The Interaction Between Monetary Policy
and the Expectation Hypothesis
of the Term Structure of Interest Rates in a
N-Period Rational Expectation Model

by

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THE INTERACTION BETWEEN MONETARY POLICY AND THE EXPECTATION HYPOTHESIS OF THE TERM STRUCTURE OF INTEREST RATES IN A N-PERIOD RATIONAL EXPECTATION MODEL

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Abstract

In the present paper we set up an N-period Rational Expectation model for the interaction between an interest rate smoothing monetary policy rule and the Expectation Theory of the term structure of interest rates. The model is a generalisation of McCallum(1994) in that we introduce a time-varying term premium, which includes McCallum’s as a special case. In fact, the term premium is assumed to consist of two parts: one is autoregressive of order one (as in McCallum), the other is a function of the short rate as suggested by the general equilibrium models of the term structure.

The consideration of an N-period model allows to solve for the dynamics of short and long rates as a function of the term structure spread, the autoregressive part of the term premium and white noise terms.

The more general characterisation of the term premium allows to get richer results as for the information content of the spread on short and long rates. The model still accounts for those empirical findings which are at odds with the Expectation Theory of the term structure (as in McCallum) and yet supports validity of usual regressions performed to test the information content of the term structure (unlike McCallum). We show that McCallum’s conclusion about information tests of the term structure derives, among other things, from having assumed the term premium to be purely autoregressive.

Final conclusions depend essentially on the relative magnitude of the parameters characterising the time-varying term premium and the monetary policy rule and have to be left to further empirical investigations. Our results are also compared with Kugler(1996), where an N-period version of McCallum model is set up and tested for four countries.

(*) L. Malaguti acknowledges financial support from GNAFA-CNR and MURST 40% , C. Torricelli from CNR96.01630.CT10 and MURST 40%-95.
1. INTRODUCTION

McCallum (1994) proposes a model for the interaction between an interest rate smoothing monetary policy rule and the Expectation Theory (ET) of the term structure of interest rates (TS), modified by the existence of a time-varying autoregressive term premium, in a two- and in a N-period setting. As for the two-period model, he essentially argues that usual regressions of the change in the short rates on the TS spread are inappropriate, since the regression coefficient goes to zero under plausible assumptions for the model parameters. Therefore he concludes that "it is misleading to think of the expectation theory in terms of the "predictive content" of the spread for future changes of the short rate.". The N-period model on the other hand, allows the author to look at the dynamics of long rates and to account for a common US empirical finding which is at odds with the ET of the TS, i.e. the negative sign of the slope coefficient in the regressions of the changes in a long rate on the TS spread.

In a previous work, Malaguti and Torricelli (1997), we have extended McCallum's two-period model by assuming a different type of time variation for the term premium, which, besides an autoregressive component, captures the dependence of the term premium on the short term interest rates as suggested by general equilibrium stochastic models (Cox-Ingersoll-Ross (1985) and Longstaff-Schwartz (1992)). Accordingly, the implications for the predictive content of the spread are quite different. Our results still account for empirical findings at odds with the ET and yet support the appropriateness of usual tests of the predictive content of the TS. The difference between McCallum's results and ours stems precisely from the different characterisation of the time-varying term premium.

In the present paper, in line with our previous work, we propose an extension of McCallum's N-period model and we solve for the N-period rate in order to work out the regression for the change
in the long rate on the TS spread. Our model differ from McCallum’s in the assumption made on the term premium, which, beside an autoregressive component, is assumed to depend on the short term interest rate. This once again implies quite different results as for the predictive power of the spread.

The plan of the paper is as follows. Section 2 discusses with some detail McCallum model and its results and presents an alternative solution recently proposed by Kugler (1997). In Section 3 our model is set up and special emphasis is given to the new characterisation of the term premium and to a comparative discussion of the results. The last section concludes. A technical discussion of the RE solution procedure is presented in the Appendix.


In this section we analyse step by step the setting up of McCallum’s N-period model in order to highlight some, somewhat hidden, underlying assumptions and approximations which will be relevant in the comparison with Kugler’s and with our paper.

The N-period model is essentially characterised by an equation for the TS and an equation for the monetary policy rule. As for the former, McCallum assumes that the TS can be explained on the basis of the ET, which implies that the return on a (N+1) period bond is given by:

\[ R_{t}^{N+1} = \frac{1}{N + 1} \left( r_{t} + \sum_{i=1}^{N} E_{t} r_{t+i} \right) + TP_{t}^{N+1} \]  

(1)

where:

- \( R_{t}^{N+1} \) is the return on a (N+1) period bond,
- N+1 is time to maturity of the long bond,
- \( r_{t} \) is the return on a one-period bond.

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1 Hence the model is actually a N+1-period model. Yet, in line with McCallum and given the approximations made in the following, we will talk of a N-period model throughout the paper.

2 Alternatively N+1 can be interpreted as the duration of the bond. Note that the duration of a zero coupon bond coincides with its time to maturity. Since we think that for empirical applications returns on zero coupon bond should be used (see e.g. Boero and Torricelli (1997)), we have decided to interpret N+1 as time to maturity.
$TP_t^{N+1}$ is the term premium on a N+1 maturity bond.

According to the ET, the term premium is constant and hence the slope coefficient in a regression such as eq. (9) below should have probability limit equal to 1. Since most of the evidence for the US is contrary to the theory, McCallum adheres to the explanation based on the existence of a time-varying term premium and models it as an AR(1) process, i.e.:

$$TP_t^{N+1} = \rho TP_{t-1}^{N+1} + \eta_t^{N+1}$$

(2)

where $\eta_t$ is a white noise and $|\rho| < 1$.

Moreover, according to the ET, the following must also hold:

$$NE_t R_t^{N} = \left( \sum_{i=1}^{N} E_t r_{t+i} \right) + N \rho TP_t^{N}$$

(3)

where $R_t^{N}$ is the return at time $t$ on a N-period zero coupon bond.

Combining (1) and (3) and assuming that, for $N$ large, the following hypothesis is reasonable:

$$E_t R_t^{N} = E_t R_t^{N+1}$$

(4)

eq (1) can be approximated as follows:

$$R_t - N(E_t R_{t+1} - R_t) = r_t + \xi_t$$

(5)

where from now on we drop superscripts and $R_t$ stands for the return on a (N+1) period bond.

It should be noted that $\xi_t$ is not exactly the term premium on an N+1-period bond\(^3\) and the relationship between the two is the following:

$$\xi_t = (N + 1)TP_t^{N+1} - N \rho TP_t^{N}$$

(6)

Yet, taking an approximation similar to the one taken above on the expected long rates, McCallum implicitly assumes:

$$TP_t^{N} = \rho TP_{t-1}^{N} + \eta_t^{N}$$

(7)

Thus $\xi_t$ is a linear combination of two AR(1) processes characterised by the same coefficient $\rho$:

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\(^3\) In fact McCallum, in the section on the N-period model, never addresses it as the term premium, although he actually uses the same notation introduced in the two-period model for the term premium.
\[ \xi_t = \rho \xi_{t-1} + u_t \]  
(8)

where \(|\rho| < 1\) and \(u_t\) is a white noise obtained as a linear combination of \(v_t^N\) and \(v_{t+1}^N\).

For empirical tests, eq. (2) can be more usefully rewritten as follows:

\[ N(R_{t+1} - R_t) = (R_t - r_t) - \xi_t + N\varepsilon_t \]  
(9)

where \(\varepsilon_t = R_{t+1} - E_t R_{t+1}\) is the expectational error, which under RE is uncorrelated with \(R_t\) and \(r_t\).

The monetary policy rule is, as in the two-period case, supposed to be aimed at interest rates smoothing, specifically it is assumed that:

\[ r_t = \sigma r_{t-1} + \lambda (R_t - r_t) + \xi_t \]  
(10)

with \(\sigma > 0\), which is presumed to be close to 1, and \(\lambda > 0\), to be smaller than or equal to \(1/N^4\) and \(\xi_t\) is a white noise.

Combining (5) and (10) gives:

\[ (1 + N)R_t = N E_t R_{t+1} + (1 + \lambda)^{-1} [\sigma r_{t-1} + \lambda R_t + \xi_t] + \xi_t \]  
(11)

which has to be solved for \(R_t\).

The RE solution procedure is based on the minimum-state-variable (MSV) criterion discussed by McCallum(1983), whereby the solutions is assumed to have the following form:

\[ R_t = \phi_1 r_{t-1} + \phi_2 \xi_t + \phi_3 \xi_t \]  
(12)

From (12) one works out the expression for \(E_t R_{t+1}\), which substituted, together with (12), into (11) leads to a non-linear system of three equations in the undetermined coefficients \(\phi_1, \phi_2, \phi_3\). Under reasonable assumptions, such a system has a unique solution (see McCallum(1994) page 11).

Focusing on the case of \(\sigma=1\), McCallum obtains the following solution:

\[ R_t = r_{t-1} + \frac{1 + \lambda}{1 + N - N \rho (1 + \lambda)} \xi_t + \xi_t \]  
(13)

\(^4\) McCallum(1994) stresses that such a condition "is the counterpart of \(\lambda < 2\) in the two-period case (in which \(N=1\)) and is again presumed but not strictly required."
and the relevant regression accordingly becomes:

\[ R_t - R_{t-1} = (\lambda \rho + \rho - 1)(R_{t-1} - r_{t-1}) + \frac{(1 + \lambda)}{1 + N(1 - \rho(1 + \lambda))} u_t + \zeta_t \]  \hspace{1cm} (14)

McCallum underlines that, except for very large values of \( \rho \) and/or \( \lambda \), the coefficient will be negative thus matching some empirical results for the U.S. (e.g. Evans and Lewis (1994), Campbell and Shiller (1991)) which cannot be reconciled with the constant term premium version of the EH.

2.1 KUGLER'S SOLUTION

Kugler (1997) paper presents an exact solution to the N-period McCallum’s model. Specifically, the solution to McCallum’s model hinges on the approximation (4) above, i.e.:

\[ E_t R_t^N = E_t R_{t+1}^N \]

which in fact allows him to get rid of the expected values for the short rate up to date N.

In this respect, his N-period model does not correspond exactly to his two-period model where such an approximation is not needed (indeed in the two-period model the search of the RE solutions is rather straightforward even without taking such an approximation).

On the basis of this observation, Kugler (1997) extends McCallum’s exact two period solution to the general N period long rate case, i.e. he solves McCallum’s N period model without taking the approximation (4) above. This implies that he has to look for the (N-1) RE values of the short rate up to date N.

Kugler’s RE solutions are still attained according to the MSV criterion and by means of the method of undetermined coefficients (for details on the solutions, see the original paper). Accordingly, Kugler presents the regression equation for the short rate which is the following:

\[ (r_t - r_{t-1}) = \lambda \rho (R_{t-1} - r_{t-1}) + \frac{(N + 1)\lambda}{N + 1 - \lambda \sum_{j=1}^{N} (N + 1 - j) \rho^j} u_t + \zeta_t \]  \hspace{1cm} (15)
and has essentially the same implication as the corresponding equation in McCallum’s two-period model, i.e. the information content of the spread vanishes whenever $\lambda$ or $\rho$ tends to zero.

Infact Kugler concludes on the point: "This finding can be interpreted as follows: the predictive power of the spread for the short rate is based on predictable policy reaction of the central bank to the spread. However, if $\rho$ is zero there is no predictable exogenous movements of the spread which results in predictable policy reactions."

Yet, Kugler does not present the regression equation for the long rates, which we have derived, with a little bit of algebra, from his solutions:

$$R_t - R_{t-1} = (\lambda \rho + \rho - 1)(R_{t-1} - r_{t-1}) + \frac{(1 + \lambda)}{1 - \lambda \sum_{j=1}^{N} (1 - \frac{j}{N + 1}) \rho^j} u_t + \zeta_t$$  \hspace{1cm} (16)

By comparative inspection of (14) and (16), it is clear that the exactness of the solution worked out by Kugler is relevant only for the coefficient of the white noise term $u_t$. Yet, being the coefficient of the spread the same, the implications for the tests of the ET based on the value of the spread coefficient are the same as in McCallum’s and have been recalled in the previous section.

Therefore we can presume that the approximation made by McCallum in order to get the RE solutions of the model (i.e. (4) and (7) above) do not imply a significant departure from the exact solution obtained by Kugler, which additionally requires bothersome calculations.\(^5\) That is why in the following of the present study, we will look at an approximate solution of the type proposed in McCallum(1994).

\(^5\) It should be stressed that Kugler also discusses the relevance of having assumed $\zeta_t$ to be not autocorrelated and applies the model rather successfully to recent data for four counties.
3. A GENERALISATION OF McCALLUM MODEL

In line with our previous two-period model (see Malaguti and Torricelli(1997)), in this section we propose a modification of McCallum's model by introducing a different characterisation of the term premium, which includes McCallum's as a special case.

The relationship between the time-varying nature of term premia and the empirical failures of the ET have been long debated in the literature. The point was clearly made by Mankiw and Summers(1984): "Without an explicit theory of why there is such a premium and why it varies, it has no function but tautologically to rescue the theory. [...] These results suggest the importance of developing models capable of explaining fluctuating liquidity premiums. [...] Without a satisfactory theory of liquidity premiums, predicting the effect of policies on the shape of the yield curve is almost impossible."

Hence the question is whether a theory of the term premium is now available in the literature. In our opinion, a positive answer to this question is to be found within general equilibrium stochastic models of the term structure of the type introduced by Cox, Ingersoll and Ross (1985), CIR, and further developed by Longstaff and Schwartz (1992), LS. Each of the two papers offers a precise functional form for the risk premium, which in both cases turns out to be a rather cumbersome function involving the model parameters.

Yet, to the aim of the present paper, a useful implication of the two mentioned stochastic models of the term structure (CIR and LS) is that the time-varying nature of the term premium is due its dependence on the short term interest rate.6

In what follows, we model the dependence of the term premium on the short rate in such a way as to preserve linearity of McCallum's model and to allow, at the same time, a sensible economic interpretation. To this end, we assume that the term premium, beside the autoregressive term of the

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6 LS conclude that, for a fixed maturity, the term premium is a linear function of the riskless interest rate and its volatility. Yet, the inclusion of a volatility measure would transform the basic model into a non-linear one.
type introduced by McCallum, is characterised also by a term introducing dependence on the realised and expected variation in the short rate. Specifically, our term premium on a N+1-period bond is modeled as follows:

\[ TP_{t}^{N+1} = \theta N \frac{E_{t}r_{t+1}N - r_{t-1}}{N + 1} + \beta_{t} \]  

(17)

where:

\[ \theta \geq 0 \quad \text{and} \quad \beta_{t} = \rho \beta_{t-1} + \nu_{t} \quad \text{with} \quad \nu_{t} \quad \text{white noise.} \]

Note that \( \beta_{t} \) has the same role as the AR(1) term premium in McCallum and the discussion above is valid in our model too. Therefore we omit dependence on maturity N+1.

In order to get some intuition on the first component on the r.h.s., we can rewrite it as follows:

\[ \theta N \frac{E_{t}r_{t+1}N - r_{t-1}}{N + 1} = \theta N \frac{(r_{t} - r_{t-1}) + (E_{t}r_{t+1} - r_{t}) + \ldots + (E_{t}r_{t+N} - E_{t}r_{t+N-1})}{N + 1} \]

i.e. as the simple arithmetic mean of the realised and expected one period changes in the short rate up to time N.

We can therefore interpret it as a measure of the variation in the short rates based on what happened and what is expected to happen within this model investigation horizon (i.e between \( t-1 \) and \( t+N \)). Intuitively, it is sensible to assume that long rates are requested be bigger than predicted by the ET when the average of the one-period changes in the short rate on the whole period is expected to be positive (i.e. an average increase in the short rate is expected) and vice versa. Note that our term premium collapses in McCallum's by setting \( \theta = 0 \) and also when the average of the one-period changes in the short rate on the whole period is expected to be null.

Since the other equations are the same as in McCallum's paper, our model is fully described by equations (1), (17) and (10). The first two together provide a behavioural equation for the TS which represents a modified version of the ET (from now on MET) and the latter describes the monetary policy rule (from now on MP).
3.1 AN APPROXIMATION TO THE MODEL

As McCallum, before looking for the RE solution, we rewrite the original model and we make some approximations which will ease the solution. For convenience, we report here the equations of the original model:

\[ R_t^{N+1} = \frac{1}{N+1} \left( r_t + \sum_{i=1}^{N} E_i r_{t+i} \right) + TP_t^{N+1} \]  
(1)

\[ TP_t^{N+1} = \theta N \frac{E_i r_{t+i} - r_t}{N+1} + \beta, \]  
(17)

\[ r_i = \sigma r_{i-1} + \mu (R_i - r_i) + \zeta_i \]  
(10)

At this stage we want to solve the model for its RE solutions. To this end we follow McCallum's in rewriting equation (1) as a regression (and consequently in doing some approximations discussed below).

First we evaluate (1) and (17) for the return on a bond with maturity \( t \), \( R_t^N \), and then we take its expected value, i.e.:

\[ E_t R_{t+1}^N = \frac{1}{N} \left( \sum_{i=1}^{N} E_t r_{t+i} \right) + \theta (N-1) \frac{E_t r_{t+N} - r_t}{N} + \rho \beta_t \]  

From the latter, \( \left( \sum_{i=1}^{N} E_t r_{t+i} \right) \) can be worked out and substituted into (1), which, together with (17) gives:

\[ R_t^{N+1} = \frac{1}{N+1} \left[ r_t + N E_t R_t^N - N \rho \beta_t \right] - \theta \frac{N-1}{N+1} (E_t r_{t+N} - r_t) + \theta \frac{N}{N+1} (E_t r_{t+N} - r_{t+1}) + \beta, \]  
(18)

In line with McCallum, we assume that: \( E_t R_t^N = E_t R_t^{N+1} \) which is reasonable for \( N \) big.

Moreover, we also take the following approximation: \( \frac{N-1}{N+1} \geq \frac{N}{N+1} \) which again is reasonable.
for big values of $N$ and/or small values of $\theta$ (in fact the two quantities differ for $\theta/N+1$, which vanishes when, for fixed $\theta$ $N$ goes to infinity, or for fixed $N$, $\theta$ goes to zero).

In the light of the approximations made, we can rewrite (18) in the following way:

$$N(E_{t}R_{t+1} - R_{t}) = (R_{t} - r_{t}) - \theta N(r_{t} - r_{t-1}) - \xi_{t}$$

where $\xi_{t} = (N + 1)\beta_{i} - N\rho\beta_{i}$ is an AR(1) process.

Note that we drop superscripts and $R_{t}$ stands for the return on a $(N+1)$ period bond.

Summing up so far, even though the model is represented by eq. (1), (17) and (10), the RE solution we look for will be referred to equations (19) and (10), whereby eq. (19) is related to the original model by means of the two approximations above.

3.2 THE RE SOLUTIONS AND THEIR INTERPRETATION

At this stage we are ready to look for the RE solution to the model derived in the previous section:

$$N(E_{t}R_{t+1} - R_{t}) = (R_{t} - r_{t}) - \theta N(r_{t} - r_{t-1}) - \xi_{t}$$

$$r_{t} = \sigma r_{t-1} + \lambda(R_{t} - r_{t}) + \xi_{t}$$

where:

$$\theta \geq 0 \quad \sigma = 1 \quad \lambda \geq 0$$

$$\xi_{t} = \rho \xi_{t-1} + u_{t} ; |\rho| < 1 ; u_{t} \text{ and } \xi_{t} \text{ uncorrelated white noise terms.}$$

The solution procedure (presented in detail in the Appendix) is essentially developed in two steps: first, by means of the MSV criterion, we look for the RE of the short rate, then, using MP, we derive the corresponding RE value for the long rate.

Since we aim at discussing the implications of our new assumption on the term premium as for the regression equations typically performed in the empirical literature, we will focus our attention on the equation relating the one-period change in the short and in the long rate to the yield spread.
deferring to the Appendix details on the solutions. The main regression equations turn out to have respectively the following form:

\[ \frac{1}{2}(r_t - r_{t-1}) = b(R_{t-1} - r_{t-1}) + ar + wn \]  
\[ N(R_t - R_{t-1}) = B(R_{t-1} - r_{t-1}) + ar + wn \]

where \( ar \) is some autoregressive term and \( wn \) stands for some linear combination of uncorrelated white noise terms. The values of \( b \) and \( B \) depend on the value and the relationship among the model parameters as summed up in Table 1 and 2 respectively.

**TABLE 1** - The first difference in the short rate as a function of the yield spread for all possible values of the parameters \( \theta, \lambda \) and \( \rho \).

<table>
<thead>
<tr>
<th>Case A</th>
<th>( 0 &lt; \lambda \leq \frac{1}{N} ) and ( 0 \leq \theta \leq \frac{1 - N\lambda}{N\lambda} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho \neq 0 )</td>
<td>( \frac{1}{2}(r_t - r_{t-1}) = \frac{\lambda}{2} (R_{t-1} - r_{t-1}) + \frac{\lambda}{2N(N+1-\theta N\lambda - N\lambda \rho - N\rho u_t)} ) ( \zeta_{t-1} ) + ( \frac{N+1}{2(N+1-\theta N\lambda)} \zeta_t )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case B</th>
<th>( 0 &lt; \lambda \leq \frac{1}{N} ) and ( \theta &gt; \frac{1 - N\lambda}{N\lambda} ) or ( \lambda &gt; \frac{1}{N} ) and ( \theta \geq 0 ) ( \theta \neq \frac{1 + N}{\lambda N} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho \neq 0 )</td>
<td>( \frac{1}{2}(r_t - r_{t-1}) = \frac{\lambda(N+1-\theta N\lambda)}{2N(1+\lambda)} (R_{t-1} - r_{t-1}) - \frac{\lambda}{2N(1+\lambda)} \zeta_{t-1} )</td>
</tr>
<tr>
<td>&amp; ( + \frac{\lambda}{2N(1+\rho)(1+\lambda)} u_t - \frac{\lambda \theta}{2(1+\lambda)} \zeta_{t-1} + \frac{N+1}{2N(1+\lambda)} \zeta_t )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case C</th>
<th>( \lambda &gt; 0 ) and ( \theta = \frac{1 + N}{\lambda N} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho \neq 0 )</td>
<td>( \frac{1}{2}(r_t - r_{t-1}) = \frac{-\lambda(1-\rho)}{2} (R_{t-1} - r_{t-1}) - \frac{\lambda}{2N(1+\lambda)} \zeta_{t-2} )</td>
</tr>
<tr>
<td>&amp; ( + \frac{\lambda}{2N(1-\rho)(1+\lambda)} u_t - \frac{(N+1)(1-\rho)}{2N(1+\lambda)} \zeta_{t-2} )</td>
<td></td>
</tr>
<tr>
<td>&amp; ( + \frac{N\lambda \rho - N\lambda - N - \rho}{2N(1+\lambda)} \zeta_{t-1} + \frac{N+1}{2N(1+\lambda)} \zeta_t )</td>
<td></td>
</tr>
</tbody>
</table>
TABLE 2 - The first difference in the long rate as a function of the yield spread for all possible values of the parameters $\theta$, $\lambda$ and $\rho$.

<table>
<thead>
<tr>
<th>Case</th>
<th>Condition</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$0 &lt; \lambda \leq \frac{1}{N}$ and $0 \leq \theta \leq \frac{1-N\lambda}{N\lambda}$, $\rho \neq 0$</td>
<td>$N(R_t - R_{t-1}) = N(\lambda \rho + \rho - 1)(R_{t-1} - r_{t-1})$ $+ \frac{N(\lambda + 1)}{N + 1 - \theta N\lambda - N\lambda \rho - N\rho} \xi_t - \theta N^2 \rho \xi_{t-1}$ $+ \frac{N(N + 1 + \theta N)}{N + 1 - \theta N\lambda} \xi_t$</td>
</tr>
<tr>
<td>B</td>
<td>$0 &lt; \lambda \leq \frac{1}{N}$ and $\theta &gt; \frac{1-N\lambda}{N\lambda}$ or $\lambda &gt; \frac{1}{N}$ and $\theta \geq 0$ and $\theta \neq \frac{1+N}{N\lambda}$</td>
<td>$N(R_t - R_{t-1}) = (1 - \theta N\lambda)(R_{t-1} - r_{t-1}) - \xi_{t-1}$ $+ \frac{1}{N(1 - \rho)} \xi_t - \theta N\rho \xi_{t-1} + \frac{1}{N\lambda} \xi_t$</td>
</tr>
<tr>
<td>C</td>
<td>$\lambda &gt; 0$ and $\lambda = \frac{1+N}{N\lambda}$</td>
<td>$N(R_t - R_{t-1}) = -N(2 + \lambda - \lambda \rho - \rho)(R_{t-1} - r_{t-1}) - \xi_{t-2}$ $+ \frac{\lambda}{(1 - \rho)} \xi_t - \frac{(N + 1)(1 - \rho)}{\lambda} \xi_{t-2} + \frac{N\lambda \rho - N\lambda - N - \rho}{\lambda} \xi_{t-1}$ $+ \frac{1}{N\lambda} \xi_t$</td>
</tr>
</tbody>
</table>

Since our main scope is that of assessing the implication of our TP on the spread coefficients $b$ and $B$ in (20) and (21), we focus on them in the following comments. Given that we have first derived the RE dynamics for the short rate and then, using MP, the dynamics of the long rate, we start by commenting the former dynamics.

INTERPRETATION OF TABLE 1

The results we obtain in Table 1 as for the short rates are essentially the N-period analogue of those we obtained in a previous paper within a two-period model (see Malaguti and Torricelli(1997)).
In case A and C, the spread coefficient is exactly the same as in the two-period model. Specifically, in case A, where the values $\theta$ can assume are smaller than in the other two cases, we essentially obtain the same result as McCallum. In case C, for a single value of the term premium parameter $\theta$, we obtain a negative relationship between the sign of the spread and the variation in the short rate.

In case B, the spread coefficient differs from the corresponding one in the two-period model, whereby differences are due to the slightly different characterisation of the term premium (as underlined in footnote 7 below) and to the approximations made to solve for the N-period case. Yet, such differences do not substantially change the implications as for the information content of the spread. In fact, in case B, our result differ from McCallum’s model and we obtain a sort of separation between the information content of the spread and that of the autoregressive component of the term premium, whereby the coefficient of the former does not vanish as $\rho$ goes to zero. Moreover, it can be either positive or negative depending on whether $\theta < \frac{N + 1}{N \lambda}$ or $\theta > \frac{N + 1}{N \lambda}$.

A deeper interpretation of these results can be found in our previous paper. In order to clarify intuitively how the three cases emerge, recall that, according to the MSV criterion, the solution is assumed to have the following form:

$$r_t = \phi_1 r_{t-1} + \phi_2 \xi_t + \phi_3 \zeta_t,$$

It is clear that the dynamics of the short rate depends primarily on the value of $\phi_1$.

When $\phi_1 = 0$ or $\phi_1 = 1$, the dynamics of the short rate depends (though differently in the two cases) only on the autoregressive component of the term premium and on the white noise term and therefore our assumption on the term premium is not relevant to the spread coefficient. On the other

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7 Note that the value of $\theta$ which discriminates between a positive and a negative spread coefficient is, given the approximation on $N$ made in the present mode, approximately the same as in the two-period model. In the particular case when $N=1$ we obtain a two-period model with a term premium which is equal to the one in Malaguti-Torriceilli(1997), with $\theta$ in place of $2\alpha$. It is interesting to notice that the value of $\theta$ which discriminates between a positive and a negative spread is the same in the two models.
hand, when \( \phi_i \neq 0 \) and \( \phi_i \neq 1 \), the RE value of the short rate depends also on its lagged value and hence our assumption on the term premium plays a role and more interesting dynamics can emerge, as in case B. The points just made are central to the understanding of the results obtained in Table 2.

**INTERPRETATION OF TABLE 2**

Our results for the long rates can be better interpreted by recalling the solution procedure (see Appendix for details) and the possible dynamics for the short rate just described. In fact, since we have first derived the RE dynamics for the short rate and then, using MP, the dynamics of the long rate, it is clear that the dynamics of the long rate follows that of the short rate.

Therefore it is not surprising that in case A we obtain again the same result as in McCallum N-period model, given that our assumption on the term premium is, in such a case, not substantially relevant to the short rate dynamics.

The interesting case is Case B, where the results lead themselves to a simple intuitive interpretation as follows. As starting point, it is necessary to investigate the interaction between the MET and the MP characterising our model. This can be done by looking at the two model equations before substituting for the RE solutions. For convenience, we rewrite (19) and (10) as follows:

\[
(E, R_{t+1} - R_t) = \frac{1}{N} (R_t - r_t) - \theta (r_t - r_{t-1}) - \frac{1}{N} \xi_t
\]

\[r_t - r_{t-1} = \lambda (R_t - r_t) + \zeta,
\]

where the latter has been obtained, following McCallum, by assuming \( \sigma = 1 \) in (10).

It is therefore clear that MP modifies the implication of the MET via the second term on the r.h.s. of MET.

In fact, substituting MP in MET, we have:

\[
(E, R_{t+1} - R_t) = \frac{1}{N} (R_t - r_t) - \theta \lambda (R_t - r_t) - \frac{1}{N} \xi_t - \theta \zeta,
\]
where the first term on the r.h.s. captures the classical implication of the ET as for the information content of the spread (a positive spread should predict an increase in the long rate), while the second term modifies the impact of the former in that implies that a positive spread should predict a decrease in the long rate. Note that this second term, capturing a negative relationship between the long rate dynamics and the spread, is due to the interaction between the MP rule and our assumption on the time-varying term premium. Specifically, the MP implies a positive relationship between the short rate dynamics and the spread, while the autoregressive part of the term premium implies a negative relation between the long rate dynamics and the short rate one. In order to understand this latter point, which is specific to our model, it is useful to decompose the second term on the r.h.s. of MET as follows:

\[-\theta(r_t - r_{t-1}) = -\theta[(E_t r_{t+N} - r_{t-1}) - (E_t r_{t+N} - r_t)]\]

where the first term in square brackets is proportional to the term premium at time t and the second term is proportional to the expected value of the term premium at time t+1. When the latter is bigger than the former the expected long rate is bigger than the current long rate (as far as this component of the term premium is concerned) and in fact the term above is positive, and vice versa.

The overall information content of the spread as for the expected long rate depends on the relative magnitude of the two coefficients of the spread. Three cases may realise:

i) \(\theta = 1/\lambda\) : the joint effect of MP and our assumption on the term premium exactly offsets the classical ET effect, hence there is no information content in the spread;

ii) \(\theta < 1/\lambda\) : the joint effect of MP and our assumption on the term premium diminishes the typical implication of the ET, without offsetting it completely;

iii) \(\theta > 1/\lambda\) : the joint effect of MP and our assumption on the term premium more than offsets the ET effect and hence the classical implication is reversed, i.e. a positive spread predicts a decrease in the long rate.
Yet, these statements have to be better specified since the RE solution procedure imposes constraints on the model parameters. In particular, case B can be discussed only for $\phi_1 > 0$ or $\phi_1 = 0$ and we fall either in case C or A) and hence the statement sub ii) actually holds for $1 - \frac{N\lambda}{N\lambda_t} < \theta < \frac{1}{N\lambda}$. 

Moreover $\theta = \frac{1 + N}{\lambda N}$ has also to be excluded from case B (otherwise $\phi_1 = 0$): in fact in such a case we get the results reported in case C, where, under the very specific assumption on $\theta$, the spread coefficient may be either positive or negative depending on the relationship between $\rho$ and $\lambda$. Hence statement iii) holds for $\theta < \frac{1}{\lambda N}$ and $\theta \neq \frac{1 + N}{\lambda N}$.

Finally, we make a few remarks on the two limiting cases, i.e. $\lambda = 0$ and $\rho = 0$, which are different in nature and yet lead to the same conclusions. As for the former, which has to be excluded in all cases from both Tables, it is clear from MP that, when $\lambda = 0$, the first difference in the short rate behaves just as a white noise. It follows that the same behaviour carries over to the non-autoregressive part of the TP (see eq. (17) and below) and MP interacts with MET simply by means of a white noise term. Loosely speaking, monetary policy is precisely aimed at making the one-period dynamics in the short rate unpredictable. Hence there is no scope in looking at the information content of the spread.

$\rho = 0$ has been excluded from both Tables only in case A, which is characterised by $\phi_1 = -1$. Therefore, when $\rho = 0$ and recalling equation (8), the MSV solution becomes:

$$r_i = r_{i-1} + \phi_2 u_i + \phi_3 \zeta_i,$$

which again implies that the short rate dynamics is a white noise (specifically is the sum of two uncorrelated white noise terms). The same conclusions as in the case $\lambda = 0$ follow, which means that the spread does not contain information as for future rates.
Summing up we can conclude by saying that while McCallum’s model mainly rationalises empirical findings contrary to the EH of the term structure (i.e. those for the US), our model gives rise to a variety of results which account also for empirical findings in line with the EH (e.g. Boero, Madjlessi and Torricelli, 1996).

4. CONCLUSIONS

In the present paper we have extended McCallum(1994) N-period Rational Expectation model for the interaction between an interest rate smoothing monetary policy rule and the Expectation Theory of the term structure of interest rates by introducing a different characterisation of the time-varying nature of the term premium. In fact, our term premium is assumed to consists of two parts: one is autoregressive of order one (as in McCallum), the other is a linear function of the difference between the short rate and its expected value at time \( t+N \).

We have solved for the dynamics of both the short and the long rate as a function of the term structure spread, the autoregressive part of the term premium and white noise terms. The more general characterisation of the term premium allows to get richer results as for the information content of the spread on short and long rates, which include McCallum and Kugler(1996) results as special cases (Case A in Table 1 and 2).

As for the dynamics of the short rate, the results we obtained in Table 1 are essentially a generalisation of our previous work to the N-period case (see Malaguti and Torricelli, 1997), whereby conclusions on the information content of the spread on future short rates depend on the relationship between the model parameters. In particular, for a given value of the monetary policy parameter \( \lambda \), McCallum’s negative conclusions on the information content of the spread obtain when our new characterisation of the term premium is not relevant (\( \Theta \) small or nought). By contrast,
in the other cases, the coefficient of the spread can be either positive or negative thus rationalising empirical findings at odds with the ET of the term structure.

The N-period model allows to investigate the information content of the spread on the long rate too.\(^8\) Also in this case final conclusions depend on the relative magnitude of the relevant model parameters and McCallum's results hold, for a given monetary policy, when the term premium parameter \(\theta\) is small or nought. For most other values of \(\theta\) (i.e. case B), conclusions on the information content of the spread over the long rate, depend on whether the joint working of monetary policy and the time-varying term premium, which is peculiar to our model, exactly compensate or not the classical implication of the ET. So our results can rationalise regression tests obtaining small or even negative values for the spread coefficient (among others see Boero, Madjlessi and Torricelli(1996), Campbell and Shiller(1991), Evans and Lewis(1994)).

In summary, our N-period model still accounts for those empirical findings which are at odds with the Expectation Theory of the term structure (as McCallum and Kugler) and yet supports validity of usual regressions performed to test the information content of the term structure (unlike McCallum), whereby McCallum's argument against information tests of the term structure derives, among other things, from having assumed the term premium to be purely autoregressive. Final conclusions depend essentially on the relative magnitude of the parameters characterising the time-varying term premium and the monetary policy rule. Therefore it will be interesting to investigate the empirical performance of our model and compare it with Kugler paper where the exact solution to McCallum's model is tested rather successfully to recent data for four countries. These type of models, by taking into account the interaction between the ET of the term structure and a monetary policy rule, may account for an empirical result emerging in recent tests of the ET: a "U-shaped" pattern of the predictive ability of the yield curve, i.e. the spread coefficient is lower at intermediate

\(^8\) A regression such as eq. (21) can be derived also within a two-period model. Yet, it turns out to be less interesting since in a two period model the long rate is bound to be defined on a maturity which is the double as the short rate one.
maturities which are indirectly controlled by the central bank (see, among others, Rudebusch(1995), Boero and Torricelli(1997)). In other words an interest rate targeting monetary policy rule (as opposed to a monetary targeting one) seems to be responsible for the empirical failure of the ET at mid-term maturities.

Given the sensitivity of the results obtained to the monetary policy rule assumed, it will be interesting to analyse the effect of alternative, and possibly more realistic, policy rules (see Rudebusch(1995) page 247). Finally, the absence of nominal variable in the models analysed in the present paper, naturally raises the opportunity of bringing the price level into the model. The latter could also, in some time- and country-specific set up, play a role in the monetary policy rule.
APPENDIX

In this part we describe the method used to obtain all relations in Tables 1 and 2, that is to obtain the first difference, both in the short and in the long rate, for all admissible values of the parameters.

We recall that our model is given by eqs. (19) and (10) (see Section 3.2) and it involves rational expectations. As it as been pointed out by many authors (see McCallum(1983) for a discussion on the subject), such kind of models, even when they are linear as in our case, may have infinite solutions. In order to eliminate such multiplicity we shall use a solution procedure, proposed by McCallum(1983), assuming that a minimal set of state variables be employed in agents forecasting rules and that solution formulae be valid for all possible parameter values. The method, known as minimal state variable criterion (MSV), actually allows to eliminate such multiplicity in a wide class of linear expectation models; in particular it works in our case.

In Section 3.2 we have already noticed that, when $\lambda=0$, it is not possible to get any information from the spread, hence in the following we shall always assume $\lambda \neq 0$. In particular, when $\lambda \neq 0$ the long rate $R_t$ may be derived in eq. (10) from the short one; this justifies our technique which consists in getting first the dynamic for the short rate $r_t$, that is relations in Table 1, and then in using (10) to derive the relations in Table 2.

Combining eqs. (10) and (19) we have:

$$(2N + N\lambda + 1 - \theta N\lambda)r_t = (N + 1 - \theta N\lambda)r_{t-1} + N(1 + \lambda)E_t r_{t+1} + \lambda \xi_t + (1 + N)\zeta_t$$ (A-1)

so the short rate $r_t$ only depends on the variables $r_{t-1}$, $\xi_t$ and $\zeta_t$. According to the MSV criterion we look for a solution for $r_t$ which is a linear combination of such functions, that is:

$$r_t = \phi_1 r_{t-1} + \phi_2 \xi_t + \phi_3 \zeta_t$$ (A-2)

with the coefficients $\phi_1$, $\phi_2$ and $\phi_3$ to be determined.

By rational expectation theory, since $\xi_t$ is an AR(1) process, from (A-2) we get:
$E_r r_{t+1} = \phi_1^2 r_{t-1} + (\phi_1 + \rho) \phi_2 \xi_t + \phi_1 \phi_3 \zeta_t$  \hspace{1cm} (A-3)

Substituting (A-2) and (A-3) in (A-1) we then obtain:

$$
\left(2N + N\lambda + 1 - \theta N\lambda \right) \left( \phi_1 r_{t-1} + \phi_2 \xi_t + \phi_3 \zeta_t \right) = \\
\left( N + 1 - \theta N\lambda \right) r_{t-1} + N(1 + \lambda) \phi_1^2 r_{t-1} + N(1 + \lambda) \phi_3 \phi_2 \xi_t + N(1 + \lambda) \phi_1 \phi_3 \zeta_t + \lambda \xi_t + (1 + N) \zeta_t .
$$

In order for (A-2) to be a solution for the short rate $r_t$, the coefficient of $r_{t-1}$ both in the r.h.s. and in the l.h.s. of the above equation must be necessarily equal and the same has to be true also for the coefficients of $\xi_t$ and $\zeta_t$. This is to say that the following identities must hold:

$$
\begin{align*}
\left(2N + N\lambda + 1 - \theta N\lambda \right) \phi_1 &= N + 1 - \theta N\lambda + N(1 + \lambda) \phi_1^2 \\
\left(2N + N\lambda + 1 - \theta N\lambda \right) \phi_2 &= N(1 + \lambda) (\phi_1 + \rho) \phi_2 + \lambda \\
\left(2N + N\lambda + 1 - \theta N\lambda \right) \phi_3 &= N(1 + \lambda) \phi_1 \phi_3 + 1 + N 
\end{align*}
$$

Solving the first equation in (A-4) we have:

$$
\phi_1 = \frac{2N + N\lambda + 1 - \theta N\lambda \pm \sqrt{\theta N\lambda + N\lambda - 1}}{2N(1 + \lambda)}. \hspace{1cm} (A-5)
$$

To get a unique value for $\phi_1$ we need to choose between the two possibilities in (A-5). Looking at (A-1) it is easy to see that, when $N\theta\lambda = N + 1$ the short rate $r_t$ does not depend on $r_{t-1}$, so in this special case we expect that $\phi_1 = 0$. On the other hand, when $N\theta\lambda = N + 1$ such value for $\phi_1$ may be obtained only when the negative square root is assumed in (A-5); since we require that a unique solution formula be valid for all possible values of the parameters, we take the negative square root in (A-5) in all cases and this determines the coefficient $\phi_1$. Substituting the value for $\phi_1$ in the other two equations of the system, which are linear in $\phi_2$ and $\phi_3$ respectively, it is possible to get a unique solution for (A-5); the following table summarizes such solution for all values of the parameters.
TABLE 1-A - The solution of system (A-4)

<table>
<thead>
<tr>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ 0 &lt; \lambda \leq \frac{1}{N} \text{ and } 0 \leq \theta \leq \frac{1 - \lambda N}{N\lambda} ]</td>
<td>[ 0 &lt; \lambda \leq \frac{1}{N} \text{ and } \theta &gt; \frac{1 - \lambda N}{N\lambda} \text{ or } \lambda &gt; \frac{1}{N} \text{ and } \theta \geq 0 \text{ and } \theta \neq \frac{1 + N}{N\lambda} ]</td>
<td>[ \lambda &gt; 0 \text{ and } \theta = \frac{1 + N}{N\lambda} ]</td>
</tr>
</tbody>
</table>

| \( \phi_1 \) | \( \frac{N + 1 - \theta N\lambda}{N(1 + \lambda)} \) | 0 |
| \( \phi_2 \) | \( \frac{\lambda}{N + 1 - \theta N\lambda - N\lambda \rho - N\rho} \) | \( \frac{\lambda}{N(1 - \rho)(1 + \lambda)} \) |
| \( \phi_3 \) | \( \frac{N + 1}{N + 1 - \theta N\lambda} \) | \( \frac{N + 1}{N(1 + \lambda)} \) | \( \frac{N + 1}{N(1 + \lambda)} \) |

Notice, in particular, that the denominators of \( \phi_2 \) and \( \phi_3 \) in Case A vanishes respectively when
\[ \theta = \frac{N + 1 - N\lambda \rho - N\rho}{N\lambda} \text{ and } \theta = \frac{N + 1}{N\lambda} \text{ but such values never occur in this case; also the denominator of } \phi_2 \text{ in Cases B and C never vanishes, since } \rho < 1, \text{ so no further condition on the parameters has to be added.} \]

Now we are able to derive all relations in Table 1. To this aim it is convenient to write both the first difference in the short rate \( r_t - r_{t-1} \) and the spread \( R_t - r_t \) as functions of the fundamental variables of the model. As for the first difference in the short rate, from (A-2) we get:
\[
r_t - r_{t-1} = (\phi_1 - 1)r_{t-1} + \phi_2 \xi_t + \phi_3 \zeta_t = (\phi_1 - 1)r_{t-1} + \phi_2 \rho \xi_{t-1} + \phi_3 \zeta_t. \tag{A-6}
\]

According to (10) and (A-2) we have:
\[
R_t = \left( \frac{\phi_1}{\lambda} + \phi_1 - 1 \right) r_{t-1} + \left( \frac{\phi_2}{\lambda} + \phi_2 \right) \xi_t + \left( \frac{\phi_3}{\lambda} + \phi_3 - 1 \right) \zeta_t;
\]
consequently it holds:
and substituting in (19) we obtain the following relation for the spread:

\[ R_t - r_t = N \left( \frac{1}{\lambda} - \theta \right) r_{t-1} + N \left( \theta + \phi_1 - \frac{2}{\lambda} - 1 \right) \left( \phi_1 r_{t-1} + \phi_2 \xi_{t-1} + \phi_3 \zeta_{t-1} \right) + \left( \frac{N}{\lambda} \rho \phi_2 + N \rho \phi_2 + 1 \right) \xi_{t-1} + \frac{N}{\lambda} \zeta_{t-1}. \]

Indeed we are interested in the spread at time \( t-1 \), for which we have:

\[ R_{t-1} - r_{t-1} = N \left( \frac{1}{\lambda} - \theta \right) r_{t-2} + N \left( \theta + \phi_1 - \frac{2}{\lambda} - 1 \right) \left( \phi_1 r_{t-1} + \phi_2 \xi_{t-1} + \phi_3 \zeta_{t-1} \right) + \left( \frac{N}{\lambda} \rho \phi_2 + N \rho \phi_2 + 1 \right) \xi_{t-1} + \frac{N}{\lambda} \zeta_{t-1}. \]  

(A-7)

Looking at (A-6) and (A-7), it is clear that, in order to state a relation between \( r_t - r_{t-1} \) and \( R_{t-1} - r_{t-1} \), different situations occur according to different values for \( \phi_1 \). More precisely, we have the following three cases:

1. \( \phi_1 = 1 \), which happens when the parameters \( \lambda, N \), and \( \theta \) satisfy the conditions in Case A. According to (A-7), the spread in this case actually depends on \( r_{t-1} - r_{t-2} \) and from (A-6) such difference is a function only of \( \xi_{t-1} \) and \( \zeta_{t-1} \). Substituting in (A-7) and extracting \( \xi_{t-1} \) we obtain:

\[ \xi_{t-1} = \left( N + 1 - \theta N \lambda \right) \left( r_{t-1} - r_{t-2} \right) + \frac{\theta N}{N + 1 - \theta N \lambda} \zeta_{t-1}. \]  

(A-8)

When \( \rho \neq 0 \), combining (A-8) and (A-6), we have the first relation in Table 1. Otherwise (see Section 3.2), \( \rho = 0 \) gives rise to a first difference \( r_t - r_{t-1} \) which is a white noise term, so there is no possibility to state a relation between it and the spread at time \( t-1 \).

2. \( \phi_1 \neq 1 \) and \( \phi_1 \neq 0 \) that is, \( \theta, N \), and \( \lambda \) verifying the constraints in Case B. Here the problem is that the spread at time \( t-1 \) really depends on \( r_{t-2} \); otherwise, since \( \phi_i \neq 0 \), from (A-2) evaluated at \( t-1 \) instead of \( t \), it is possible to derive \( r_{t-2} \) as a function of \( r_{t-1} \); substituting such an expression in (A-7) we get that the spread \( R_{t-1} - r_{t-1} \) depends only on \( r_{t-1}, \xi_{t-1} \) and \( \zeta_{t-1} \); extracting \( r_{t-1} \) we obtain:

\[ r_{t-1} = \frac{\lambda (N + 1 - \theta N \lambda)}{1 - N \lambda - \theta N \lambda} \left( R_{t-1} - r_{t-1} \right) + \frac{\lambda}{(1 - \rho)(N \lambda + \theta N \lambda - 1)} \xi_{t-1} + \frac{\theta N \lambda}{N \lambda + \theta N \lambda - 1} \zeta_{t-1}; \]  

(A-9)
and combining (A-9) and (A-6), we have the second relation in Table 1.

\( \phi_1=0 \), that is \( \theta, N \) and \( \lambda \) satisfying the condition defining Case C; according to (A-2) the short rate \( r_t \) depends only on the variables \( \xi^1_t \) and \( \zeta^1_t \).

Substituting (A-2), both written for time \( t-1 \) and for time \( t-2 \) in (A-7), we get that also the spread \( \Delta r_{t-1} - r_{t-1} \) depends only on the same variables but evaluated at times \( t-1 \) and \( t-2 \). In particular we may extract \( \xi_{t-1}^1 \) from such a relation obtaining:

\[
\xi_{t-1}^1 = N(1-\rho)(1+\lambda)(R_{t-1}-r_{t-1}) + \xi_{t-2}^1 + \frac{(N^2-1)(1-\rho)}{\lambda} \xi_{r-1}^1 + \frac{(1+N)(1-\rho)}{\lambda} \zeta_{r-2}.
\]  

(A-10)

Substituting (A-10) in (A-6) we have the last relation in Table 1.

Since eq. (10) implies the following:

\[
\lambda R_t - R_{t-1} = -\frac{1}{\lambda} (r_{t-1} - r_{t-2}) + \left(1 + \frac{1}{\lambda}\right) (r_t - r_{t-1}) - \frac{1}{\lambda} (\xi_t - \xi_{t-1}),
\]

in order to derive the relations in Table 2, it is sufficient to get an expression for the difference \( r_{t-2} - r_{t-1} \) as a function of \( R_{t-1} - r_{t-1} \); but, again from (10), we have:

\[
r_{t-1} - r_{t-2} = \lambda (R_{t-1} - r_{t-1}) + \xi_{t-1}^1
\]

so all relations in Table 2 follow quite easily from the corresponding ones in Table 1.
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