Does Detrending Matter for the Determination of the Reference Cycle and the Selection of Turning Points?

by

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Abstract
This paper examines the sensitivity of turning points classification to different detrending methods and compares the characteristics of the implied reference cycles to those compiled by NBER or DOC researchers. Two turning point dating rules are considered. I show that turning point dates are broadly insensitive to detrending with one dating rule but extremely sensitive to detrending with another. With this latter rule many of the detrending procedures generate false alarms and miss several commonly classified turning points. Amplitude and duration properties are also sensitive to both detrending and dating rules. The reference cycles generated with the Hodrick and Prescott filter and a frequency domain masking of the low frequency components of the series closely mimic NBER and DOC cycles, regardless of the dating rule used.

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1 Introduction

Is the dating of business cycle turning points sensitive to the choice of detrending? Are the amplitude, duration and persistence characteristics of the resulting reference cycle robust? Is there any detrending method which produces a reference cycle whose turning points match NBER or Department of Commerce (DOC) turning points and replicates features of the US reference cycle?

This paper attempts to shed some light on these three issues. There are several reasons why these questions may be important for business cycle researchers. First, although there is a long history dating business cycle extremes using level data, since Mintz (1969) it has become more standard to select turning points and classify business cycle phases using a growth-cycle approach, i.e. using fluctuations around the trend of the series (see e.g. Zarnowitz (1991b) or Niemera (1991)). However, as Zarnowitz (1991a) has pointed out, trends vary over time, may interact in a nontrivial way with the cyclical component of the series and be difficult to isolate and measure given the size of the available samples and existing econometric techniques. In standard practice NBER or DOC growth cycles are extracted using elaborate and ad-hoc procedures which are hard to reproduce, involve a substantial amount of judgmental decisions by the researchers and a number of ex-post revisions as more information is obtained over time. It is therefore worthwhile to study, on one hand, whether any well known mechanical detrending procedure can provide a simple rationale for these complicated and subjective approaches and, on the other, whether there is a class of detrending methods which produce reference cycles with “desirable” properties. Canova (1993) showed that different trend removal procedures, all of which are reasonable given existing empirical evidence and available econometric tools, induce different properties in the moments of the cyclical component of several real macroeconomic series and different implications for how we perceive the economy to work. It is therefore interesting to check whether the path properties of the cycles induced by different detrending methods are also substantially different, thus providing a more complete perspective on the macroeconomic implications of different trend-removal procedures. The ability to reproduce NBER or DOC turning points and well known characteristics of the US reference cycle can be used as a limited information test to discern among a variety of detrending procedures which a-priori would be on an equal footing. Second, many researchers have examined the statistical features of the NBER reference cycle over the pre and post WWII period, in particular the amplitude and duration properties (see e.g. Diebold and Rudebush (1990) and (1992), Romer
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(1994) or Watson (1994)). It is therefore worthwhile to study whether the statistical features they unveil persist when mechanical detrending procedures are used to construct reference cycles. Third, there exists a large branch of the literature which deals with the question of turning point predictions (see e.g. Wecker (1979) or Zellner and Hong (1991)) and how to better evaluate the record and the quality of turning point forecasts (see e.g. McNees (1991)). However, the conclusions of this literature hinge on having available a "correct" notion of reference cycle at hand. Therefore, our study may also help researchers studying this problem to select one concept of cycle over another in deciding the validity of various forecasting approaches.

In examining the questions posed in this introduction, the paper focuses on 12 widely used detrending methods (linear and segmented detrending, first order differencing, frequency domain filtering, Hodrick and Prescott filtering, detrending with the Beveridge and Nelson model, with an unobservable components model, with Hamilton's 2-state model, with a one dimensional index model, with Blanchard and Quah's model, with King, Plosser and Rebelo's model and with King, Plosser, Stock and Watson's model). To classify turning points and to construct business cycle phases, I consider two standard mechanical dating rules. The first rule defines a trough as a situation where two consecutive quarter declines in the reference cycle are followed by an increase. Likewise, a peak is defined by two consecutive increases followed by a decline. The second rule selects a quarter as a trough (peak) if there have been at least two consecutive negative (positive) spells in the reference cycle over a three quarter period. Although the search across detrending methods and dating rules is not exhaustive and more complicated dating rules may generically improve the quality of the outcomes, our work provides a first step in systematically addressing these issues and methodically studying these features of the data.

The results of the paper complement those of Canova (1994). There I showed that turning point classification is essentially robust to detrending with the first dating rule but not with the other. In this paper I qualify this statement by showing that with this latter dating rule many standardly reported turning points are missed and many false alarms appear with the majority of the detrending methods. For those turning points which are correctly identified I find that the majority of methods produce dates which slightly lead NBER peaks and troughs and lead DOC peaks but coincide with DOC troughs.

In addition, I demonstrate that the statistical properties of the generated reference cycles are sensitive to both the detrending procedure and the dating rule. With the first rule, regardless of the
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detrending procedure used, cycles are slightly asymmetric and the duration of expansions exceeds, on average, the duration of contractions. We also show that there exists only a moderate degree of variability in the duration of each phase and in the amplitude of contractions. Furthermore, we find little evidence that peak dates can be predicted using the information contained in past durations while trough dates are predictable with at least 7 methods. The statistical properties of the various reference cycles are much more heterogeneous with the second rule. In general, the average duration of contractions exceeds the average duration of expansions and there is a large degree of variability in the duration of each phase and in the amplitude of contractions. Peak dates are more predictable than trough dates but differences across detrending methods are substantial. The only regularity that is robust to both the choice of detrending method and dating rule is that there appears to be very little persistence in business cycle phases: the amplitude of contractions is in fact uncorrelated with both the duration of contractions and of peak-to-peak cycles.

Overall, two detrending procedures produce reference cycles which come closest in reproducing the statistical features and the reference dates of standard NBER or DOC reference cycles with both rules: the Hodrick and Prescott filter and a frequency domain (MA) filter.

I conclude that, in general, statements concerning the location of turning points and the properties of the reference cycle are not independent of the statistical assumptions needed to extract the trend of the series. While this outcome is somewhat disappointing, our exercise also provides important information on the characteristics of various detrending procedures and on the types of cycles they generate. When we take the ability to reproduce the characteristics of NBER or DOC growth cycles as a limited information test to select a class of detrending procedures over another, the paper indicates that standard methods employed by RBC researchers (see e.g. Hodrick and Prescott (1980) or Baxter and King (1994)) are in fact generating cycles capturing the essence of what the community perceives as business cycle fluctuations. This obviously does not mean that these filters are appropriate for all purposes, as they may wipe out cycles with important economic features (see Canova (1993)) and may induce spurious patterns in series which do not display any form of classical cyclical fluctuations (see e.g. Cogley and Nason (1995)). But they appear to provide (i) a solid rationale for the current NBER or DOC practice and (ii) a clear standard to study the path properties of time series generated by dynamic general equilibrium models.

The rest of the paper is organized as follows: the next section describes the various detrending procedures employed in the paper. Section 3 presents the data, the dating rules and the statistics
used to characterize the properties of the reference cycle. Section 4 discusses the results. Section 5 concludes describing the implications of the results for current macroeconomic practice.

2 Alternative Detrending Methods

This section reviews the procedures I use to extract trends from the observable time series. I divide the methods into two broad categories: "statistical" methods, which assume that the trend and the cycle are unobservable but use different statistical assumptions to identify the two components, and "economic" methods, where the choice of trend is dictated by an economic model, by the preferences of the researcher or by the question being asked. Since only trend and cycle are assumed to exist, all procedures implicitly assume that either data has previously been seasonally adjusted or that the seasonal and the cyclic component of the series are lumped together and that irregular (high frequency) fluctuations play little role.

2.1 Statistical Procedures

Most of the procedures in this class assume that

\[ y_t = x_t + c_t \]  

(1)

where \( y_t \) is the natural logarithm of a time series, \( x_t \) its trend and \( c_t \) its cyclic component.

2.1.1 Polynomial Functions of Time

This procedure is the simplest and the oldest one. It assumes that trend and cycle of the (log) of the series are uncorrelated and that \( x_t \) is a deterministic process which can be approximated with polynomial functions of time. These assumptions imply a model for \( y_t \) of the form

\[
x_t = a + \sum_{j}^{q} b_{1j} f_{j}(t-t_0) \quad \text{if } t \leq t
\]

\[
x_t = a + \sum_{j}^{q} b_{2j} f_{j}(t-t_1) \quad \text{if } t + 1 \leq t \leq T
\]

(2)

where \( q \) is typically chosen to be small, \( t_0 \) and \( t_1 \) are given points in time scaling the origin of the trend. In (2) I allow for the possibility of a structural break in the secular component at a known time \( t \). I present results for \( q = 1 \). \( f_{1}(t-t_0) = t \) and either \( t = T \) (LT in the tables), or \( t_1 = t = 1973.3 \) (SEG in the tables). The trend is estimated by fitting \( y_t \) to a constant and
to scaled polynomial functions of time using least squares and by taking the predicted value of the regression. The cyclical component is the residual from (1). The results I present are broadly insensitive to the choice of $t$ in the range [1973.1-1975.1].

2.1.2 First Order Differencing

The basic assumptions of a first order differencing procedure (FOD in the tables) are that the secular component of the series is a random walk with no drift, the cyclical component is stationary and that the two components are uncorrelated. In addition, it is assumed that $y_t$ has a unit root which is entirely due to the secular component of the series. Therefore $y_t$ can be represented as:

$$y_t = y_{t-1} + \epsilon_t$$

(3)

the trend is defined as $T_t = y_{t-1}$ and an estimate of $\epsilon_t$ is obtained as $\hat{\epsilon}_t = y_t - y_{t-1}$.

2.1.3 Beveridge and Nelson's Procedure

The key identifying assumption of Beveridge and Nelson's (1981) procedure is that the cyclical component of the series is stationary while the secular component accounts for its nonstationary behavior. Let $w_t = (1 - \ell) y_t$ be a stationary ARMA process with moving average representation

$$w_t = \mu + \gamma(\ell) \xi_t$$

where $\xi_t \sim i.i.d.(0, \sigma^2)$ and $\gamma(\ell) = \phi(\ell)^{-1} \theta(\ell)$ is a polynomial in the lag operator with the roots of $\phi(z) = 0$ outside the unit circle.

Beveridge and Nelson show that the secular component of a series can be defined as the long run forecast of $y_t$ adjusted for its mean rate of change $k\mu$; i.e

$$x_t = y_t + \hat{w}_t(1) + \cdots + \hat{w}_t(k) - k\mu$$

(4)

with $\hat{w}_t(i) = E_t(w_{t+i} | y_t, y_{t-1}, \cdots)$ $= \mu + \sum_{i=1}^{\infty} \gamma_i \xi_{t-j}$ so that $x_t = y_t + \sum_{j=0}^{k-1} (\sum_{i=j+1}^{j+k} \gamma_i) \xi_{t-j}$. For $k$ sufficiently large, (4) collapses to: $x_t = x_{t-1} + \mu + (\sum_{i=1}^{\infty} \gamma_i) \xi_t$. The cyclical component of the series is then

$$c_t = \hat{w}_t(1) + \cdots + \hat{w}_t(k) - k\mu = \chi(\ell) \xi_t$$

(5)

Two characteristics of this decomposition should be noted. First, since trend and cycle are driven by the same shock, this decomposition has the remarkable property that the secular and the cyclical components are perfectly correlated. Second, since estimates of the $\gamma$'s and forecasts $\hat{w}_t(i)$
are obtained from an ARIMA model, the problems inherent to ARIMA specifications are carried over to this method also (see e.g. Christiano and Eichenbaum (1990)).

Because the results vary considerably with the choice of $\theta(\ell)$ and $\phi(\ell)$, both in terms of the magnitude of the fluctuations and of the path properties of the data, I examined various ARIMA specifications. Here I present results obtained using $\theta(\ell) = 1 \forall \ell, \phi(\ell) = 1 + \ell + \ldots + \ell^5$, the actual value of GNP at 1955.2 as the initial condition and the quick computational approach of Coddington and Winters (1987) (BN in the tables).

2.1.4 Unobserved Components Model

The key identifying assumptions of this procedure are that the secular component follows a random walk with drift and that the cyclical component is a stationary finite order AR process. Also, contrary to a FOD procedure, an UC approach allows for correlation between the trend and the cycle. The most recent Unobservable Components (UC) literature assumes that the drift term in the random walk may drift over time as well (see e.g. Harvey and Jaeger (1993)). However, since the task here is to compare methodologies, not to find the best model specification, I do not consider this possibility. UC models are usually cast in a state space framework (see Harvey (1985) and Watson (1986) among others). The measurement equation is given by

$$y_t = x_t + c_t + \epsilon_t, \quad t = 1, \ldots, T.$$  \hspace{1cm} (6)

where $\epsilon_t \sim N(0, \sigma^2)$ for all $t$ and $E(\epsilon_t \epsilon_{t-1}) = 0$ for $i \neq 0$. The transition equations are

$$x_t = x_{t-1} + \delta + u_t,$$

$$c_t = \phi(\ell)c_{t-1} + \nu_t,$$  \hspace{1cm} (7)

where $\delta$ is a drift parameter and the $q$ roots of $\phi(z) = 0$ lie outside the unit circle. The properties of $x_t$ and $c_t$ are fully characterized by the assumption that the distribution of $u_t$ and $\nu_t$ are jointly normal with covariance matrix $\Sigma$ and by the fact that $\epsilon_t$, $u_t$ and $\nu_t$ are all pairwise uncorrelated. The parameters $\beta = (\sigma^2, \sigma_u^2, \sigma_c^2, \delta, \phi_j, j = 1, \ldots, q)$ are typically estimated using the prediction error decomposition of the likelihood and a smoothing algorithm that revises recursive estimates (see, e.g. Harvey (1985)). To simplify, estimates of $\beta$'s are obtained using the autocovariances of $w_t = (1 - \ell)y_t$ (see Carvalho, Grether and Nerlove (1979)). Given the estimates of $\beta$ and a zero
mean and a diagonal covariance matrix with large but finite elements as initial conditions, recursive estimates of the state vector $\alpha_t = [x_t, c_t, \ldots, c_{t-\eta}, 1]^T$ are obtained with the Kalman filter.

Here I report results obtained using 2 lags for $\phi(t)$ with no smoothing of recursive estimates (UC in the tables). The results I report are not very sensitive to the choice of lag length for $\phi(t)$ in the range [2, 4].

### 2.1.5 Frequency Domain Methods

The frequency domain procedure employed here draws from Sims (1974). The procedure assumes that the cyclical and secular components of the series are independent, that the secular component has most of its power in a low frequency band of the spectrum and that away from zero the power of the secular component decays very fast. The identification assumptions do not restrict the trend to be either deterministic or stochastic and allows for changes in the trend over time as long as the changes are not too frequent. The secular component can be recovered from $y_t$ using

$$a(\omega)F_y(\omega) = F_x(\omega)$$

where $a(\omega) = 1_{[\omega_1, \omega_2]}$ is the Fourier transform of a "low" pass filter, $1$ is the indicator function for the interval $[\omega_1, \omega_2]$ and $F_y(\omega)$ and $F_x(\omega)$ are the Fourier transforms of $y_t$ and $x_t$. In the time domain the polynomial $a(\ell)$, the inverse Fourier transform of $a(\omega)$, has the form:

$$a(\ell) = \frac{\sin(\omega_2 \ell) - \sin(\omega_1 \ell)}{\pi \ell}$$

(see e.g. Priestley, 1981, p.275) where $\omega_1$ and $\omega_2$ are the upper and lower limits of the frequency band where the secular component has all its power. An estimate of the cyclical component is then $(1-a(\ell))y_t$. The key to this procedure is the appropriate selection of the upper and lower limits of the frequency band. Following the NBER taxonomy, which describes as business cycle those fluctuations with 2-6 years periodicity, and the conventional wisdom that no complete cycle has exceeded 8 years in length, I chose $\omega_1 = 0$ and $\omega_2 = \frac{\pi}{15}$. Since the spectrum is symmetric around the origin, this filter wipes out all the power of the series in the band $(-\frac{\pi}{15}, \frac{\pi}{15})$ and cycles with length less that 30 quarters are all assumed to belong to the cyclical component of $y_t$ (FREQ in the tables). The results I present are not too sensitive to choices of values for $\omega_2$ that leave in $c_t$ cycles with maximum length between 20 and 30 quarters. Baxter and King (1994) provide a time domain version of this filter and study its implication for stylized facts of the business cycle.
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2.1.6 Hamilton's 2-State Markov Chain

Hamilton (1989) assumes that although the trend is characterized by a unit root, its shifts are drawn from a binomial distribution. The key identifying assumptions of this procedure are that trend and cycle of the log of the series are independent and that both components are nonstationary. Because the nonstationarity of the cyclical component is an odd feature of this decomposition, Lam (1990) suggests an alternative, but more complicated specification which extracts cyclical components which are stationary. Because for our analysis it is irrelevant whether \( c_t \) is stationary or not, we employ Hamilton's original approach. The model for \( x_t \) and \( c_t \) is given by:

\[
\begin{align*}
x_t &= a_0 + a_1 s_t + x_{t-1} \\
s_t &= (1 - q) + \lambda s_{t-1} + \epsilon_t \\
c_t &= c_{t-1} + \phi(t)(c_{t-1} - c_{t-2}) + \epsilon_t
\end{align*}
\]

where \( s_t \) is a two state Markov chain uncorrelated with \( \epsilon_t \) whose transition matrix has diagonal elements \( p, q; \lambda = p + q - 1 \). \( \phi(t) \) is a polynomial in the lag operator of order \( r \) with all roots outside the unit circle, \( \epsilon_t \sim iid \mathcal{N}(0, \sigma^2_t) \) and \( (v_t|S_{t-1} = 1) = 1 - p \) with probability \( p \) and \( -p \) with probability \( 1 - p \) and \( (v_t|S_{t-1} = 0) = -(1 - q) \) with probability \( q \) and \( q \) with probability \( 1 - q \).

Given an initial condition \( x_0 \) and estimates of \( a_0, a_1 \) and \( s_t \), an estimate of the cyclical component of the series can be obtained recursively from (10)-(12). Estimates of \( a_0, a_1 \) and \( s_t \) are obtained using Hamilton's EM algorithm when \( \phi(t) \) is a second order polynomial (HAMIL in the tables).

2.1.7 One Dimensional Index Model

The final procedure in the statistical group is multivariate and assumes that while each series is trending, either deterministically or stochastically or both, some linear combination of them does not have trends (see e.g. Stock and Watson (1989)). The key assumption is that in the low frequencies of the spectrum there exists a one dimensional process (a secular component) which is common to all series (see Quah and Sargent (1993) for a two-index model). This process is characterized by the property that it has all its power at low frequencies and that away from zero it decays very fast. The model for the \( n \times 1 \) vector \( y_t \) is given by (1) and

\[
\begin{align*}
x_t &= Az_t \\
z_t &= z_{t-1} + \delta + \eta_t
\end{align*}
\]

where \( s_t \) is a two state Markov chain uncorrelated with \( \epsilon_t \) whose transition matrix has diagonal elements \( p, q; \lambda = p + q - 1 \). \( \phi(t) \) is a polynomial in the lag operator of order \( r \) with all roots outside the unit circle, \( \epsilon_t \sim iid \mathcal{N}(0, \sigma^2_t) \) and \( (v_t|S_{t-1} = 1) = 1 - p \) with probability \( p \) and \( -p \) with probability \( 1 - p \) and \( (v_t|S_{t-1} = 0) = -(1 - q) \) with probability \( q \) and \( q \) with probability \( 1 - q \).
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\[ c_t = φt c_{t-1} + c_t \]  \hspace{1cm} (15)

where \( z_t \) is a scalar process, \( A \) is an \( n \times 1 \) vector of loadings and \( x_t \) is an \( n \times 1 \) vector independent of \( c_t \). An estimate of \( x_t \) is obtained using a multivariate version of the procedure used for the UC model and \( \hat{c}_t \) is obtained residually from (1) (MINDEX in the tables). The vector \( y_t \) is composed in our case of GNP, Consumption, Investment, Real Wage and Capital.

2.2 Economic Procedures

2.2.1 A Model of Common Deterministic Trends

King, Plosser and Rebelo (1988) present a neoclassical model of capital accumulation with labor supply choices where there is deterministic labor augmenting technical progress. Their model implies that all endogenous variables have a common deterministic trend (the growth rate of labor augmenting technical progress) and that fluctuations around the common linear trend are all of a transitory nature. Each time series is therefore generated by a model like (1) where the secular and cyclical components are independent, where \( x_t \) is common to all series and given by

\[ x_t = x_0 + δt \]  \hspace{1cm} (16)

where \( δ \) is the growth rate of technological progress. To construct a deterministic trend which is common to all series I use data on GNP, Consumption, Investment, Real Wage and Capital and select \( x_0 \) to be an estimate of the unconditional mean of each series. The resulting estimate of \( δ \) is 0.7%. This differs from the one of King, Plosser and Rebelo (0.1%) because they employ per-capita variables, do not use the capital stock in the calculations and they employ a different sample (CDT in the tables).

2.2.2 A Model of Common Stochastic Trends

King, Plosser, Stock and Watson (1991) propose a version of King, Plosser and Rebelo's (1988) model where the long run properties of the endogenous variables are driven by the same nonstationary technological shock. The corresponding statistical common trend representation, developed in Stock and Watson (1988), implies that all the endogenous variables have a common trend. This approach produces, as a by-product, a decomposition into secular (nonstationary) and cyclical (stationary) components which is the multivariate counterpart of the method of Beveridge and
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Nelson. Let \( w_t \) be an \( n \times 1 \) vector of time series, \( w_t = (1 - \ell)y_t \) with moving average representation \( w_t = \delta + C(\ell)\epsilon_t + B(\ell)z_t \) where \( \alpha'C(1) = 0 \), \( \epsilon_t = C(1/2)v_t \) with \( v_t \sim iid \ (0, I) \) and \( \alpha \) is a set of cointegrating vectors. Stock and Watson show that the model implies that:

\[
x_t = y_0 + A\tau_t = y_0 + \delta t + C(1)\zeta_t
\]

(17)

\[
c_t = D(\ell)\epsilon_t
\]

(18)

where \( A \) is an \( n \times k \) vector, \( \tau_t = \mu + \tau_{t-1} + \eta_t \), \( \eta_t \) is a serially uncorrelated random noise, \( dim(\tau_t) = k \leq n \), \( D_j = -\sum_{i=1}^{\infty} C_i \) and \( \zeta_t = \sum_{s=1}^{\ell} \epsilon_s \). Rather than testing whether there is a cointegrating vector \( z_t \), I estimate a vector error correction model (VECM) and use one lag of two cointegrating vectors (GNP/consumption, GNP/investment) to obtain estimates of \( \delta, C(\ell) \) and \( \epsilon_t \). An estimate of the transitory component is obtained by taking \( \hat{\epsilon}_t = y_t - y_0 - \hat{\delta}t - \hat{C}(1)\hat{\zeta}_t \).

As in the Beveridge-Nelson decomposition, estimates of \( x_t \) and \( \epsilon_t \) differ for different specifications of the VECM model (both in terms of the number of variables and lag length). Here I present the results obtained using data on GNP, Consumption, Investment, Hours, Real Wage and Capital and five lags for each variable (COIN in the tables).

2.2.3 The Blanchard and Quah Approach

Blanchard and Quah (1989) propose a procedure to decompose bivariate systems into trend and cycle which has its economic justification in a version of Fischer's (1977) model of staggering contracts. Their behavioral model delivers a representation for the bivariate vector \( X = (\Delta y, u) \), where \( y \) represents GNP and \( u \) unemployment, of the form:

\[
X_t = A(\ell)\epsilon_t, \quad \epsilon_t \sim (0, I)
\]

(19)

where the upper left entries of \( A(\ell) \) sum to zero, i.e. \( a(1)_{11} = 0 \). This implies that one shock has long run effects on GNP and the other does not, while neither have long run effects on \( u \). The implied trend-cycle decomposition satisfies the assumption that the two components are uncorrelated and that the trend has unit root like behavior. To recover the behavioral model from an unrestricted VAR the following restrictions must be satisfied:

\[
\epsilon_t = A(0)\epsilon_t
\]

(20)

\[
A_j = C_j A(0)
\]

(21)
where $v_t$ are the innovations of the unrestricted VAR representation of the data and $C_t$ is the matrix of moving average coefficients from the data. In practice, the Blanchard and Quah decomposition requires the estimation of an unrestricted VAR and the computation of the structural coefficients and innovations using (20) and $A_1 = C_1 A(0)$. An estimate of the trend in GNP is $\hat{T}_t = \sum_j \hat{a}_{12}(j)\hat{e}_2(t - j)$ and an estimate of the cycle is $\hat{c}_t = y_t - \hat{T}_t$.

One appealing feature of this decomposition is that it generates an unrestricted permanent-transitory decomposition for series with unit roots which overcomes the lack of economic interpretability of decompositions like UC or BN. One important drawback is that the properties of the cyclical component of GNP are sensitive to both the dimension of the VAR, to the variables included and to the number of shocks impinging on the actual economy (see e.g. Faust and Leeper (1993)). The results presented here pertain to the same specification employed by Blanchard and Quah (1989) (BQ in the tables).

2.3 The Hodrick and Prescott’s Filter

The Hodrick and Prescott (HP) (1980) filter has two justifications: one intuitive and one statistical.

In the Real Business Cycle (RBC) literature the trend of a time series is not intrinsic to the data but is a representation of the preferences of the researcher and depends on the economic question being investigated (see also Maravall (1993)). The popularity of the HP filter among applied macroeconomists results from its flexibility to accommodate these needs since the implied trend line resembles what an analyst would draw by hand through the plot of the data (see e.g. Kydland and Prescott (1990)).

The selection mechanism that economic theory imposes on the data via the HP filter can be justified using the statistical literature on curve fitting (see e.g. Wabha (1980))\(^1\). In this framework the HP filter optimally extracts a trend which is stochastic but moves smoothly over time and is uncorrelated with the cyclical component. The assumption that the trend is smooth is imposed by assuming that the sum of squares of the second differences of $x_t$ is small. An estimate of the secular component is obtained by minimizing:

$$\min_{[c_t]_{t=1}^T} \left[ \sum_{t=1}^T c_t^2 + \lambda \sum_{i=2}^T ((x_{i+1} - x_t) - (x_t - x_{t-1}))^2 \right] \quad \lambda > 0$$  \hspace{1cm} (22)

where $T$ is the sample size and $\lambda$ is a parameter that penalizes the variability of trend. As $\lambda$

\(^1\)Harvey and Jaeger (1993) offer an unobservable component interpretation of this filter.
increases, the penalty imposed for large fluctuations in the secular component increases and the path for $\hat{x}_t$ becomes smoother. In this context, the "optimal" value of $\lambda$ is $\lambda = \frac{\sigma_x^2}{\sigma_e^2}$, where $\sigma_x$ and $\sigma_e$ are the standard deviations of the innovations in the trend and in the cycle.

Users of the HP filter select $\lambda$ a-priori to isolate those cyclical fluctuations which belong to the specific frequency band the researcher wants to investigate. With quarterly data, $\lambda = 1600$ is typically chosen which results in a filter that leaves in the data cycles of average duration of 4-6 years. While this approach is meaningful from the point of view of a business cycle researcher, the assumed magnitude of $\lambda$ is debatable. Nelson and Plosser (1982) estimated $\lambda$ to be in the range $[16, 1]$ for most of the series they examine. Harvey and Jeager find values in the range $[1, 8]$. This implies that much of the variability that the Hodrick and Prescott filter attributes to the cyclical component is, in fact, part of the trend. To investigate this possibility I experiment with two values of $\lambda$: a standard one (HP1600 in the tables) and one obtained by assuming that the relative standard deviation of the innovations in the components is 2 (HP4 in the tables). Results obtained when $\lambda$ is estimated by maximum likelihood are intermediate between these two and not reported.

In practical terms the procedure involves the solution of a system of $T$ linear simultaneous equations in $T$ unknowns, of the form $A\hat{x} = y$ where $x = [x_1, x_2, \ldots, x_T]'$ and $y = [y_1, y_2, \ldots, y_T]'$ and $A$ is a sparse matrix. An estimate of the cyclical component is obtained from (1).

Some of the properties of the HP filter when $T - \infty$ and the penalty function is two-sided have been highlighted by Cogley and Nason (1991) and King and Rebelo (1993). Some of the similarities between the HP procedure and seasonal adjustment procedures and some of the drawbacks of the approach are discussed by Maravall (1993). The relationships between the HP and exponential smoothing (ES) filters have been investigated by King and Rebelo (1993).

2.4 A Word of Caution

Before proceeding with the analysis it is useful to stress three important facts which may make the approaches not exactly comparable. First, the information used to compute the trend of the series differs across detrending methods. While most procedures employ information up to the end of the sample, FOD, UC and HAMIL only use the information available at $t - s$ to compute the trend for $t - s + 1$. This may generate a more imprecise estimate of the trend and, as a consequence, produce cyclical components which are more erratic than those obtained with other methods. As a consequence, most methods date peaks and troughs having available data for the entire
time span, while others "call them out as they go". Second, while most methods use maximum likelihood procedures to estimate the parameters, others use only approximate maximum likelihood techniques and with three procedures (FOD, HP and FREQ) no parameter is estimated from the data. Because the sample size is relatively short, this may induce small sample differences in the estimates of the cyclical components. These differences should be kept in mind when comparing turning point dates and the amplitude properties of the estimates of the cyclical component across detrending methods. Third, because the UC model assumes the presence of both an irregular and a cyclical component, care should be exercised in comparing the path properties of $c_t$ (and the record of turning point classification) obtained with UC and other methods since the UC cyclical component is likely to be much smoother than others.

3 The Data, the Dating Rules and Summary Statistics

3.1 The Data

The data used in the exercise is taken from the Citibase Tape. The results refer to the logarithm of seasonally adjusted quarterly US series for the period 1955:3-1990:1. For all univariate procedures we use real gross national product in 1982 dollars (Citibase name: GNP82). For multivariate procedures we add to GNP consumption expenditure by domestic residents on nondurables and services (Citibase names: GSC82+GCN82), fixed investment in plants and equipment plus consumer durables (Citibase names: GINPD82+GCD82), total number of hours of labor input as reported by establishment survey data (Citibase name: LPHU), real wage constructed as the ratio of nominal total compensation of nonagricultural employees and the CPI (Citibase names: GCOMP/PUNEW) and a capital stock series constructed using the net capital stock for 1954, the quarterly series for investment and a depreciation rate of 2.5% per quarter. For the BQ decomposition I use, in addition to GNP, the seasonally adjusted unemployment rate for males, age 20 and over, as reported by the US Bureau of Labor Statistics (Table A-39).

3.2 Determining the Reference Cycle

The first step in examining the properties of the cycle is to delineate periods of economic expansions and contractions. According to NBER practices as set out by Burns and Mitchell (1943), this is typically done by examining the behavior and the comovements of a variety of series, many of which are not perfectly in phase, and constructing an index of cyclical movements (the reference cycle).
3 THE DATA, THE DATING RULES AND SUMMARY STATISTICS

From this information a set of reference dates, which specify turning points in aggregate economic activity, are selected and business cycle phases are automatically constructed. This process has the drawback of being time consuming and involving a considerable amount of subjective judgement in selecting reference dates.

In this paper I depart from the standard Burns and Mitchell approach in several ways. First, as in Simkins (1991) and King and Plosser (1991), instead of constructing an index of cyclical fluctuations, I use the cyclical component of real GNP as a simple measure of the reference cycle. Although it has been suggested that using the cyclical component of GNP to proxy for the reference cycle fails to capture certain contractions (see e.g. Zarnowitz and Moore (1991)), our choice has the advantage of eliminating judgmental aspects present in the standard procedure and of being easily reproducible. In addition, because a large number of economic variables appear to be procyclical and coincident with GNP, this choice of reference cycle should only minorly distort the dating of turning points even though the amplitude characteristics of the cycle and the severity of contractions may be misrepresented. Finally, the four multivariate procedures do use the information contained in several additional series. Therefore, by comparing the dating record obtained with univariate and multivariate methods we can check whether the information contained in GNP is sufficient to accurately characterize turning points and describe the properties of the reference cycle.

Second, as many have done in this literature, I use mechanical rules to select the reference cycle turning points. However, contrary to e.g. Simkins (1991) or King and Plosser (1991), which use variants of the Bry and Boschen (1971) algorithm, I use two simple and commonly used rules to date turning points. The first classification rule I use is very standard (see e.g. Wecker (1979) or Zellner and Hong (1991)). It defines a trough as a situation where two declines in the cyclical component of GNP are followed by an increase, i.e., at time $t$, $c_{t+1} > c_t < c_{t-1} < c_{t-2}$. Similarly, a peak is defined as a situation where two consecutive increases in the cyclical component of GNP are followed by a decline, i.e., at time $t$, $c_{t+1} < c_t > c_{t-1} > c_{t-2}$. The second classification rule is less standard but it has some appealing features (see e.g. Webb (1991)). It selects quarter $t$ as a trough (peak) if there have been at least two consecutive negative (positive) spells in the cyclical component of GNP over a three quarter period, i.e. when $c_t < (>)0$ and $c_{t-1} < (>)0$ or when $c_{t+1} < (>)0$ and $c_t < (>)0$.

The first classification rule emphasizes primarily the duration characteristics of the cycle (no mention of severity is made) and therefore may pick up mild contractions and mild recoveries, while
3 THE DATA, THE DATING RULES AND SUMMARY STATISTICS

this is not necessarily the case with the second rule. On the other hand, also the second classification may suffer from amplitude misspecifications if there are multiple sequential peaks (and troughs) in the reference cycle (for an example of this type see figure 1). In general, the first rule may signal the presence of a turning point earlier than the second one. Therefore the two rules balance the scope for an early recognition of the phenomena (at the cost of possible false alarms) vs. its more accurate description (at the cost of a later discovery). Also, it is important to emphasize that we make no adjustments for situations where the reference cycle may reach a plateau around a turning point.

One reason for using these two rules instead of others is that several authors (Wecker (1979), Webb (1991)) have shown that when they are applied to a standardlly constructed (level) reference cycle they generate turning points which match NBER dates. There are variants and combinations of these two rules which can reduce the frequency of missing signals and discount false alarms (see e.g. Hymans (1973) or Zarnowitz and Moore (1991)) and one can design sequential dating procedures (as in Zarnowitz and Moore (1982)), or rules examining the path properties of the reference cycle over longer spans of time (see e.g. McNees (1991) or Stock and Watson (1990)) and add to the mechanical recognition of the extremes of the cycle the flexibility of ex-post adjustments to improve the overall dating record (see Romer (1994)). However, we restrict attention to these two because they are simple, easily reproducible and provide a useful benchmark to compare the properties of the various reference cycles. These two rules are also preferable to the Bry and Boschen algorithm for our purposes because the latter is sufficiently complicated to render the comparison across detrending methods less transparent. In addition, since the Bry-Boschen algorithm computes turning point dates by detrending the data with a series of MA filters, it appears inappropriate to apply it in its original form to detrended data.

As a term of reference in our exercises we use the dating reported by the Center for International Business Cycle Research for the NBER (NBER) and by the Department of Commerce (DOC) (both of which are taken from Niemera (1991)). The procedure the NBER employs to construct growth cycles is complicated and involves the calculation of the trend of a vector of series by piecewise smooth interpolation of segments of a series obtained by filtering the original data with long term moving averages (see Zarnowitz (1991a)). The DOC reference growth cycle, on the other hand, is constructed by detrending the reference index using an exponential smoothing method (Higgins and Poole procedure, see Niemera (1991)). In both cases, turning points and cycle phases are
identified using a mixture of mechanical rules and subjective intuition.

3.3 Summary Statistics

To analyze the statistical features of the reference cycle and how generated contractions and expansions match up with standardly reported business cycle phases, I compile a number of statistics. To evaluate the dating record of a procedure it is quite common to use simple summary statistics of the differences (in quarters) between the signal and the NBER or DOC turning point. Because such an approach wastes useful information, I employ a different summary statistic based on the timing of the event. I rank the signal as false if a NBER or a DOC turning point does not appear within a ± 3 quarter interval around the selected date and missing if no signal appears within a ± 3 quarter interval around the actual NBER or DOC turning point. The proportion of false alarms and missing signals to the total number of turning points gives an idea of how each detrending procedure trades off the two types of losses. For those turning points which are correctly identified, I also record the proportion of cases where the selected date is leading, coincident or lagging the corresponding NBER or DOC date. This information may suggest whether some detrending method generates a systematic bias in recognizing standardly classified turning points with one of the two rules.

Together with the dating record of turning points I also present five statistics summarizing the statistical properties of each reference cycle: the average amplitude of contractions, the maximum amplitude and the date at which it occurs, the average duration of expansions and contractions (trough-to-peak (TP) and peak-to-trough (PT) half-cycles) and the percentage of times the economy is expanding. While it is typical to measure the severity of recessions using the distance between the peak and the trough of the cycle, I define severity using the distance of the troughs from the trend line, an approach which is more consistent with the growth-cycle approach adopted in the paper. This measure of the severity is clearly imperfect, but gives a rough idea of how the different detrending methods picture contractions with each of the two dating rules. The percentage of times the economy is in an expansion, on the other hand, is a useful statistic to gauge whether the reference cycle generated by each detrending method and each rule is symmetric or not (see e.g. Klein and Moore (1985)) \(^2\).

\(^2\)As an alternative, it would be instructive to compute, for each method and each dating rule, the ratio between the percentage of dates the economy is above the trend and below the trend. Although the absolute magnitude of the numbers differ from the one reported, the relative ranking across detrending methods and dating rules is unaltered.
To examine whether there is a tendency for contractions and expansions to terminate the longer they have lasted, a question recently investigated using NBER dates by Diebold and Rudebush (1990) and (1992). I also computed a nonparametric test for duration dependence of each business cycle phase. This test formally examines whether contractions or expansions have a recurrent and stable (periodic) structure with any detrending method or dating rule, a feature which would facilitate the prediction of turning points. The test, developed by Stephens (1978), is exact even for samples of three durations and incorporates the idea that there is a minimum duration of each phase. In our case the minimum duration \( \gamma \) is two quarters for each phase. This selection is based on the criteria used by NBER researchers in dating contractions and expansions and on the characteristics of the two dating rules we employ. The results we report, however, are not too sensitive to the choice of this parameter within a reasonable range. The statistic used to test for duration dependence is given by:

\[
W(t_0 = \gamma) = \frac{(\sum_{i=1}^{N} z_i - \gamma)^2}{N[(N + 1) \times (\sum_{i=1}^{N} (z_i - \gamma)^2 - (\sum_{i=1}^{N} z_i - \gamma)^2)]}
\]  

(23)

where \( z_i \) is the \( i \)-th ordered duration for each detrending procedure and each dating rule. The \( W(t_0 = \gamma) \) statistic for \( N \) durations has an exact small sample distribution which can be recovered from Shapiro and Wilks' (1972) tables using the line corresponding to \( N+1 \) durations.

Finally, to study whether there is any systematic relationship between the severity of contractions and either their duration or the duration of full peak-to-peak cycles, I computed Spearman's rank correlation coefficient between the amplitude of contractions and the two types of cycles and tested if they are different. Burns and Mitchell (1943) and Moore (1958) suggested that, because of the way recessions spread in the economy, the association between the severity of the contractions and peak to peak cycles should be stronger than the association between the severity and the duration of contractions. Knowing the severity of a contraction is therefore considered an important ingredient to predict how long it would take the economy to reach another peak level. Also, as emphasized by Romer (1994), the association between the severity and the duration of contractions may indicate how rapidly the effects of a contraction are undone, thereby providing a rough measure of the persistence of this business cycle phase.
4 The results

The results of the investigation appear in tables 1 and 2. Table 1, Panel A reports, for each detrending method, the number of troughs (column 1) and of peaks (column 7) found, the percentage of false alarms and missing signals for troughs (columns 2-3) and for peaks (columns 8-9) and for correct signals if they are leading, coincident or lagging the NBER classification (columns 4-6 and 10-12) for each of the two dating rules. Panel B of the table reports the same information when the DOC classification is used as a term of comparison.

Table 2 presents, for each detrending method, the average severity of contractions and its standard deviation (column 1), the maximum amplitude of the contractions and the date at which it occurs (columns 2-3), the percentage of times the economy is in an expansion phase over the sample period (column 4), the average duration of contractions and expansions and their standard deviations (columns 5 and 7), the values of the Stephen’s test for duration dependence (columns 6 and 8), the rank correlation coefficient between the amplitude of contractions and the duration of full peak to peak cycles and between the amplitude and the duration of contractions (columns 9 and 10) for each of the two dating rules.

4.1 Dating Turning Points

The main features of table 1 are the sensitivity of turning point classification to detrending and dating rules and, to some extent, the dependence of the results on the reference dating employed. The lack of robustness in the characterization of the extremes of the reference cycle appears in several aspects of the table. First, in agreement with McNees (1991) and Zarnowitz and Moore (1991), the number of complete cycles identified depends on the detrending methods and the dating rule. With the first rule, all methods select at least 8 peaks and 8 troughs (with a maximum of 11), while with the second rule the range of extremes identified by the various detrending methods is much larger (between 2 and 15 peaks and 2 and 16 troughs).

Second, the percentage of false alarms varies substantially with the detrending method and the dating rule. For troughs, the percentage of false alarms is between 25 and 90% with the first dating rule and 0 and 80% with the second when the NBER classification is used and between 37.5 and 87.5% with the first dating rule and 0 and 75% with the second when the DOC classification is used. For peaks the heterogeneity is even more evident. When we use the NBER classification as
a reference the percentage of false alarms is between 37.5 and 88.8% with the first dating rule and 0 and 100% with the second dating rule. When we use the DOC classification as a reference the percentage of false alarms is between 25 and 77.7% with the first dating rule and 0 and 100% with the second dating rule.

Third, the percentage of missing signals depends on the detrending method and differs significantly between troughs and peaks. For example, when the NBER classification is used as a benchmark the range of missing troughs is between 11.2 and 85.7% with both dating rules, while the range of missing peaks is between 11.2 and 83.4% with the first rule and between 0 and 100% with the second one. Interestingly, there are two NBER troughs (61.1 and 75.1) and one DOC trough (75.2) which are missed by practically all methods. Note that the 1975 recession was a multiple dips recession in which the growth rate of output was positive during two quarters so that both dating rules find it difficult to appropriately identify the trough date. Notice also that, generally speaking, all methods are worse off in dating peaks than troughs with the second rule. This may be due to the fact that peaks appear more as plateau than sharp edges and the second rule finds it difficult to clearly pick a turning date in this situation.

Fourth, for those turning points which are identified within the chosen confidence interval, there are differences across detrending methods, types of turning points and, to some extent, benchmark classification. In general, when NBER benchmark is used many detrending methods generate troughs which lead or coincide and peaks which lead the standard classification regardless of the dating rule employed. The exceptions are BQ and BN detrending which produce reference cycles whose turning points lag NBER dates both for peaks and troughs. When DOC reference is used, the results are more heterogeneous. With the first dating rule all detrending methods produce troughs which lead or coincide with DOC troughs while with the second dating rule trough dates generally lead DOC troughs. On the other hand, the reported peak dates generally lag standard DOC dates with the first dating rule, but lead with the second. Also with this classification, both the BQ and BN methods produce turning points which tend to lag benchmark dates.

Furthermore, univariate procedures generate trough dates which anticipate benchmark trough dates in several instances, while multivariate procedures select trough dates which, in general, coincide with benchmark dates when the first dating rule is used. This heterogeneity is less evident with the second dating rule but this may be due to the fact that the number of correctly recognized turning points is typically smaller. Overall, with the second dating rule all detrending methods
generate turning point dates which lead by about 1-2 quarters, regardless of the benchmark classification used. On average and regardless of the dating rule employed, each method appears to produce smaller discrepancies relative to the DOC reference classification.

Fifth, although multivariate detrending procedures employ more information to construct the reference cycle than univariate ones, they do not provide a necessarily superior picture in dating business cycle phases. In particular, these methods produce reference cycles whose turning points do not match NBER or DOC dates and for three out of the four detrending methods, the dating performance is definitively inferior relative to the one of univariate procedures with at least one dating rule. While the relevance of this finding clearly depends on the variables included in the econometrician’s information set and different information sets may give different conclusions, the results suggest that the loss of information incurred in constructing reference cycles using real GNP alone may be small.

In conclusion, the dating of turning points appears to be sensitive to the choice of detrending. Differences emerge in the dates selected, in the number of cycles discovered and in the number of false alarms and missing signals they generate.

4.2 The Statistical Properties of Reference Cycles

Next we study the statistical properties of generated reference cycles. In particular, I am interested in the amplitude characteristics of contractions and in the duration and persistence of various business cycle phases, as these are the statistics typically employed in the literature to summarize the properties of reference cycles.

4.2.1 Amplitudes

Amplitude measures display significant differences across detrending methods and dating rules. With the first rule the largest average amplitude is -5.0% which is obtained with BQ, while the others range from -0.1% obtained with MLT to -2.4% obtained with MINDEX. For two methods (UC and SEGM) the average amplitude of contractions is positive, i.e. on average, contractions were mere slowdowns of economic activity which did not involve crossing below the trend of the real GNP series. With the second rule the average amplitude of contractions is, in general, smaller. The maximum value is -2.5% obtained, once again, with BQ, while the others range from -0.1% obtained with UC to -1.1% obtained with FOD filter.
Additional information on the amplitude of the resulting cycles can be obtained by examining the timing and the severity of the worst contraction. With the first dating rule, the severity of the worst contraction varies substantially with the detrending method, ranging from -0.4% with UC to -11.3% with BQ, with most of the other methods producing a drop of approximately 4.0-5.0% below the trend line. Out of the 13 detrending procedures, 5 picked 1982.1 as the worst time and one 1982.1, while the remaining 7 methods selected dates from 1957 to 1960. Interestingly enough, no method except UC selected a date in the middle of the 1970's as the worst time in the sample. Once again, there is much more homogeneity in the results with the second rule: the range for amplitude of the worst contraction is between -0.2% with UC and -6.4% with BQ, with most other methods producing a maximum fall of 1.5-2.0% below the trend line. This homogeneity however is more the result of the poor dating record of many procedures rather than an intrinsic similarity of the reference cycles generated with this dating rule. This impression is confirmed by the considerable variety of dates picked by each method as the worst contraction date. Four methods selected dates between 1957 and 1958 and two picked 1980.2, but for the rest there appears to be little congruence. Finally, note that LT selected 1975.1 as the worst recession and at this date the cyclical component of GNP was about 2.5% below the trend.

4.2.2 Durations

The average duration of expansions is not sensitive to detrending when the first rule is used: the range is between 6.6 and 10.1 quarters (with a standard deviation of about 5 quarters) and, except marginally for BQ, there is no evidence of duration dependence for this phase. That is, there is no evidence that expansions tend to terminate the longer they have lasted. A somewhat different picture emerges when we look at contraction cycles. In this case the range of average durations is slightly larger varying from 3.8 to 9.25 quarters but for 7 out of the 13 methods, the null hypothesis of no duration dependence of contractions is rejected. However, there seems to be no relationship between the average duration of contractions and the rejection of the hypothesis of no duration dependence. Therefore, in agreement with Diebold and Rudebush (1990), the prediction of peak dates is problematic, given the highly irregular nature of expansion phases, but it appears to be easier to predict trough dates. This is generally true regardless of the detrending method.

The average duration of expansions exceeds the average duration of contractions for all reference cycles except those generated with FOD and COIN. Typically, expansions last about 1.5 times
longer than contractions. In addition, all reference cycles indicate that the economy is expanding 5-15% more times than contracting, a result in broad agreement with those of Klein and Moore (1985).

With the second rule the features of the durations of business cycle phases strongly depend on detrending. The average duration of expansions ranges from 4.5 quarters with FOD to 36 quarters with MINDEX, while the average duration of contractions ranges from 4 quarters with HP4 to 36.3 quarters with UC. The range of standard deviations is large as well ranging from 4 to 24 quarters for expansions and from 1.6 to 21.1 for contractions. With this second dating rule there is some evidence of duration dependence of expansion cycles for five methods, while contractions display duration dependence only with LT. Moreover, for 7 detrending methods the average duration of contractions exceeds the average duration of expansions and except for HP4, LT, HAMIL and MLT the economy is contracting in more than 50% of the time periods. Finally, there are strong asymmetries in the duration of business cycle phases with FOD and UC detrended data.

4.2.3 Persistence

Burns and Mitchell (1943), Moore (1958) and others have argued that the severity of contractions is an important ingredient to know how long it will take to the economy to recover to its previous peak level. A direct test of their conjecture is impossible within the present context because their analysis did not distinguish the trend from the cycle. As a close substitute, I examine the relationship between various business cycle phases and the severity of contractions. The hypothesis then states that the deeper is the contraction (as measured here by the amplitude of the trough relative to the trend), the longer is the duration of the complete peak to peak cycle. On the other hand, there should be no systematic relationship between the depth of contractions and their duration. (See Romer (1994) for an alternative view regarding the relationship between the depth of the contraction and their duration).

Table 2 indicates that the conjecture is not supported in the data. However, the results should be interpreted with caution because of the small number of durations available with many reference cycles, especially with the second rule. In general, although the correlation between the severity of contractions and the duration of full peak to peak cycles appears to be stronger than the correlation between the severity of contractions and their duration for all reference cycles, the differences are statistically insignificant. Moreover, in both cases, the rank correlation coefficients are not signif-
4 THE RESULTS

icantly different from zero and this is true regardless of the detrending method used to construct growth cycles and the dating rule employed to classify turning points.

To summarize, the amplitude and duration properties of the business cycle phases depend, as in the case of turning point classification, on the detrending methods and on the dating rule. However, the persistence properties of contractions and peak-to-peak cycles are robustly unrelated to the severity of contractions.

4.3 Comparison with Benchmark Growth Cycles

We next turn to the final question addressed in this paper, i.e., is there a detrending method which reproduces the features of standard reference cycles regardless of the dating rule employed?

Tables 1 and 2 indicate the HP filter and the FREQ filter come closest to do the job. In particular, they are the detrending procedures which minimize the unweighted sum of false alarms and missing signals, regardless of the dating rule or the benchmark classification used. These methods are conservative in the sense that the implied reference cycles are sufficiently smooth to avoid the generation of too many false alarms while avoiding missing important signals. As a matter of fact, the HP1600 filter and the FREQ filter capture all DOC peaks with the first dating rule while HP4 captures all NBER and DOC dates with the second dating rule. On average the turning points they generate slightly lead NBER turning points and are coincident with DOC turning points.

The similarities between HP1600 and FREQ filters we unveil confirm, on one hand, the low band-pass features of the HP filter highlighted by King and Rebelo (1993) and, on the other, the MA features of the FREQ filter (see also Baxter and King (1994)).

Among the other methods, it is worth noting that the HQ approach does well both in terms of false alarms and missing signals with the first dating rule but is clearly inappropriate with the second dating rule. The Hamilton filter also performs very poorly with the second dating rule where it either misses or incorrectly identifies 13 of the 15 turning points of the sample regardless of the benchmark classification employed. This, however, is not surprising since the method was designed to be optimal with a probabilistic dating rule.

The statistical properties of the various business cycle phases generated with these two methods are also broadly consistent with both NBER and DOC cycles. In particular, the HP1600 filter

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3 According to Hamilton's rule the economy is in a contraction if there is at least 50% probability of being in a low state.
5 CONCLUSIONS

generates cycles which are slightly asymmetric as are the NBER cycles, while the FREQ filter cycle closely replicates the more symmetric pattern of DOC cycles. Moreover, the amplitude characteristics of both benchmark cycles are sufficiently well approximated by the reference cycles generated by these methods.

The worst performers in this comparison with benchmark growth cycles are FOD, LT, SEGM and HAMIL. To investigate why these procedures fail to generate cycles that resemble the ones identified by NBER and DOC researchers I present in figure 2 the time paths of the reference cycles generated by these four detrending methods. Shaded regions represent contractions according to the NBER classification. The reference cycle generated by FOD is very erratic, in many standardly classified contractions it is above the trend and in others it does not conform to the two-quarter-declines-over-three rule. The other three methods produce reference cycles which are visually similar even though the amplitude of the fluctuations differ. Note that these reference cycles are persistently on one side of the trend line for long periods of time producing infrequent shifts in the turning point indicator when the second dating rule is used. Furthermore, the average duration of a cycle is 1.5 years with FOD, 7.5 years with HAMIL and more than 8 years with LT and SEGM. Therefore, none of these detrending methods produce cyclical components whose average duration matches the average duration of NBER growth cycles (which have a periodicity of about 4-6 years).

5 Conclusions

This paper examined three questions concerning (i) the sensitivity of turning points classification to different detrending methods and dating rules, (ii) the robustness of the time series properties of the implied reference cycles and (iii) the ability of different methods to replicate NBER or DOC dating. We use a variety of detrending methods to separate the trend from the cycle in the data and two different dating rules to select turning points and construct business cycle phases.

Overall, the results indicate that the dating of turning points is sensitive to detrending and dating rules and that both the amplitude and duration properties of the growth cycles generated with alternative detrending methods significantly differ. These results confirm the findings of Canova (1993), who shows that the second moments of the cyclical component of several US real macroeconomic variables are very sensitive to detrending. The sensitivity of outcomes to detrending is easily interpretable since different detrending methods leave cycles of different average duration
5 CONCLUSIONS

in the data. What is surprising is that differences in the average duration of cycles are somewhat irrelevant when the first dating rule is used. That is, while the second moment properties of the data vary with detrending, the time paths of the various cyclical components are not too different. These differences are however amplified with the second rule because the crucial factor for dating turning points and selecting business cycle phases is whether the reference cycle is above or below the trend line. In this case, asymmetries emerge because the average spans of time spent above and below the trend line differ across detrending methods.

Is there any sensible way to reduce the range of outcomes by eliminating some detrending methods as “unreasonable”? If we take the ability to reproduce a standard turning points classification as a limited information test to select a class of detrending methods, then the results suggest that HP and FREQ filters are those which should be selected as they come closest in reproducing standard dating and business cycle features. Turning points line up in the right way and, regardless of the dating rule, the features of the implied cycle resemble those of NBER or DOC growth cycles. This apparent superiority of this class of low band-pass filters, however, should be weighed against the drawbacks noted by Cogley and Nason (1991), Harvey and Jeager (1993) and Maravall (1993). For a more complete answer on the subject it is therefore necessary to confront the various detrending procedures with alternative and, possibly, more powerful tests.

This paper did not address questions concerning the construction of leading indicators and of useful statistics to evaluate the record and the quality of turning point forecasts. In the literature on the subject (see e.g. Wecker (1979), McNees (1991) or Zellner and Hong (1991)), the results generally hinge on having available a “correct” reference cycle. Therefore, the results contained in this paper are of interest to researchers engaged in these important activities as they may give a rationale for choosing one concept of cycle or one dating rule over another. On the other hand, forecasting exercises comparing both the record and the quality of turning point selections may be a useful class of tests to examine the superiority of one trend specification over another. We plan to conduct these experiments in future research.
REFERENCES

References


REFERENCES


REFERENCES


Table 1, Panel A
Business Cycle Chronology using NBER Dates as Reference
Sample 1953-90.1

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Filter Rule 2

Note: With Filter Rule 1 a trough occurs at $t$ if $c_{t+1} > c_t < c_{t-1} < c_{t-2}$ and a peak if $c_{t+1} < c_t > c_{t-1} > c_{t-2}$. With Filter Rule 2 a trough occurs at $t$ if $c_t < 0$ and $c_{t-1} < 0$ or if $c_{t+1} < 0$ and $c_t < 0$ and a peak if $c_t > 0$ and $c_{t-1} > 0$ or if $c_{t+1} > 0$ and $c_t > 0$. A false alarm occurs if there is no turning point within ±3 quarters of the reference date. A missing signal occurs if the method does not signal a turning point within ±3 quarters of the XBER date. In the XBER classification there are 7 troughs and 7 peaks. LE stands for leading, CO for coincident and LA for lagging.
## REFERENCES

Table 1. Panel B

<table>
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<tr>
<th>Method</th>
<th>Troughs</th>
<th>Peaks</th>
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**Note:** With Filter Rule 1 a trough occurs at \( t \) if \( c_t > c_{t-1} \), \( c_{t-1} < c_{t-2} \), and a peak if \( c_{t-1} < c_t > c_{t-1} \) \( c_{t-1} < c_{t-2} \). With Filter Rule 2 a trough occurs at \( t \) if \( c_t < 0 \) and \( c_{t-1} < 0 \) or if \( c_{t+1} < 0 \) and \( c_t < 0 \) and a peak if \( c_t > 0 \) and \( c_{t-1} > 0 \) or if \( c_{t+1} > 0 \) and \( c_t > 0 \). A false alarm occurs if there is no turning point within \( \pm 3 \) quarters of the reference date. A missing signal occurs if the method does not signal a turning point within \( \pm 3 \) quarters of the DOC date. In the DOC classification there are 8 troughs and 7 peaks. LE stands for leading, CO for coincident and LA for lagging.
Table 2: Statistics of the Reference Cycle, Sample 55.3-90.1

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Filter Rule 1

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## REFERENCES

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### Filter Rule 1

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### Filter Rule 2

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<th>Troughs</th>
<th>Peaks</th>
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Note: With Filter Rule 1 a trough occurs at $t$ if $c_{t-1} > c_t < c_{t+1} < c_{t+2}$ and a peak if $c_{t+1} < c_t < c_{t-1} > c_{t+2}$. With Filter Rule 2 a trough occurs at $t$ if $c_t < 0$ or $c_{t-1} < 0$ or if $c_{t+1} < 0$ and $c_t < 0$ and a peak if $c_t < 0$ and $c_{t-1} < 0$ or if $c_{t+1} < 0$ and $c_t < 0$. NBER refers to the NBER chronology reported by the Center for International Business Cycle Research at Colombia University. DOC refers to the Higgins and Poole chronology compiled from the DOC composite index of leading indicators. Both are taken from Niemera (1991) and checked against those reported by Simkins (1994). A "*" indicates that the previous or the next turning point is censored.
27. Giovanna Procacci [1988] “The State and Social Control in Italy During the First World War”, pp. 18
43. Giovanni Procacci [1989] “State coercion and worker solidarity in Italy (1915-1918): the moral and political content of social unrest”, pp. 41
44. Carlo Alberto Magni [1989] “Reputazione e credibilità di una minaccia in un gioco bargaining”, pp. 56
55. Paolo Silvestri [1990] “Sull’autonomia finanziaria dell’università”, pp. 11