IRR, ROE and NPV: a Systemic Approach

by

Carlo Alberto Magni

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Università degli Studi di Modena
Dipartimento di Economia Politica
Viale Berengario, 51
41100 Modena (Italia)
e-mail: magni@unimo.it
IRR, ROE AND NPV: A SYSTEMIC APPROACH

CARLO ALBERTO MAGNI
University of Modena

ABSTRACT. The net present value (NPV) of an investment is the algebraic sum of the cash flows discounted at a rate $i$. The NPV rule states that, among two or more projects, the investor will accept the one with the highest NPV.

The internal rate of return (IRR) of an investment is that rate $x$ at which NPV $= 0$. The IRR rule asserts that, among two or more projects, the investor will select the one which shows the highest IRR.

The return on equity (ROE) is the ratio of net profit to ownership equity. This tool of performance analysis is often used as a rule for capital budgeting: an investor will prefer the project with the highest ROE.

Financial analysts and researchers have always regarded the three rules as different. Also, the NPV rule is regarded as the most reliable criterion for capital budgeting. This paper aims to show that their differences are merely cognitive and that we can unify the three criteria following a systemic approach. In particular, IRR and ROE are the same index. Also, the NPV rule is shown to be a particular case of the IRR rule.

1. IRR, NPV and ROE

Consider an investment with cash flow $a_s$ at the time $t_s = s$, $s = 0, 1, \ldots, n$, $a_s \in \mathbb{R}$. The net present value is the sum

$$G(i) = \sum_{s=0}^{n} a_s (1 + i)^{-s}$$

where $i$ is an interest rate. The internal rate of return (IRR), often called Yield, is defined as the rate $x$ at which

$$G(x) = 0.$$ 

If two projects, $A$ and $B$, are being evaluated, the IRR rule suggests the investor to accept the project with the highest rate of return.\(^1\) In the sequel we will always refer to decision processes under certainty, so that all forecastings are correct and no risk is involved. Assume that the investor wishes to invest $1000 for three years and let $A$ and $B$ have the following graphical representation:

\(^1\)We can think of the investor either as an individual or as a company.
Project A’s IRR is $x_A = 40\%$, project B’s IRR is $x_B = 30\%$; then, the preferred alternative should be A. But A vanishes at the end of the second year, while the investor wishes to maximize his wealth at the end of the third period. The decision-maker will therefore reinvest $\$1960$ for one year. The NPV rule, according to this line of argument, states that the correct comparison is

$$1960(1 + i) \leq 2197$$

or else

$$-1000 + \frac{1960}{1 + i} \leq -1000 + \frac{2197}{(1 + i)^2}$$

where $i$ is the rate at which the decision-maker will reinvest the sum. Consider now the following case:

**project C**

<table>
<thead>
<tr>
<th>time</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>cash flow</td>
<td>-1000</td>
<td>0</td>
<td>0</td>
<td>1331</td>
</tr>
</tbody>
</table>

**project D**

<table>
<thead>
<tr>
<th>time</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>cash flow</td>
<td>-1500</td>
<td>0</td>
<td>0</td>
<td>1890</td>
</tr>
</tbody>
</table>

The two IRRs are, respectively, $x_C = 10\%$ and $x_D = 8\%$. The IRR rule suggests to choose investment C. On the contrary, the NPV rule suggests to accept C only if

$$1331 + 500(1 + i)^3 > 1890$$

or else

$$-1000 + \frac{1331}{(1 + i)^3} > -1500 + \frac{1890}{(1 + i)^3}.$$
In general, when coping with two projects with cash flows $a_s$ and $b_r$ respectively, $s = 0, 1, \ldots, n$, $a_s \in \mathbb{R}$, $r = 0, 1, \ldots, m$, $b_r \in \mathbb{R}$, if $I_0$ denotes the investor's net worth at time $t_0 = 0$ and $T$ is a fixed horizon, $T \geq \max \{n, m\}$, $A$ is to be accepted if and only if

$$I_0(1 + i)^T + \sum_{s=0}^{n} a_s(1 + i)^{T-s} > I_0(1 + i)^T + \sum_{r=0}^{m} b_r(1 + i)^{T-r}$$

(1)

where $i$ denotes a rate of interest for reinvestment (if $a_s$ or $b_r$ is positive) and for financing (if $a_s$ or $b_r$ is negative). The rate $i$ is the well-known opportunity cost of capital and it suggests the idea of a standard business, in (from) which the investor commonly invests (draws) funds. This standard business is a sort of ‘current account’ the investor can turn to whenever it is necessary. Dividing both sides of (1) by $(1 + i)^T$ we obtain

$$\sum_{s=0}^{n} a_s(1 + i)^{-s} > \sum_{r=0}^{m} b_r(1 + i)^{-r}.$$  

(2)

It is worthwhile noting that (1) turns out to be (2) for the only reason that the NPV rule makes two implicit assumptions: (a) the investor has a single standard business, (b) the investor can invest and draw funds at the same rate $i$.

An alternative approach to capital budgeting derives from a tool of performance analysis: the return on investment (ROI). To calculate this profitability index it is necessary to divide the profits of a project by the capital invested in the project. It is a ratio frequently used by analysts as an investment rule where the returns are expressed in terms of estimated profits. A variant of this ratio is the so-called return on equity (ROE), often referred to as return on (net) capital employed. It is given by

$$\frac{\text{net profit}}{\text{equity}}$$

and in case of multiperiodic projects it is obtained by dividing the average forecasted profits by the initial net worth (equity).² By comparing the ROEs of project $A$ and project $B$ the investor is able to select the most profitable one. Let $\$1000$ be the initial outlay of project $A$ lasting one year and let $\$100$ be the total forecasted profit. The annual ROI will simply be

$$\frac{100}{1000} = 0.1;$$

if the investment is partly financed by liabilities and the debt amounts to $\$200$ with a debt rate of 9%, then the net worth invested in project $A$ is $\$800$ and the prospective ROE is

$$\frac{100 - 18}{1000 - 200} = 0.1025.$$  

If, for instance, the alternative $B$ has a higher ROE, the investor will choose the latter.

²In the sequel we will use the terms net worth, equity, ownership equity, net capital employed as synonymous. By them we mean the whole wealth of the investor, that is the total monetary value of assets less liabilities.
2. Drawbacks

Most analysts and researchers consider the IRR rule a bad rule. The major drawbacks are the following:

(a) it makes the implicit hypothesis that the cash flows of the project will be reinvested at the internal rate of return;
(b) some projects have more than one IRR and some others have no one;
(c) it has an ambiguous meaning.

The ROI (ROE) rule is considered a bad rule, too. It relies on accounting data, which may differ very often from cash flows. Also, even when returns are expressed as cash flows rather than profits, the ROI (ROE) rule is totally misleading in evaluating multiperiodic projects because it does not consider discounting, and therefore it takes no account of time. It also breaks down when the initial cash flow is positive (or is negative but very low), and the others alternate in sign: it fails to recognize a specific value of the capital invested. In fact, if we have

<table>
<thead>
<tr>
<th>time</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>cash flow</td>
<td>900</td>
<td>-1200</td>
<td>+1000</td>
<td>-900</td>
<td>+400</td>
</tr>
</tbody>
</table>

or

<table>
<thead>
<tr>
<th>time</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>cash flow</td>
<td>-90</td>
<td>+1200</td>
<td>-1000</td>
<td>+400</td>
<td>+400</td>
</tr>
</tbody>
</table>

what is the value of the capital invested?

The NPV rule is regarded by all financial analysts and mathematicians as the most reliable criterion for capital budgeting. But, as we will see, it has its drawbacks as well. *In primis*, it assumes that the opportunity cost of capital is the same for both investment and financing. But we stress that the major problems stem from the fact that it cannot cope with investors who hold more than one standard business. This means that the rule conceives cash flows only from a *diachronic* point of view, forgetting their *synchronic* dimension. As a matter of fact, it does not take into consideration the opportunity, for the investor, of reinvesting cash flows in (drawing funds from) more than one business. The financial system of an investor is composed by many businesses. It can be represented by a balance sheet comprehending many accounts and each of them can be considered a sort of business which the investor can apply to for reinvestment and financing.

The three rules give in general different answers when compared one another so they are considered incompatible. In the following example the three methods are applied to three

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3 See Teichroew, Robichek e Montalbano (1965a, 1965b)).
4 See also Magni (1997d)
projects, named $E$, $F$, $G$, with a net capital employed of 20 (the first row of the matrix shows the time, the other three show the corresponding net cash flows):

<table>
<thead>
<tr>
<th>time</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>-20</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>$F$</td>
<td>-20</td>
<td>10</td>
<td>8</td>
<td>5</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>$G$</td>
<td>-20</td>
<td>4</td>
<td>24</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Assuming that the opportunity cost of capital is $i = 0.05$ the value of the three measures are

<table>
<thead>
<tr>
<th></th>
<th>$E$</th>
<th>$F$</th>
<th>$G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRR</td>
<td>0.15</td>
<td>0.18</td>
<td>0.1</td>
</tr>
<tr>
<td>ROE_{ov}</td>
<td>0.5</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>ROE_{av}</td>
<td>0.1</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>NPV</td>
<td>5.97</td>
<td>5.21</td>
<td>5.57</td>
</tr>
</tbody>
</table>

where ROE_{ov} and ROE_{av} are the overall ROE and the average annual ROE respectively.\(^5\) They imply the following ranking of preferences:

<table>
<thead>
<tr>
<th></th>
<th>$E$</th>
<th>$F$</th>
<th>$G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRR</td>
<td>$F \succ E \succ G$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ROE_{ov}</td>
<td>$E \succ F \sim G$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ROE_{av}</td>
<td>$G \succ F \sim E$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NPV</td>
<td>$E \succ G \succ F$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Our aim is to show that a different cognitive perspective can give an explanation of the following assertions:

i. the IRR and the NPV give rise to an equivalent rule;
ii. the ROE is just the IRR;
iii. the IRR (ROE) rule is equivalent, under the NPV assumptions, to the NPV rule;
iv. the differences between the three methods derive from a bias.

3. Changing perspective

The dichotomy of the IRR rule and the NPV rule is based on a particular description and interpretation of a project. The two criteria give the same answers whenever the projects are *homogeneous*, that is they have the same initial outlay, the same length and only two cash flows of different sign. In fact,

$$M^1 \leq M^2 \iff -I + \frac{M^1}{(1+i)^T} \leq -I + \frac{M^2}{(1+i)^T} \iff x_1 \leq x_2 \quad \forall i > -1$$

\(^5\)Both ROE_{ov} and ROE_{av} derive from an incorrect application of the ratio net profit/equity (see next section.)
where

\[ I = \text{initial outlay} \]
\[ M^1 = \text{compound amount of project 1} \]
\[ M^2 = \text{compound amount of project 2} \]
\[ T = \text{length of the projects} \]
\[ x_1 = \text{internal rate of return of project 1} \]
\[ x_2 = \text{internal rate of return of project 2} \]
\[ i = \text{opportunity cost of capital}. \]

If the projects under consideration are heterogeneous (they offer different patterns of cash flow over time or have different outlay or length) the two criteria differ (see the examples in sections 1. and 2.). As most projects in an economic environment are heterogeneous, the IRR rule is considered unreliable in ranking projects.

As for the divergences between these rules and the ROE rule, the latter is basically a tool of performance analysis for a single period; it therefore neglects, as an investment rule, the relevant information of time considering the cash flows as if they were generated at the same instant.

A basic principle of capital budgeting is: 'consider only differential cash flows when evaluating an investment'. So the graphical representation of a project descends from a diachronic description of the cash flows. Such a description is based on a biased cognitive interpretation which gives rise to the splitting between the IRR rule and the NPV rule, and is the source of the incorrect use of the ROE. The appraisal of an investment is therefore always based on the description of time and corresponding cash flows, i.e.

<table>
<thead>
<tr>
<th>time</th>
<th>( t_0 )</th>
<th>( t_1 )</th>
<th>( t_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>cash flows</td>
<td>( a_0 )</td>
<td>( a_1 )</td>
<td>( a_n )</td>
</tr>
</tbody>
</table>

If we describe projects as above, we take no account of the wealth of the investor. The wealth can be viewed as a system whose structure articulates in a plurality of elements: they are different businesses for the investor. The financial representation forgets the different modalities of reinvestment and withdrawal of the cash flows among the elements of the system at each stage \( t_s = s \) of the investment. The synchronic effect of these actions are totally disregarded in the IRR rule and in the ROE rule, while the NPV rule de-structures the system assuming for it a single element. This causes the differences among the three methods.

The structural change of the system alters the value of the net worth. Receipts and expenditures of an investment influence the structure of the system. Their impact on the latter determines its evolution and therefore the rate at which the net worth increases.
The system is well represented by a balance sheet comprehending all assets and liabilities expressed in term of monetary value. The cognitive and graphical representation as described above can be replaced by sequences of balance sheets (each one relating to a single project) where the value of the "businesses" (assets and liabilities) are estimated. In this sense we can easily manage both the diachronic and the synchronic aspect of the structure. The former is given by the stream of the net cash flows generated by the projects, the latter is determined by the interaction among the accounts of the balance sheets, that is by the modality of reinvestment and withdrawal of funds at any single stage.

From this point of view a project is simply a modality of evolution of the system and the capital employed is the ownership equity. As a consequence, the investment consists of two cash flows of different signs: the ownership equity at time \( t = 0 \) and its compound amount at a fixed date \( T > 0 \).

Suppose then that an investor has a net worth which is structured, at a date 0, in \( q \) accounts:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C^1 )</td>
<td>( C^{k+1} )</td>
</tr>
<tr>
<td>( C^2 )</td>
<td>( C^{k+2} )</td>
</tr>
</tbody>
</table>
| ... | ...
| ... | ...
| ... | \( C_q \)
| \( C^k \) | \( I_0 \)

where \( C^j \geq 0, j = 1, \ldots, q \), denotes the value of the \( j \)-th account, and \( I_0 \) is the net worth of the investor at time 0.

The decision-maker has the opportunity to invest part of the net worth in a project \( A \) with equidistant cash flows \( a_s \in \mathbb{R}, s = 0, 1, \ldots, n \). If the decision-maker chooses the investment undertaking, the structure of the system at time 0 changes depending on the policy of withdrawal of \( a_0 \).

The double entry system is based on the fundamental accounting equation

\[
\sum \text{Assets} = \sum \text{Liabilities} + \text{Net Worth.} \tag{7}
\]

For the project at hand, this implies, for the date 0, increases and/or decreases in assets and liabilities. In other words, the investor must selects a combination of businesses (the accounts of the balance sheet) in order to finance (if \( a_0 < 0 \)) or invest (if \( a_0 > 0 \)) the first cash flow. \( C_0^j \) denotes the new value of the \( j \)-th account:

\[
C_0^j = C^j + a_0 j \quad j = 1, \ldots, q
\]
where $a_{0j}$ is that part of $a_0$ invested in (or drawn from) the $j$-th account. In general, at time $s$, $s = 0, 1, \ldots n$ the cash flow $a_s$ is distributed among the accounts in such a way that

$$\sum_{j=1}^{q} a_{sj} = a_s$$

where $a_{sj} \in \mathbb{R}$ is the value assigned to the $j$-th account. The balance sheet becomes

\begin{align*}
\text{Assets} & \quad \text{Liabilities} \\
C_s^1 + a_{s1} & \quad C_{s+1}^1 + a_{s,k+1} \\
C_s^2 + a_{s2} & \quad C_{s+2}^2 + a_{s,k+2} \\
\vdots & \quad \vdots \\
\vdots & \quad \vdots \\
C_s^k + a_{sk} & \quad C_s^q + a_{sq} \\
A_s & \quad I_s
\end{align*}

where $A_s$ is the value of the project at the date $s$ and $^6$

$$C_s^j = C_{s-1}^j (1 + i_s^j)$$

with $i_s^j (>-1)$ being the corresponding rate of interest or else the internal rate of the businesses which compose the financial system of the investor, in the $s$-th interval. Let $x^A$ denote the internal rate of return of $A$ (which we suppose constant). We have then

$$I_s = I_{s-1} + \text{net profit of the } s\text{-th period} = I_{s-1} + \sum_{j=1}^{k} i_s^j C_{s-1}^j + x^A A_{s-1} - \sum_{j=k+1}^{q} i_s^j C_{s-1}^j. \quad (8)$$

In this way, drawing up $(n+1)$ prospective balance sheets, every element of the system is considered and explicitly represented. This systemic approach differs from the financial perspective, because the latter does not describe the elements of the system. The impact of the project on the net worth (the capital invested) is then correctly represented by the periodic alteration of the structure of the system.

If the investor has $l$ alternatives under consideration, $l \geq 2$, the systemic approach allows to choose the most profitable one by comparing the corresponding balance sheets at a fixed horizon $T$. Thus, the investor compares $l$ homogeneous alternatives, which show the same

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$^6$We have supposed that a reinvestment in project $A$ is not possible.
length \((T)\), the same initial outlay \((I_0)\), and no intermediate cash flows (these are always included in the system); so we have the following graphical description of a project \(A\):

<table>
<thead>
<tr>
<th>time</th>
<th>0</th>
<th>(T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cash flows</td>
<td>(-I_0)</td>
<td>(I_T^A)</td>
</tr>
</tbody>
</table>

The same description is required for a project \(B\), replacing \(I_T^A\) by \(I_T^B\). The investor only needs to compare the compound amounts of the net worth for each alternative \(A\) and \(B\). From the equations

\[
-I_0 + \frac{I_T^A}{(1 + x_A)^T} = 0 \tag{9a}
\]

and

\[
-I_0 + \frac{I_T^B}{(1 + x_B)^T} = 0 \tag{9b}
\]

we get the internal rates of return \(x_A\) and \(x_B\) and, thanks to (6), we can replace the comparison between compound amounts by the comparison between internal rates of return, obtaining the same ranking of preferences.

We show now that \(x_A\) and \(x_B\) are the well-known returns on equity of \(A\) and \(B\), expressed in monetary value and as average periodic indexes. In fact from (8) we have

\[
I_s = I_{s-1} \left(1 + \frac{\sum_{j=1}^k C_{s-1}^j + x^A A_{s-1} - \sum_{j=k+1}^q i_l C_{s-1}^j}{\sum_{j=1}^k C_{s-1}^j + A_{s-1} - \sum_{j=k+1}^q i_l C_{s-1}^j}\right) = I_{s-1}(1 + x_s) \tag{10}
\]

where \(x_s\) is the ROE for the investor in the \(s\)-th period. Then the following equality holds:

\[
I_T = I_0 \prod_{s=1}^T (1 + x_s) \tag{11}
\]

or

\[
I_T = I_0 (1 + x)^T \tag{12}
\]

with

\[
x = \left(\prod_{s=1}^T (1 + x_s)\right)^{1/T} - 1; \tag{13}
\]

dividing both sides of (12) by \((1 + x)^T\) we obtain equations (9): the IRR and the ROE are therefore the same parameter.
4. The equivalence between the IRR-ROE rule and the NPV rule

We have seen that IRRs and ROEs can be viewed as the same indexes and that the corresponding rules are equivalent. They also offer the same answers as the comparison between the compound amounts of the net capital employed in the projects. What about the relationships with the NPV rule? We just have to think that the NPV rule makes the implicit assumption

\[ q = 1 \]

which leads to (2). This means that the system of the net worth is de-structured in a single element, that is the balance sheet of the investor is composed of one account, which acts as an asset or as a liability according to its value (positive or negative). Consider project A with cash flows \( a_s, s = 0, 1, 2 \ldots n \) and project B with cash flows \( b_r, r = 0, 1, 2 \ldots m \). The investor's net worth consists of a "current (or standard) business" whose value is subject to the equation

\[ C_s = C_{s-1}(1 + i) + a_s \]

or

\[ C_r = C_{r-1}(1 + i) + b_r \]

depending on the choice selected, where \( i \) is the rate of interest of the business. This hypothesis modifies the fundamental equation (7) to

\[ C_s + A_s = I_s^A \quad \text{or} \quad C_r + B_r = I_r^A \quad \forall s, r \geq 1 \]

with

\[ C_T = I_T \]

since \( A_s = B_r = 0 \) for \( s \geq n \) and \( r \geq m \). The choice is determined by the comparison

\[ I_T^A \leq I_T^B \]

which results, as we have seen, in

\[ I_0(1 + x_A)^T \leq I_0(1 + x_B)^T \]

But

\[ I_s^A = C_s + A_s = C_{s-1}(1 + i) + a_s + A_s \quad C_0 = I_0 \]

\[ I_r^B = C_r + B_r = C_{r-1}(1 + i) + b_r + B_r \quad C_0 = I_0; \]

We can rewrite equations (16) as

\[ I_s^A = C_0(1 + i)^s + \sum_{h=0}^{s} a_h (1 + i)^{s-h} + A_s \]
IRR, ROE AND NPV

\[ I_r^B = C_0(1 + i)^T + \sum_{h=0}^{r} b_h(1 + i)^{r-h} + B_r. \] (17b)

It follows that

\[ I_0(1 + x_A)^T = I_0^A = I_0(1 + i)^T + \sum_{s=0}^{T} a_s(1 + i)^{T-s} \] (18a)

\[ I_0(1 + x_B)^T = I_0^B = I_0(1 + i)^T + \sum_{r=0}^{T} b_r(1 + i)^{T-r}; \] (18b)

the comparison \textit{sub} (15) becomes then

\[ (I_0 + a_0)(1 + i)^T + \sum_{s=0}^{n} a_s(1 + i)^{T-s} \leq (I_0 + b_0)(1 + i)^T + \sum_{r=0}^{m} b_r(1 + i)^{T-r}. \] (19)

Dividing both sides by \((1 + i)^T\) we obtain the NPV rule

\[ \sum_{s=0}^{n} a_s(1 + i)^{-s} \leq \sum_{r=0}^{m} b_r(1 + i)^{-r}; \] (20)

hence

\[ x_A \lesssim x_B \iff \sum_{s=0}^{n} a_s(1 + i)^{-s} \lesssim \sum_{r=0}^{m} b_r(1 + i)^{-r}. \] (21)

The coincidence between the IRR-ROE rule and the NPV rule is finally accomplished.

As for the examples carried out in section 2., (4) and (5) reveal that the description of project \(E, F,\) and \(G\) leads to inconsistent results. A systemic approach allows to calculate the correct IRRs and ROEs (which coincide). We have the equations

\[ I_0(1 + x_E)^T = I_0 + 5.97(1.05)^T \]
\[ I_0(1 + x_F)^T = I_0 + 5.21(1.05)^T \]
\[ I_0(1 + x_G)^T = I_0 + 5.57(1.05)^T \]

from which we get

\[ x_E > x_G > x_F \quad \forall T, I_0 > 0. \]

The ranking of preferences for the criteria is the same, precisely

\[ E > G > F. \]

We stress that the equivalence is assured only under the strong assumption of the NPV rule

\[ q = 1. \]
If we bring out this assumption the NPV is not applicable. As a matter of fact, the IRR-ROE rule is a more general criterion, for it is able to cope with the more realistic assumption

\[ q > 1. \]

This argument leads to the following conclusion: the NPV rule is not so reliable as the IRR-ROE rule which is able to take into consideration both the synchronic and the diachronic aspects of the cash flows of a project. In fact, let \( i^j_h \) be the rate of interest (or else the opportunity cost) of the \( j \)-th account in the \( h \)-th period: each cash flow in the NPV rule is evaluated by the product

\[ a_s \prod_{h=1}^{\infty} (1 + i^j_h)^{-1} \]

which gives it a diachronic dimension, while in the systemic IRR-ROE rule each cash flow is evaluated by the sum

\[ \sum_{j=1}^{q} a_{sj} \prod_{h=s+1}^{T} (1 + i^j_h) \]

which, in addition, considers the synchronic dimension of the phenomenon.

5. ROE as an opportunity cost of capital

Some might think that one can salvage the NPV rule by taking the ROE as the opportunity cost of capital, that is assuming

\[ i = \text{ROE}. \]

But this is not the case. Consider the mutually exclusive projects \( A \) and \( B \), lasting one period, let \( A_0, B_0 \) be the initial outlays of the projects and \( A_1, B_1 \) their compound amounts. If we use the NPV rule, according to the above assumption we have the comparison between the present values

\[ -A_0 + \frac{A_1}{1+i} \leq -B_0 + \frac{B_1}{1+i} \]

which can be written as

\[ (I_0 - A_0)(1+i) + A_1 \leq (I_0 - B_0)(1+i) + B_1. \]  \[ (22) \]

In this case

\[ i = \frac{\sum_{j=1}^{k} i^j C^j - \sum_{j=k+1}^{q} i^j C^j}{\sum_{j=1}^{k} C^j - \sum_{j=k+1}^{q} C^j} \]  \[ (23) \]

where \( i^j \) is the rate of interest of the \( j \)-th account in the single period; \((22)\) is then incorrect, because it assumes that the structure of the system is not altered by the undertaking of the projects and that the ROE for the investor remains the same. When accomplishing
a project the investor modifies the applications and the sources of funds, since the outlay is financed by an increase of sources and/or a decrease of applications. The net worth is unvaried but its composition changes, giving rise to a different ROE. \( i \) is therefore the ROE under the hypothesis of rejecting both projects. The ROEs in case of a project undertaking, are, respectively,

\[
i_A = \frac{\sum_{j=1}^{k} i^j (C^j - A_0^j)}{\sum_{j=1}^{k} (C^j - A_0^j)} - \frac{\sum_{j=k+1}^{q} i^j (C^j + A_0^j)}{\sum_{j=k+1}^{q} (C^j + A_0^j)}
\]

\[
i_B = \frac{\sum_{j=1}^{k} i^j (C^j - B_0^j)}{\sum_{j=1}^{k} (C^j - B_0^j)} - \frac{\sum_{j=k+1}^{q} i^j (C^j + B_0^j)}{\sum_{j=k+1}^{q} (C^j + B_0^j)}
\]

subject to

\[
\sum_{j=1}^{q} A_0^j = A_0 \quad \sum_{j=1}^{q} B_0^j = B_0 \quad A_0^j, B_0^j \geq 0
\]

where \( A_0^j \) and \( B_0^j \) reveal the policy of financing of the investor (decreasing the applications and/or increasing the sources). The correct comparison is therefore given by the three compound amounts

\[
I_0 (1 + i_A); \quad I_0 (1 + i_B); \quad I_0 (1 + i)
\]

which relate to the three strategies: (a) accept project A (b) accept project B (c) reject both project, and which cannot be reduced to a comparison between present values.

The multiperiodic projects are a mere generalization of this case and give the same results; we cannot compare present values by discounting cash flows at the ROE since the structure of the system changes in consequence of the choice among the alternatives.\(^7\) It is also easy to show that the use of the Weighted Average Cost of Capital for discounting the cash flows cannot salvage the NPV rule.\(^8\) The latter must be abandoned in most cases, when financing and reinvestment decisions concern more than one account and replaced by the more flexible IRR-ROE rule.

6. The bias

We have seen that a particular graphical description of a project can deeply influence the cognitive interpretation of a project.\(^9\) The radical modification of the conceptual framework has shown that the differences among the three methods are merely illusory. Financial mathematicians and analysts frame the description of the decision process using the term ‘investment’ in an ambiguous way; they describe it graphically in such a way that the net worth of the investor is totally neglected, focusing their attention on the diachronic aspects of the decision process. This framing leads to a misunderstanding and to an incorrect use of

\(^7\) See Magni (1997d) for details about multiperiodic projects.

\(^8\) See Magni (1997d) and Peccati (1996b).

\(^9\) It would be interesting to properly understand the relationships between the cognitive interpretation and the graphical description of the phenomenon.
the internal rate of return and the return on equity. If we just use the term investment not as a synonymous of 'project' but relate it to the net worth of an investor, then we rise at a superior conceptual framework, which sees the project as a single element of the financial system of the investor. The system shows the 'investment' of the net worth which evolves according to the project selected and according to the policy of financing and reinvestment carried out by the decision-maker. Therefore the capital invested is not the initial outlay of the project but the net worth. The cash flows of the project in the graphical representation suddenly disappear: diachronically, only two cash flows are considered for decision, the net capital employed and its compound amount. Synchronically, the balance sheet shows the cash flows of the projects and their relationships with the other elements of the system. The alternatives are therefore homogeneous and no difference can be found between IRR and ROE. Furthermore the two measures are more general than the NPV rule, in opposition to what is commonly believed.

7. Conclusions

The paper has shown how to reconcile three methods of investment appraisal which are so far considered different. In particular, it has shown that:

(a) a different description of a decision process is possible and consistent with an omni-comprehensive framework which makes the project an element of the investment (the net worth);
(b) adopting the systemic perspective the IRR and the ROE are the same parameter and the NPV leads to the same results as the IRR-ROE rule under the NPV rule assumptions;
(c) the NPV is a particular case of the IRR-ROE rule;
(d) the divergences of the three methods derive from a bias, which frames the description of the decision problem focusing a diachronic point of view and disregarding the synchronic aspects;
(e) the suggested approach naturally reconciles accounting measures with financial ones, following the fundamental accounting equation: Assets=Liabilities+Ownership Equity.

Further researches can lead, consistently to this conceptual framework, to a new definition of investment, a stronger integration between accounting and financial analysis, and a deeper understanding of the logical implications of the NPV assumptions. A generalization of this approach can be thought for decision making under uncertainty, studying the implications of different frameworks.

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10 See Magni (1997e).
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