D'Amato, Mario Forni, Marco Lippi.
In general, the first specification is derived in a framework of competitive market and the second in a context of imperfect competition. However, for different functional forms of the production function, both under monopolistic competition and perfect competition, it is possible to express marginal productivity of labour in terms of average productivity and hence of output level. Thus the mixed specification is compatible both with a market clearing hypothesis and with a disequilibrium hypothesis in which the representative firms can be constrained by demand or labour costs.

Both specifications have often been estimated by imposing a priori theoretical constraints about the determinants of the employment and on the structure of the adjustment that are rarely tested. On the contrary, cointegration analysis enables testing of the long run relationships implied by economic theory and lays the foundations for a correct dynamic analysis. The link between cointegration and dynamic specification is made thanks to the Representation Theorem (Granger 1981, Engle and Granger 1987): error correcting behaviour on the part of economic agents will induce co-integrating relationships among the corresponding time series and vice versa.

This result leads to a re-evaluation of the econometric modelling based on the adjustment costs (Davidson et al. 1978, Hendry 1980, Hendry et al. 1980) that emphasises the use of an Error Correction Mechanism to achieve the short run consistency of an economic relation with the steady state equilibrium of the variables in it. One important result of this literature concerns the microfoundation of the Error Correction Models. Nickell (1985) and Salmon (1982) demonstrate how an Error Correction Model is consistent with optimising behaviour on the part of economic agents. This is also true for the labour demand derived as a two step maximisation problem of a firm that optimises a quadratic loss function to adequate the current employment to the optimal one (Nickell 1986). This micro result is in general tested at macro level under the assumption that agents are equal and that the information on the shape of the micro relationships can be transferred to the macro equation. If this were true we would be able to find at any aggregation level the same information as within the macro relationship.
In this work the Italian data show that the result of cointegration among aggregate employment, output and product real wage, interpreted as labour demand as an ECM form\(^1\), can be accepted on a more solid ground as compared with the results obtained with disaggregated data relative to agriculture, industry, services and public administration. Moreover, the data also show that the macro model has a more complex dynamics. For these reasons the ECM, implicit in the aggregate result of cointegration, does not seem to be the outcome of maximising behaviours of a representative firm, as in the customary interpretation of the aggregate ECM's. Instead, such a dynamic representation seems to emerge as a spurious result of the aggregation process.

The structure of the work is as follows. Section 2 discusses the data and the behaviour of the series. In Section 3 outlines the method of estimation that is based on the multivariate cointegration procedure as in Johansen (1988, 1991a). The estimation of the cointegration space in each sector, in the aggregate and some structural hypotheses of economic interest within this space are presented. The sectoral variables result badly-cointegrated; the reason should be that the deterministic trend contained in the series does not cancel out in the cointegration relationship. Hence attention is paid to the inclusion of a time trend in the cointegration vectors in order to take into account the possibility of "stochastic" cointegration (cointegration up to a deterministic trend). Attention is also paid to the weakly exogeneity tests on the estimated adjustment coefficients in order to establish the existence of a labour demand equation with an ECM form. In this section the comparison of macro and micro results is also performed and some theoretical results about aggregation and cointegration are presented. Moreover, an explanation is suggested in order to illustrate how badly-cointegrated or non-cointegrated sectoral variables can generate cointegrated variables by aggregation. Section 4 concludes.

\(^1\)While this specification is common for the analysis of the labour demand in industry, it is not so for services and agriculture, where the movements of employment could follow simpler schemes. In services the movements of employment could be related to the anti-cyclic adjustment, while in agriculture they could be negatively correlated to the movements of the extra-agricultural employment and to an autonomous exodus from this sector. In public administration the labour input is not generally determined by an optimisation problem, but jobs are frequently created for political rather than economic reasons. However, these simplifications can be neglected here since the purpose of this work is not to seek for a better labour demand specification, but rather to offer an empirical contribution to the issue of aggregation.
2. Data, behaviour of the series and variable definitions

In this work a new Italian data set (Rossi N, Sorgato A. and Toniolo G. 1992) is used. These annual data consist of a statistical reconstruction of the Italian accounts between 1890 and 1990 that is consistent with the revision of the national accounts recently undertaken by the Italian Statistical Office (ISTAT 1989). The data set contains the main supply and demand components of GNP, both at current and constant prices, an appraisal of capital stock, a reconstruction of the hours worked by fully-employed male equivalents (homogeneous labour units) in the private and public sector and the hourly wage rate at current prices, both gross and net of the social security contribution.

The series utilised are disaggregated into the main branches of the economy: agriculture (including forestry and fishing), industries (mining and quarrying, manufacturing, electricity gas and water, construction), services (trade, transport and communication, finance and insurance, miscellaneous services, housing) and public administration. For each of these sectors there are series for gross domestic product at constant (1985) factor prices as a measure of the output (demand), the deflator of GNP as an index of output price, the gross nominal wage and the gross real wage (producer wage) constructed as nominal wage over output price as a measure of the cost of labour for the firm, the total hours worked as employment. The capital formation and export series are not used because they are not disaggregated across sectors.

By using total hours worked instead of employment, we are implicitly assuming that both the variables have the same returns and the same "adjustment mechanism". The first assumption has been made quite extensively in the literature (Dhrymes 1969 and McCarthy 1975) even if there is some evidence of different elasticity with respect to output. A greater elasticity for the hours is normally estimated (Feldstein 1967). The second assumption is more restrictive: in fact in Italy there is a wide flexibility in adjusting the hours worked and high rigidity in employment because of the presence of organised unions and a more labour-protectionist legislation with respect to other European countries.
The period of estimation used in this analysis is from 1950 to 1990 as the employment series contains a lot of omitted observations for the period 1893 to 1910 and during the two world wars. All the variables are in logarithms.

On inspection, the first differences of all the variables seem to exhibit a non-zero drift. The long run trend for agricultural employment is downwards. This suggests that a redistribution of workers from the agricultural sector to non-agricultural activities happened in the period under consideration. This redistribution is attributable in particular to technological progress. The increase in productivity is due to the use of less labour and more fixed capital because of an increase in agricultural wages with respect to prices of machines, involving labour saving (Ricardian assumption). The price of machines is related to technological innovation in the industrial sector. It is important to note that the real wage in the agricultural sector increases very fast with respect to the other sectors, thus reducing the gap with the other wages (Fig.3). This fact could confirm the Ricardian substitution effect. Another reason for the negative trend in agricultural employment could be the increase in the labour demand in the other sectors due to the increase in relative outputs. The output growth in agriculture is substantially inferior to that in other sectors (Fig.2), in particular the change in agricultural employment is inversely correlated with industrial production. So the reduction in the agricultural labour supply may be due to the higher wages in the non agricultural sectors. Both the effects contribute to determine a marked exodus from the agricultural sector.

The basic trend in industrial employment is upward, but it tends to stabilise in the 1970s until the 1980s, when it becomes downward as in many European countries. Industrial employment varies over the cycle, with the movements of output. It shows more oscillation than the other employment series in accordance with the movements of the relative output. In particular, in the 1980s the decrease in industrial employment could be due both to the decrease in the rate of growth of income and to the high real wages. Employment in the private-sector services has an upward trend with slope larger than that of the industrial sector, principally during the 1970s. In the 1980s when industrial employment is decreasing, employment in services
and public administration is once more increasing through less than in the 1970s (Fig.1). This phenomenon has been defined "dynamic transfer" between industry and services and probably is due to the increase in the services for industrial firms as indicated by Momigliano and Siniscalco (1982). Actually the series for services output jumps less than the series for industrial output in the early 1980s and the gap between industrial production and output in the services increases markedly throughout the 1980s. However, this phenomenon could also be explained by the influence of relative real wages. In fact, industrial real wages increase when employment is decreasing, while the stability of employment in services is accompanied by reduction in the relative real wages (Fig 1.3).

The trend in public administration employment has been upward for a long time. During the post-war period public employment has grown by over 2% a year, though in recent years this trend has tended to slow down, probably because of the budget deficit. This positive trend is due, on the one hand, to the constant increase in the functions performed by the public sectors, while, on the other hand, increase in public employment may be due to political reasons.

2.1 Variable definitions

- EA: agricultural employment, EI: industrial employment, ES: employment in services, EP: employment in public administration;
- YA: agricultural real output, YI: industrial real output,YS: real output in services, YP: real output in public administration;
- WRA: real wage in agriculture, WRI: real wage in industry, WRS: real wage in services, WRP: real wage in public administration;
- ET: total employment, YT: total output, W: representative real wage for the economy;
- L indicates the logarithm of the series and Δ is the differencing operator.
Fig. 1 Sectoral employment

Fig. 2 Sectoral output

Fig. 3 Sectoral real wage
3. Analysing the long run relationships

In this section the long run relationship between employment, output and real wage in each sectors is analysed. The concept of cointegration provides a framework for testing and estimating long run equilibrium among these non-stationary variables. Such variables are called cointegrated if they are individually I(1), but there exists a linear combination of them that is stationary, I(0). So these variables do not tend to wander but move together in the long run.

The cointegration analysis performed here is based on the assumption that all the variables are integrated of order one. A series is integrated of order one if contains only one unit root. All the series utilised show an evident profile of non stationarity. So they could contain one or more unit roots. Consequently, the first step is to test the order of integration of the variables. The Dickey-Fuller tests and the Phillips-Perron tests\(^2\) are conducted to verify the presence of unit roots. The testing strategy, adopted here, is that proposed by Perron (1988). It consists of a sequence of t-tests and F-tests starting with a general autoregressive specification with trend and drift and testing down the presence of unit root separately from and jointly with the significance of the drift and the trend until a more parsimonious model is accepted.

The outcome from the unit root tests is broadly consistent with the visual inspection of the first difference of all the variables. Both the disaggregated and the aggregate series seem to be integrated process, I(1). The Dickey-Fuller and the Phillips-Perron tests are available on request.

The estimation procedure proposed by Johansen (1988, 1991a) and Johansen and Juselius (1990)\(^3\) consists of a maximum likelihood estimation of a VAR, reparametrized in an ECM-form, which contains \(n\) variables, all of which I(1), if some of the series in the system are integrated of a higher order than one, e.g. I(2), then a more complicated estimation procedure is required to analyse the problem (Johansen 1991b, 1992). The ECM reparametrization is

\(^2\)The Phillips Perron procedure modifies the statistics through a non-parametric correction of the covariance matrix of the Dickey Fuller regression residuals to take into account of autocorrelation or heteroscedasticity. The computer program used to perform the tests is Shazam-Econometrics Computer Program-McGraw-Hill.

\(^3\)The maximum likelihood estimation has been treated in Johansen (1988) for a VAR without a constant and by Johansen (1991a) for a VAR with a constant.
1) \[ \Delta y_t = \mu + \sum_{i=1}^{k-1} \Gamma_i \Delta y_{t-i} + \Pi \Delta y_{t-k} + u_t \]

where \( \Gamma_i \) and \( \Pi \) are matrices of unknown parameters, \( \Delta y_t \) and \( \Delta y_{t-1} \) are vectors of I(0) variables, while the \( \Delta y_{t-k} \) is a vector of I(1) variables. \( \mu \) is a drift parameter, capturing the role of technical progress and \( u_t \) is a vector of disturbances. The system is balanced, in terms of degree of integration, only if \( \Pi = 0 \), in this case the variables are not cointegrated, or if the long run parameters of \( \Pi \) are such that \( \Pi \Delta y_{t-k} \) is also I(0). The latter case implies that the variables are cointegrated of order \( n-r \), where \( r \) is the number of cointegration vectors. If the variables are cointegrated the \( \Pi \)-matrix can be decomposed in the following way:

\[ \Pi = \alpha \beta' \]

where \( \beta \) is the matrix of the cointegration vectors and \( \alpha \) is the weight of the cointegration vectors in each equation of the VAR. So a low \( \alpha \) indicates slow adjustment towards the estimated equilibrium state and a high coefficient indicates rapid adjustment.

Before applying the Johansen procedure, it is necessary to determine the lag length of the VAR that ensures residuals approximately white noise normal. Estimating the equation (1) the data suggest that a VAR with two lags is sufficient to make the residuals vector non-autocorrelated and normal in agriculture, services and public administration. A problem of non-normality arises in industry (Table 1). 

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4The computer program used to perform the cointegration analysis is PcFiml 8.0 - Interactive Econometric Modelling of Dynamic System- International Thomson Publishing.
TABLE 1
Diagnostic checking: mis-specification tests

<table>
<thead>
<tr>
<th>Equations</th>
<th>A p-value</th>
<th>ARCH p-value</th>
<th>N p-value</th>
<th>VA p-value</th>
<th>VARCH p-value</th>
<th>VN p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECM1: lea</td>
<td>0.62</td>
<td>0.46</td>
<td>0.96</td>
<td>0.94</td>
<td>0.81</td>
<td>0.97</td>
</tr>
<tr>
<td>ECM2: lya</td>
<td>0.17</td>
<td>0.40</td>
<td>0.69</td>
<td>0.94</td>
<td>0.81</td>
<td>0.97</td>
</tr>
<tr>
<td>ECM3: lwea</td>
<td>0.94</td>
<td>0.85</td>
<td>0.96</td>
<td>0.94</td>
<td>0.81</td>
<td>0.97</td>
</tr>
<tr>
<td>ECM1: lei</td>
<td>0.42</td>
<td>0.60</td>
<td>0.15</td>
<td>0.06</td>
<td>0.94</td>
<td>0.01</td>
</tr>
<tr>
<td>ECM2: lyi</td>
<td>0.54</td>
<td>0.27</td>
<td>0.03</td>
<td>0.06</td>
<td>0.94</td>
<td>0.01</td>
</tr>
<tr>
<td>ECM3: lwri</td>
<td>0.16</td>
<td>0.93</td>
<td>0.17</td>
<td>0.06</td>
<td>0.94</td>
<td>0.01</td>
</tr>
<tr>
<td>ECM1: les</td>
<td>0.15</td>
<td>0.71</td>
<td>0.006</td>
<td>0.07</td>
<td>0.003</td>
<td>0.17</td>
</tr>
<tr>
<td>ECM2: lys</td>
<td>0.04</td>
<td>0.32</td>
<td>0.02</td>
<td>0.07</td>
<td>0.003</td>
<td>0.17</td>
</tr>
<tr>
<td>ECM3: lwrs</td>
<td>0.005</td>
<td>0.12</td>
<td>0.25</td>
<td>0.07</td>
<td>0.003</td>
<td>0.17</td>
</tr>
<tr>
<td>ECM1: lep</td>
<td>0.34</td>
<td>0.40</td>
<td>0.08</td>
<td>0.55</td>
<td>0.84</td>
<td>0.11</td>
</tr>
<tr>
<td>ECM2: lyp</td>
<td>0.09</td>
<td>0.03</td>
<td>0.42</td>
<td>0.55</td>
<td>0.84</td>
<td>0.11</td>
</tr>
<tr>
<td>ECM3: lwrp</td>
<td>0.24</td>
<td>0.74</td>
<td>0.31</td>
<td>0.55</td>
<td>0.84</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Note: A: test for serial correlation, F(2, 29); VA: vector test for serial correlation, F(18, 65)
ARCH: Autoregressive conditional heteroscedasticity, F(1, 29); VARCH: vector test, F(72, 76)
N: test for normality, χ²(2); VN: vector test, χ²(6)
p-value: probability of a type I error

The likelihood ratio test statistics for the determination of the cointegration rank of each Π of the VARs in each sector are given in Table 2. It also reports the eigenvalues (denoted λ) and the LR-tests both for the λₘₐₓ statistics, calculated as −TN(1 − λᵢ) for the eigenvalue of the r th cointegration vector, and for the Trace statistics, calculated as −TN(1 − λᵢ). The 95% and 90% quantiles for testing the order of r from the Osterwald-Lenum (1992) tables are reported.

If we consider λₘₐₓ test statistics, the hypothesis of one cointegration vector is accepted in agriculture at 95%, while in public administration the same hypothesis is accepted at 90%. The Trace statistics (trace test is more reliable when the eigenvalues are similar) leads to accept one cointegration vector in industry at 90% and public administration at 95%, while in the services we accept one cointegration vector at the boundary of 90%. In general the tests do not give a clear-cut result because of their low power, when the cointegration relation is close to the non-stationary boundary. In other words, the result may be contradictory when the speed of adjustment to the hypothetical long run equilibrium is slow. Hence, in agriculture and public
administration the cointegration is stronger than in industry and services because both the tests enable us to accept at least one cointegration vector.

**TABLE 2**
Testing the rank of Π matrix: agriculture, industry, services, public administration.

<table>
<thead>
<tr>
<th>H_0</th>
<th>H_1</th>
<th>λ_{max}</th>
<th>95%</th>
<th>90%</th>
<th>Trace</th>
<th>95%</th>
<th>90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>r = 0</td>
<td>r = 1, r ≥ 1</td>
<td>24.2109</td>
<td>20.9670</td>
<td>18.5980</td>
<td>44.4452</td>
<td>29.6800</td>
<td>26.7850</td>
</tr>
<tr>
<td>r ≤ 1</td>
<td>r = 2, r ≥ 2</td>
<td>12.6113</td>
<td>14.0690</td>
<td>12.0710</td>
<td>20.2343</td>
<td>15.4100</td>
<td>13.3250</td>
</tr>
<tr>
<td>r ≤ 2</td>
<td>r = 3, r ≥ 3</td>
<td>7.6230</td>
<td>3.7620</td>
<td>2.6870</td>
<td>7.6230</td>
<td>3.7620</td>
<td>2.6870</td>
</tr>
</tbody>
</table>

| Industry (λ_1 = 0.33989, λ_2 = 0.25510, λ_3 = 0.042876) |
|-----|---------|-----|-----|------|-----|
| r = 0 | r = 1, r ≥ 1 | 15.7833 | 20.9670 | 18.5980 | 28.6398 | 29.6800 | 26.7850 |
| r ≤ 1 | r = 2, r ≥ 2 | 11.1912 | 14.0690 | 12.0710 | 12.8565 | 15.4100 | 13.3250 |
| r ≤ 2 | r = 3, r ≥ 3 | 1.6652 | 3.7620 | 2.6870 | 1.6652 | 3.7620 | 2.6870 |

| Services (λ_1 = 0.37025, λ_2 = 0.19753, λ_3 = 0.017297) |
|-----|---------|-----|-----|------|-----|
| r = 0 | r = 1, r ≥ 1 | 17.5723 | 20.9670 | 18.5980 | 26.5075 | 29.6800 | 26.7850 |
| r ≤ 1 | r = 2, r ≥ 2 | 8.3622 | 14.0690 | 12.0710 | 9.0252 | 15.4100 | 13.3250 |
| r ≤ 2 | r = 3, r ≥ 3 | 0.6630 | 3.7620 | 2.6870 | 0.6630 | 3.7620 | 2.6870 |

| Public Administration (λ_1 = 0.40893, λ_2 = 0.25081, λ_3 = 0.042710) |
|-----|---------|-----|-----|------|-----|
| r = 0 | r = 1, r ≥ 1 | 19.9812 | 20.9670 | 18.5980 | 32.6129 | 29.6800 | 26.7850 |
| r ≤ 1 | r = 2, r ≥ 2 | 10.9731 | 14.0690 | 12.0710 | 12.6317 | 15.4100 | 13.3250 |
| r ≤ 2 | r = 3, r ≥ 3 | 1.6586 | 3.7620 | 2.6870 | 1.6586 | 3.7620 | 2.6870 |

Critical values: Osterwald-Lenum 1992
H_1: r = 0, 1... is the alternative hypothesis for the λ_{max} test
H_1: r ≥ 0, 1... is the alternative hypothesis for the Trace test.

Table 3 reports the stationary cointegration relations, that can be derived from the Trace tests, and the corresponding weights (α vectors). All the cointegration vectors reflect the positive relationship between employment and output and a negative link between employment and real wage. The positive coefficient of the real wage in the public sector seems not significantly different from zero. We will test this hypothesis later.
The sign on output in the agricultural cointegration vector is positive even if agricultural employment has a negative trend while the output has a positive trend. However, from the plot of the series it is evident that the output in agriculture increases less than output in the other sectors, whereas the rapid growth of the agricultural real wage markedly reduces the gap with the other wages. Hence the negative trend in agricultural employment is partially captured, in the cointegration vector, by the increase of the wage with respect to output. Moreover, the principal factor accounting for the negative trend in employment is the growth of productivity that is well captured by the constant in the VAR. This is another fact explaining the estimated positive elasticity between employment and output in the long run relationship.

The estimated cointegration vectors presented above are obtained from the estimation of equation (1) under the assumption that the deterministic trend contained in the series, captured by the constant of the VAR, cancel out in the cointegration relationship. If this is not the case a deterministic trend should have been restricted to enter in the cointegration vector (Campbell and Perron 1991, Johansen 1991c, 1991d, Ogaki and Park 1990). In our case this experiment may be required because of the weakness of the sectoral cointegration results due to the non-uniformity of the tests. This non-uniformity would suggest that the series are cointegrated up to a deterministic trend (stochastic cointegration): in other words, there is some linear growth, which our model cannot totally explain. We performed the experiment in the data but the inclusion of the trend in the cointegration vectors does not lead to a significantly different from zero coefficient of the trend and does not improve the result of cointegration.
TABLE 3
Estimated cointegration vector and corresponding weight

<table>
<thead>
<tr>
<th>Variables</th>
<th>( \beta_1(A) )</th>
<th>( \beta_1(I) )</th>
<th>( \beta_1(S) )</th>
<th>( \beta_1(P) )</th>
<th>( \alpha_1(A) )</th>
<th>( \alpha_1(I) )</th>
<th>( \alpha_1(S) )</th>
<th>( \alpha_1(P) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LE</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-0.23</td>
<td>0.070</td>
<td>0.075</td>
<td>0.077</td>
</tr>
<tr>
<td>LY</td>
<td>0.53</td>
<td>1.36</td>
<td>0.81</td>
<td>1.11</td>
<td>-0.12</td>
<td>0.25</td>
<td>0.08</td>
<td>-0.22</td>
</tr>
<tr>
<td>LW</td>
<td>-0.60</td>
<td>-1.39</td>
<td>-0.77</td>
<td>0.03</td>
<td>0.58</td>
<td>0.32</td>
<td>0.27</td>
<td>-0.18</td>
</tr>
</tbody>
</table>

Note: \( \beta(A) \) : represents the cointegration vector in agriculture
\( \beta(I) \) : represents the cointegration vector in industry
\( \beta(S) \) : represents the cointegration vector in services
\( \beta(P) \) : represents the cointegration vector in public administration
\( \alpha(.) \) : represents the corresponding weight

3.1 Weak exogeneity, long run exclusion and some structural hypotheses

Having assumed one long run relationship in each sector, we may test restrictions on the cointegration vector and on the relative adjustment vector \( (\alpha) \) by the likelihood ratio test in order to test the null hypothesis that the restriction is valid. The likelihood ratio test is

\[
I.R(\alpha, \beta^\prime) = -T \sum_{i=1}^{r} \ln \left( \frac{(1 - \lambda^R_i)}{(1 - \lambda^U_i)} \right)
\]

where \( r \) is the order of cointegration vector established through the Trace tests and/or the \( \lambda_{\text{max}} \) tests, \( \lambda^R \) represents the estimated characteristic roots from the restricted model and \( \lambda^U \) represents the roots from the unrestricted model. Under the null that the restriction is valid, the test is asymptotically distributed as a \( \chi^2 \) \( (rs) \), where \( s \) is the number of restrictions imposed on the cointegration vectors.

The concept of weak exogeneity can be utilised to motivate the reduction of the dimension of the VAR. The condition for the variables to be weakly exogenous for \( \beta \) is that the associated coefficient of adjustment, \( \alpha \), be equal to zero, implying that the long run parameters \( \beta \) can be estimated efficiently without the equation for the weakly exogenous variables. In our case the coefficients of the \( \alpha \) vectors are small in the cases of the labour equation in the industrial sector, in services and the public sector and in the output equation in the services.
The weak exogeneity tests confirm that all these coefficients are not significantly different from zero (Table 4). In fact, the tests that $\alpha_{1,1}(l) = 0$, $\alpha_{1,1}(S) = 0$, $\alpha_{1,1}(P) = 0$ yield respectively a likelihood ratio test LR, which compared with the 5% critical value, $\chi^2(1) = 3.84$, enables us to accept the restriction in industry, services and public administration. This means that the labour demand equation, in these sectors, does not adjust to the long run target, i.e. employment enters as an independent variable in the other equation of VAR. Instead, the employment equation in agriculture is not weakly exogenous, in fact it is easy to reject the null $\alpha_{1,1}(A) = 0$. For services we can also test if output is weakly exogenous, for $\alpha_{1,2}(S)$ is small 0.08. The test $\alpha_{1,2}(S) = 0$ yields a LR = 0.789 and a t-statistic = 0.888 that enables us to accept the null. Moreover, the joint restriction $\alpha_{1,1}(S) = 0$ and $\alpha_{1,2}(S) = 0$ yields a LR = 2.59, which compared with the 5% critical value, $\chi^2(2) = 5.99$, enables us to accept the restriction. So in the services there are two weakly exogenous variables Les and Lys. Tests of the significance of the other adjustment coefficients were also performed and can be read in Table 4.

In conclusion, there is an ECM in agriculture for employment and real wage conditioning on output, an ECM in industry for real wage conditioning on employment and output, an ECM in the private sector for real wage conditioning to employment and output, and finally an ECM in the public administration for output conditioning to employment and real wage. This means that the labour demand in industry, services and public administration does not adjust to the long run equilibrium and it is possible to specify only an equation in the difference to take into account the short-run movements of the employment.
**TABLE 4**  
Weak exogeneity tests on the $\alpha$ vector

<table>
<thead>
<tr>
<th>Restrictions</th>
<th>LR($\alpha$)</th>
<th>$\chi^2(\cdot)$</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{1,1}(A) = 0$</td>
<td>5.02</td>
<td>$\chi^2(1) = 3.84$</td>
<td>2.24</td>
</tr>
<tr>
<td>$\alpha_{1,2}(A) = 0$</td>
<td>0.54</td>
<td>$\chi^2(1) = 3.84$</td>
<td>0.73</td>
</tr>
<tr>
<td>$\alpha_{1,3}(A) = 0$</td>
<td>8.48</td>
<td>$\chi^2(1) = 3.84$</td>
<td>2.91</td>
</tr>
<tr>
<td>$\alpha_{1,1}(I) = 0$</td>
<td>0.98</td>
<td>$\chi^2(1) = 3.84$</td>
<td>0.99</td>
</tr>
<tr>
<td>$\alpha_{1,2}(I) = 0$</td>
<td>1.27</td>
<td>$\chi^2(1) = 3.84$</td>
<td>1.12</td>
</tr>
<tr>
<td>$\alpha_{1,3}(I) = 0$</td>
<td>4.56</td>
<td>$\chi^2(1) = 3.84$</td>
<td>2.13</td>
</tr>
<tr>
<td>$\alpha_{1,1}(S) = 0$</td>
<td>1.20</td>
<td>$\chi^2(1) = 2.71$</td>
<td>1.09</td>
</tr>
<tr>
<td>$\alpha_{1,2}(S) = 0$</td>
<td>0.78</td>
<td>$\chi^2(1) = 3.84$</td>
<td>0.88</td>
</tr>
<tr>
<td>$\alpha_{1,3}(S) = 0$</td>
<td>3.67</td>
<td>$\chi^2(1) = 3.84$</td>
<td>1.97</td>
</tr>
<tr>
<td>$\alpha_{1,1}(S) = 0, \alpha_{1,2}(S) = 0$</td>
<td>2.59</td>
<td>$\chi^2(2) = 5.99$</td>
<td>-----</td>
</tr>
<tr>
<td>$\alpha_{1,1}(P) = 0$</td>
<td>1.81</td>
<td>$\chi^2(1) = 3.84$</td>
<td>1.34</td>
</tr>
<tr>
<td>$\alpha_{1,2}(P) = 0$</td>
<td>0.31</td>
<td>$\chi^2(1) = 3.84$</td>
<td>0.55</td>
</tr>
<tr>
<td>$\alpha_{1,3}(P) = 0$</td>
<td>3.67</td>
<td>$\chi^2(1) = 2.71$</td>
<td>1.97</td>
</tr>
</tbody>
</table>

*Note: t-statistic is calculated by taking the square root of the $\chi^2(1)$ likelihood ratio statistic.*

5% critical value: $\chi^2(1) = 3.84$, 10% critical value: $\chi^2(1) = 2.71$, 5% critical value: $\chi^2(2) = 5.99$

The difficulty in identifying the labour demand could be due to the interaction between demand and supply shocks. If the supply shocks prevail with respect to the demand shocks it is easier to identify a labour demand and a negative relation between employment and real wage, while if the demand shocks prevail it is possible to identify a wage equation, i.e. a positive relation between real wage and employment. This could be our case because an endogenous real wage is found. However, the real wage relation in industry and services captures a negative long run relation between wages and employment and a positive long run relation between output and employment, so it is difficult to interpret these relations in terms of a wage equation because of
the incorrect sign of employment. In public administration instead we find a simple output
equation capturing the positive relation between output and employment.

Different linear restrictions on each cointegration vector have also been performed, to
verify the significance of the coefficients. Actually it is possible that some variables are not
relevant for the long run relations but are important for the short run behaviour of the dependent
variables. In this case they are excluded from the cointegration vector. The likelihood ratio tests
and the derived t-statistics\(^5\) enable us to say that the coefficients on real wages, except in public
administration, and on output are significant in each sector, compared with the 10% critical value
in industry and private services. The hypothesis that the real wage in the public sector is zero, \(H_0\)
p(-1, b, 0) with an estimated \(b = 1.16\) (Table 6), yields an LR= 0.41 which compared with the
10% critical value, \(\chi^2(1)= 2.71\), enables us to accept the restriction. The coefficient on
employment is significant in each sector, compared with the 10% critical value in agriculture,
industry and services.

Table 5 reports the normalisation of the cointegration vectors with respect to
"endogenous" variables found with the \(\alpha\)-restrictions. It also reports the t-statistics for the
significance of the coefficients. The long run equations can be written as follows

<table>
<thead>
<tr>
<th>TABLE 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalised cointegration vectors based on the (\alpha)-restriction</td>
</tr>
<tr>
<td>Employment</td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td>Agriculture 1</td>
</tr>
<tr>
<td>(1.82)</td>
</tr>
<tr>
<td>Agriculture 2</td>
</tr>
<tr>
<td>(1.82)</td>
</tr>
<tr>
<td>Industry</td>
</tr>
<tr>
<td>(1.85)</td>
</tr>
<tr>
<td>Services</td>
</tr>
<tr>
<td>(1.86)</td>
</tr>
<tr>
<td>Publ. Admin.</td>
</tr>
<tr>
<td>(2.52)</td>
</tr>
</tbody>
</table>

Note: t-statistics in brackets derived from the LR tests on the cointegration vector coefficient
Agriculture 1 and Agriculture 2 are two different normalisation, with respect to real wage and employment, of the
same cointegration vector.

\(^5\)The t-statistics can be derived only in the case of one cointegration vector.
Another reason for performing restrictions is connected with the possibility of finding relations that have equal coefficients with opposite signs corresponding to a long run unit elasticity (structural hypotheses). Let us indicate the hypotheses of homogeneity restrictions on the cointegration vectors as $H(E, Y, W)$, where $E$ is the coefficient of employment, $Y$ on output and $W$ on the real wage.

In agriculture the hypothesis $H_{1a}(-1, a, -a)$ yields a $LR = 0.047$, that compared with the 5% critical value, $\chi^2(1) = 3.84$, enables us to accept the restriction and yields an estimated $a = 0.63$. This implies a long run equilibrium in which the employment is determined equally by real wage and output. Normalising for the agricultural real wage, the same hypothesis becomes $H_{2a}(b, 1, -1)$, with an estimated coefficient of employment $b = -1.58$. The hypothesis $H_{3a}(-1, 1, -1)$ on both the normalisations is rejected.

In industry the hypothesis $H_{1i}(-1, 1, -1)$ yields an $LR = 2.36$, which, compared with the 5% critical value, $\chi^2(2) = 5.99$, enables us to accept the restriction. This interesting result implies that in the long run the real wage is equal to the average product of labour plus a constant, or in other words, the change in the real wage is equal to the change in productivity. This means that there is no evidence of a redistribution of income in favour of labour input, because the long run product wages and productivity move together. So in this long-run relationship it seems difficult to find the reasons behind the increase in industrial unemployment during the 1980s.

In services the same hypothesis is rejected, yielding $LR = 8.16$. Instead the hypothesis of unity elasticity between real wage and output, $H_{3s}(c, 1, -1)$, yields an $LR = 0.31$ which, compared with the 5% critical value, $\chi^2(1)$, enables the restriction to be accepted. The estimated coefficient for employment is $c = -1.24$.

These results offer no evidence for attributing a predominant role to real wage or output in the long run in explaining the movements in employment or, at least, not in agriculture and industry. The new cointegration vectors based on the latter restrictions are reported in Table 6.

---

6The same conclusion is in Zenezini (1989).
TABLE 6
Cointegration vectors based on the β-restrictions

<table>
<thead>
<tr>
<th></th>
<th>Employment</th>
<th>Output</th>
<th>Real wage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture 1</td>
<td>-1</td>
<td>0.63</td>
<td>-0.63</td>
</tr>
<tr>
<td>Agriculture 2</td>
<td>-1.58</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>Industry</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>Services</td>
<td>-1.24</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>Publ. Administ.</td>
<td>1.16</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: Agriculture 1 and Agriculture 2 are two different normalisation of the same cointegration vector.

3.2 Micro and Macro cointegration

A result of cointegration is obtained from the aggregate estimation. Table 7 reports diagnostic tests for serial correlation and normality from equation (1) for aggregate series. This shows that three lags make the vector of residuals serially uncorrelated and only the output equation fails the test of normality due to fat tails in the distribution. The different lag length of the VAR confirms the important result in Lippi (1988) that the aggregation induces more dynamically complex macro equation. Thus a different dynamic structure from micro to macro is a first indicator of the weakness of the theoretical position of those who consider the aggregate Error Correction Model as being based on the maximising representative agent.

TABLE 7
Diagnostic checking for the aggregateVAR(3): mis-specification tests

<table>
<thead>
<tr>
<th>Equations</th>
<th>A p-value</th>
<th>ARCH p-value</th>
<th>N p-value</th>
<th>VA p-value</th>
<th>VH p-value</th>
<th>VN p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECM1: 1st</td>
<td>0.21</td>
<td>0.78</td>
<td>0.95</td>
<td>0.21</td>
<td>0.99</td>
<td>0.02</td>
</tr>
<tr>
<td>ECM2: 2nd</td>
<td>0.06</td>
<td>0.70</td>
<td>0.04</td>
<td>0.21</td>
<td>0.99</td>
<td>0.02</td>
</tr>
<tr>
<td>ECM3: 3rd</td>
<td>0.64</td>
<td>0.93</td>
<td>0.20</td>
<td>0.21</td>
<td>0.99</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Note: A: test for serial correlation F(2,25); VA: vector test for serial correlation, F(18,54)
ARCH: Autoregressive conditional heteroscedasticity, F(1,25); VH: vector heteroscedasticity test, F(108,24)
N: test for normality, χ²(2); VN: vector test, χ²(6)
p-value: probability of a type I error
From the likelihood ratio tests (Table 8) there is clearly one cointegration vector. The first estimated long run relationship normalised with respect to employment is (-1, 0.95, -0.82). The signs are correct and reflect the expected negative relation between employment and real wage and the positive relation with the output. The $\alpha$-vector contains adjustment coefficients that are significantly different from zero: (0.29, 0.43, 1.23). So, in this case it is possible to specify an ECM equation for employment to take into account the short-run dynamics and the adjustment to the long run target.

The hypothesis of equality between real wage and average product of labour is rejected since $H(-1, 1 -1)$ leads to an LR = 18.24 which, compared with the 5% critical value, $\chi^2(1)$, enables us to reject the restriction. On the contrary, $H(-1, 1, -0.82)$ leads to an LR = 1.67 which enables acceptance of unity elasticity between employment and output. This restriction describes a long run relation between average product of labour and wage in which the redistribution of income is in favour of workers, even if from the disaggregated analysis this restriction could not be accepted. This is a second indicator of the weakness of the aggregate result.

<table>
<thead>
<tr>
<th>$H_0$</th>
<th>$H_1$</th>
<th>$\lambda_{\text{max}}$</th>
<th>95%</th>
<th>Trace</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r = 0$</td>
<td>$r = 1, r \geq 1$</td>
<td>33.35</td>
<td>21.0</td>
<td>48.29</td>
<td>29.7</td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>$r = 2, r \geq 2$</td>
<td>11.41</td>
<td>14.1</td>
<td>14.94</td>
<td>15.4</td>
</tr>
<tr>
<td>$r \leq 2$</td>
<td>$r = 3, r \geq 3$</td>
<td>3.53</td>
<td>3.8</td>
<td>3.53</td>
<td>3.8</td>
</tr>
</tbody>
</table>

$H_1$: $r = 0, 1...$ is the alternative hypothesis for the $\lambda_{\text{max}}$ test
$H_1$: $r \geq 0, 1...$ is the alternative hypothesis for the Trace test.

The differences between the aggregate and disaggregate result can be summarised as follows

1) The sectoral cointegration is not strong owing to the non-uniformity of the result of the $\lambda_{\text{max}}$ and Trace tests. Instead the aggregate result shows a stronger cointegration relation. Hence the macro cointegration probably emerges empirically as an aggregation effect from badly-
cointegrated micro variables. One reason for this could be the occurrence in the micro series of non-common, I(1), components which prevent cointegration at the disaggregate level, but nearly cancel out when aggregation is performed (Granger 1993). This point can be described by the following example.

Let \( y_{it} \) and \( x_{it} \) be two variables of sector \( i \) (say output and employment). Each series is composed of a common I(1) component, denoted by \( \tau_t \) and by an I(1) non-common component, denoted by \( \phi_{it} \) and \( \psi_{it} \) i.e,

\[
y_{it} = \tau_t + \phi_{it}
\]
\[
x_{it} = \tau_t + \psi_{it}
\]

where \( \phi_{it} \) is orthogonal to \( \tau_t \) and \( \phi_{j}, i \neq j \), while \( \psi_{it} \) is orthogonal to \( \tau_t \) and \( \psi_{j}, \)
\( i \neq j, \) (at all leads and lags). The aggregated series are

\[
y_t = \sum_{i=1}^{n} y_{it} = n\tau_t + \phi_t
\]
\[
x_t = \sum_{i=1}^{n} x_{it} = n\tau_t + \psi_t
\]

where \( \phi_t = \sum_{i=1}^{n} \phi_{it} \) and \( \psi_t = \sum_{i=1}^{n} \psi_{it} \). Furthermore, let us assume for simplicity equal variances for the changes in the non-common components of different sectors, i.e. \( \text{Var}(\Delta \phi_{it}) = \sigma_{\phi}^2 \) and \( \text{Var}(\Delta \psi_{it}) = \sigma_{\psi}^2, \forall i \). At the aggregate level we obtain

\[
\text{Var}(\Delta n\tau_t) = n^2 \sigma_{\tau}^2
\]
\[
\text{Var}(\Delta \phi_t) = n \sigma_{\phi}^2
\]
\[
\text{Var}(\Delta \psi_t) = n \sigma_{\psi}^2.
\]

Hence the variance of the change in the common component grows with \( n^2 \), whereas that of the change in the non-common components grows only with \( n \). The reduction in the relative
weights of the non-common components at the aggregate level can lead to the acceptance of cointegration of the macro variables even if n is small. If for instance $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = 1$ and $n = 4$, aggregate variances are $\text{Var}(\Delta n\tau, ) = 16$, $\text{Var}(\Delta \phi, ) = 4$ and $\text{Var}(\Delta \psi, ) = 4$. While at the micro level the fraction of the total variance explained by the common trend is only 1/2, at the macro level it is 16/20.

2) The dynamics is substantially modified in the aggregation process. The more complex dynamics of the aggregate can be found if non-common components are responsible for much of the disaggregate dynamics but loose their importance trough the aggregation. Another explanation is presented in Lippi (1988).

3) Even though the sectoral cointegration is accepted, the disaggregate variables do not cointegrate to a labour demand in three sectors: industry, services and public sector, for the respective coefficients of adjustment to the long run equilibria are not significantly different from zero. This means that the labour demand equation does not have an ECM form. In services and industry it is also difficult to identify a long run relation that makes economic sense and in the public sector we identify only a simple output equation. On the contrary at the aggregate level it is possible to specify a labour demand equation with an ECM form.

4) The accepted restriction on the cointegration vector is different in each sector and does not reflect the restriction at the aggregate level. Necessary and sufficient conditions for micro-cointegration to imply macro-cointegration and vice versa are defined in Gonzalo (1992). They are very restrictive and even more restrictive if log-linear models are involved. In Lippi (1986) these conditions are defined such that disaggregate log-linear cointegration goes through the aggregation process. In the log-linear case the relevant conditions from an economic point of view are the following: the logs of the micro variables must cointegrate in each sector with the common cointegration vector (1, -a) and the logs of the micro regressors must cointegrate across the sectors with cointegration vector (1, -1). Under these hypotheses the aggregate cointegration vector is (1, -a). If at least one of these conditions is not satisfied the macro cointegration
becomes only a chance event. The first condition can be rejected by data used in this work. This evidence suggests that the macro cointegration does not stem from micro cointegration.

4. Conclusion

For Italian data cointegration of aggregate employment, real wage and output can be accepted on a more solid ground as compared with the results obtained with disaggregated data relative to agriculture, industry, services and public administration. The dynamics is substantially modified in the aggregation process. The accepted restriction on the cointegration vector, reflecting structural hypothesis on the long run relation, is different in each sector and does not reflect the restriction at the aggregate level. The weak exogeneity tests, on the adjustment coefficients to the long run, suggest that it is not possible describe a labour demand with an ECM form in three sectors. Therefore, even though an ECM representation is implicit in the aggregate result, such a dynamic model does not seem to be the outcome of maximisation under adjustment costs as in the customary interpretation of the aggregate ECM's.
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