A note on the equivalence of long-run and short-run identifying restriction in cointegrated systems

by

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Abstract. In this paper it is shown that, given a bivariate cointegrated system, if the variable first in the causal ordering is not caused in the long-run by the second variable, then Blanchard-Quah (1989) and Sims (1980) orthogonalization are equivalent. Since no restriction on the dynamic shape of the variable ordered first is required, we generalize Cochrane’s (1994) result.

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1. Introduction

Blanchard and Quah (1989) propose a method to identify the dynamic effects of structural disturbances on real GNP. Their strategy consists in the estimation of a reduced-form bivariate VAR model, including the first difference of GNP and the rate of unemployment, on which a set of just-identifying restrictions is imposed. In particular, given orthonormal innovations, it is assumed that one structural shock has no long-run effect on the level of both variables.

This latter identifying restriction represents an important point of departure with respect to the traditional identification scheme which is based on a recursive structure (cfr. Sims 1980).

Nevertheless, in this paper we show that, given a bivariate cointegrated system, if the variable first in the causal ordering is not caused in the long-run by the second variable, then Blanchard-Quah and Sims orthogonalization are equivalent.

It is important to notice that no restriction on the dynamic shape of the variable ordered first is required. Thus, we generalize Cochrane’s (1994) result, where a random walk process for the variable ordered first is assumed.

Section 2 introduces the basic notation and definitions. Section 3 demonstrates the equivalence of the Blanchard-Quah and Sims identification in cointegrated systems. Section 4 concludes.
2. Notation, definitions and assumptions.

Let us consider a vector $x_t = (x_{1t}, x_{2t})'$ of I(1) or Difference-Stationary (DS) variables. Recall that $x_{it}$ is DS if: (1) $\Delta x_{it} = x_{it} - x_{it-1}$ is covariance stationary but $x_{it}$ itself is not; (2) let $g_{\Delta x}(\lambda)$ be the spectral density of $\Delta x_{it}$, with $-\pi \leq \lambda \leq \pi$, in $\lambda = 0$ is $g_{\Delta x}(0) \neq 0$.

Let us make the following assumption:

**Assumption A1.** The vector $x_t$ has an Error-Correction Model (ECM) representation:

$$\Delta x_t = \mu + \Gamma(L)\Delta x_{t-1} - A(1)x_{t-1} + e_t$$  \hspace{1cm} (1)

where: (i) $L$ is the lag operator; (ii) $A(1) = \alpha\beta'$ has reduced rank $k = 1$. Moreover, $\alpha$ is a $(2x1)$ vector of loadings and $\beta$ is a $(2x1)$ vector of coefficients in the cointegrating vector; (iii) $e_t$ is the $(2x1)$ vector of disturbances such that $E(e_t) = 0$ and $E(e_t e_t') = \Omega_e$.

As shown in Johansen (1991), the reduced-form Wold representation of (1) is given by:

$$\Delta x_t = \rho + C(L)e_t$$  \hspace{1cm} (2)

where: (i) $\rho = C(1)\mu$; (ii) $C(1) = \beta_\perp \gamma \alpha_\perp \gamma \mu$; and $\beta_\perp$ are, respectively, the orthogonal complements to the matrix of error correction coefficients and the matrix of cointegration vectors, i.e. $\beta_\perp^\prime \beta_\perp = 0$, $\alpha_\perp \alpha_\perp = 0$; (iii) $\gamma = \left(\alpha_\perp \Psi \beta_\perp\right)^{-1}$ where $\Psi$ is the derivative of $\Psi(z)$, the characteristic polynomial of model (1), for $z = 1$.

Let us now make the following assumption:

**Assumption A2.** $x_{2t}$ does not cause in the long-run (at frequency 0) $x_{1t}$, i.e. $\alpha_\perp = (1, 0)$.

The notion of (Granger) causality at different frequencies is explained in detail in Granger-Lin (1995). It is worth noticing that if $\alpha_\perp = (1, 0)$ then the error-correction term does not enter the equation of $\Delta x_{1t}$. Thus, assumption A2 implies that $x_{1t}$ does not adjust to the steady-state equilibrium.
Starting with the estimation of reduced-form VAR models, an important question concerns the structure the researcher wishes to impose in order to recover the structural disturbances which affect the economic system. We can think of (2) as a reduced-form relation of the following structural model:

$$\Delta x_t = \rho + B(L)\eta_t$$

where: $B(L) = C(L)B(0)$, $\eta_t = B(0)^{-1}e_t$, $E(\eta_t \eta'_t) = 1$. $\eta_t = (\eta_{1t}, \eta_{2t})'$ is a (2x1) vector of structural disturbances. Since $B(0)$ satisfies $B(0)B(0)' = \Omega_e$, this imposes three restrictions on the four elements of $B(0)$. Hence, we need one more restriction in order to achieve exact identification\(^1\).

In Sims (1980) it is assumed that $B(0)$ is lower triangular, i.e. the contemporary effect of $\eta_{2t}$ on $x_{1t}$ is restricted to zero. In a more recent paper, Blanchard and Quah (1989) propose a long-run restriction: $\eta_{2t}$ has only a transitory effect on both $x_{1t}$ and $x_{2t}$. Thus, in this case, we have a fourth restriction which is imposed on the matrix of long-run multipliers, i.e. the second column of $B(1)$ is such that $b_{12}(1) = b_{22}(1) = 0$. As far as the impulse-response functions and forecast-error variance analysis are concerned, the two methods usually lead to different results. Nevertheless, the next section presents a result of equivalence between short-run and long-run restrictions in cointegrated systems.

\(^1\) We are making the usual and implicit assumption that no zero of the determinant of the moving-average matrix polynomial is of modulus less than one. This assumption guarantees that the structural disturbances are in the space spanned by current and lagged values of $x_t$. Nevertheless, as shown in Lippi-Reichlin (1993), nonstandard moving-average representations may provide economically sensible alternatives to standard impulse-response functions.
3. The result

We are now in the position to state the following result:

**Proposition 1.** Given the vector \( X_t = (x_{1t}, x_{2t})' \), let assumptions A1 and A2 be satisfied. Then, Sims (1980) orthogonalization, with \( \Delta X_{1t} \) first in the causal ordering, and Blanchard-Quah (1989) orthogonalization, with long-run restriction, are equivalent.

**Proof.**

We obtain Sims orthogonalization in the following way. Let \( D(0) \) be the unique lower triangular matrix (Cholesky factor) such that \( D(0)D(0)' = \Omega_e \). Rewrite (2) as:

\[
\Delta X_t = \rho + D(L)e_t
\]

where: \( D(L) = C(L)D(0) \), \( e_t = D(0)^{-1}e_t \) and \( E(e_t e_t') = I \).

Representation (4) exists and is unique. Moreover, any other orthogonalization can be obtained as an orthonormal transformation of \( D(0) \) (cfr. Blanchard-Quah, 1989). In particular, the Blanchard-Quah representation is obtained by postmultiplying \( D(0) \) for a non-singular matrix \( V \) such that \( VV' = I \) and such that one shock is permanent and one is transitory. Let us notice that \( D(1) = C(1)D(0) \). Given \( \alpha'_1 = (1, 0) \) it follows that the second column of \( C(1) \) is such that \( c_{12}(1) = c_{22}(1) = 0 \). But, since \( D(0) \) is lower triangular, also the second column of \( D(1) \) has zero elements. Thus, in this case, the matrix \( V \) is the identity.

Q.E.D.

**Remark 1.** We have shown that, if \( \alpha'_1 = (1, 0) \), a shock to \( x_{2t} \) which does not contemporaneously affect \( x_{1t} \) has only a transitory effect on both \( x_{1t} \) and \( x_{2t} \). It is important to notice that no assumption on the dynamic shape of \( x_{1t} \) is required. This implies a generalization of Cochrane's (1994) result. In fact, Cochrane shows that Sims and Blanchard-Quah identification are equivalent under the condition that \( x_{1t} \), the variable ordered first, follows a random walk.
Remark 2. In a recent paper Crowder (1995) shows that the bivariate model estimated by Blanchard and Quah satisfies the restriction $\alpha' = (1, 0)$. Thus, we can infer that the same structural model could have been recovered by imposing a causal ordering with the first difference of GNP ordered first and the rate of unemployment second. It is worth noticing that such a structural model was indeed estimated by Evans (1989).

Remark 3. One might wonder whether the result is confined to the case of cointegration. Let us assume that $x_{1t}$ and $x_{2t}$ be both I(1) but do not exhibit cointegration. It is easy to show that if $\Delta x_{2t}$ does not cause $\Delta x_{1t}$ in the long-run, then Sims orthogonalization, with $\Delta x_{1t}$ ordered first, is the unique structural representation such that a shock to the variable ordered second has only a transitory effect on the variable first. Nevertheless, in this case, there is a permanent effect on $x_{2t}$.

4. Conclusions

The main contribution of this paper has consisted in the demonstration that, given a bivariate cointegrated system, the traditional identification scheme proposed by Sims (1980) and the recent method proposed by Blanchard and Quah (1989) are equivalent, provided that the variable second in the causal ordering does not cause the variable ordered first in the long-run. The important conclusion is that, in this case, a contemporary restriction is equivalent to a long-run restriction.

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2 Blanchard and Quah (1989) assume that GNP (Y) is DS, whereas the rate of unemployment (U) is covariance stationary. These assumptions imply that $Z = (Y, U)'$ is cointegrated with cointegrating vector $(0, 1)'$. 
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