Optimal Procurement in Multiproduct Monopoly

by

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Abstract: In this paper we characterize the optimal procurement policy for a multiproduct monopoly with multidimensional private information about its costs. We show that, unless correlation between costs is too large, the optimal procurement contract should regulate jointly the production of the various goods even when these goods are not linked by any technological or demand factor. The economic intuition behind this result is similar to the rent-extracting argument used to justify the optimal selling strategy of a multiproduct monopolist. In both cases a bundling strategy allows the principal to reduce the informational rents of ‘mixed type’ agents when they are more likely. The results are also applied to the case where, for each good, a verifiable quality as well as a quantity index can be contracted upon.

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1. Introduction

Both in the case of public utility regulation and in the case of procurement the firm with which the government enters a contractual relationship is often a multiproduct firm with private information about its technological capabilities. In these situations an interesting question to answer is whether an optimal policy for the government is to regulate activities, that is each line of product separately, or to regulate the firm as a single unit. In this paper we study how and under what circumstances the multiproduct nature of the firm affects the optimal regulatory policy.

The case in which the regulated firm is a single product monopoly has been modelled in recent literature as a principal agent game where the government, behaving as a Stackelberg leader, proposes a (regulatory or procurement) "take it or leave it" contract to the firm that has private information over a parameter and/or a choice variable affecting its own benefits as well as the benefits of the principal (Baron and Myerson, 1982, Laffont and Tirole 1993). In this paper we extend the analysis to the case of a multiproduct firm whose private information on costs is multidimensional, that is involves more than one parameter.¹

A similar problem has been analysed in the literature on industrial organization in the case of the pricing strategies of a multiproduct monopolist facing consumers with private information over their reservation prices. Adams and Yellen (1976), following a suggestion by Stigler, show that the firm may have an incentive to package two or more products in bundles rather than selling

¹ Models of multidimensional screening have been introduced by Mirlees (1971), (1986) in the context of optimal taxation; Laffont, Maskin and Rochet (1985) deals with a two dimensional case and fully characterizes the optimal non linear tariff for a monopolist producing a single good. McAfee and McMillan (1988) and Armstrong (1996), provide further results and solution techniques for the problem of multidimensional screening in the case of continuous types.
them separately. In particular, they show how bundling can be used as a price discrimination strategy to extract consumer's surplus when reservation prices across goods are negatively correlated. Spence (1980) and McAfee, McMillan and Whinston (1989) provide further conditions on the distribution of consumers valuations under which a bundling strategy dominates unbundled sales.

In this paper we show that a similar incentive to bundle goods arises for a government entering a contractual relationship with a multiproduct firm having private information about some of the parameters affecting its costs. To this aim we consider a discrete multidimensional screening model where the government faces a monopolistic firm producing two goods. To isolate the problem from other types of effects we assume that utility and cost functions for each good are functionally independent from each other. We show that the main principles established for optimal multiproduct monopoly pricing carry over to the case of optimal regulation so that the government has an incentive to adopt multiproduct regulation schemes by making monetary transfers to the firm dependent on quantities of both goods. In the same way as a bundling strategy for sales allows a monopolist to increase profits when consumer valuations across goods are likely to differ, in a regulatory setting the government can increase social welfare by adopting procurement contracts which bundle goods the more likely are the differences of costs across products. Finally, we apply this result also to the case in which a verifiable quality index, as well as a quantity index, can be explicitly contracted upon.

Our model draws from Spence (1980) and is similar to Dana (1993). Spence provides a useful approach to the solution of multidimensional screening problems and applies it to a case of optimal multiproduct monopoly pricing. We adopt Spence's approach to solve a different but related problem in optimal procurement.

The paper by Dana concerns the optimal organizational structure of a multiproduct industry and compares an integrated organization, in which control over both goods is given to a multiproduct firm, with a decentralized organiza-
tion where each firm produces only one good. Although there are similarities, our paper differs from Dana's in some respects. By allowing for a more general specification of cost functions we obtain more general results. We also derive a new result concerning the relationship between single and multi-product optimal contracts. Moreover, differently from Dana, we discuss the binding constraints in the regulator's problem along the lines set by Spence (1980); this approach helps to simplify subsequent proofs. Another difference with respect to Dana is that we consider a welfare function including the cost of public funding instead of distributional concerns.

The plan of the paper is as follows: in section 2 we present the model and discuss the regulator's problem; in sections 3 and 4 we derive a complete characterization of the optimal contract in a symmetric context; section 5 compares the previous scheme with optimal single product contracts, that is contracts regulating each line of product separately; section 6 shows an application of the results to the case of regulation of quantities and qualities, section 7 concludes.

2. The model

Our analysis deals with the case of procurement of non-marketable goods produced by an existing multiproduct monopolistic firm. The central authority, a benevolent regulator, is allowed to decide the quantities to be produced and the amount of monetary transfer to the firm. We consider the case of a regulated monopoly producing two goods, \( a \) and \( b \), whose quantities are denoted by \( q = (q_a, q_b) \), and assume that social benefits from consumption of the goods can be represented by an additively separable function

\[
U(q) = u_a(q_a) + u_b(q_b)
\]

where \( u_k(\cdot) \), \( k = a, b \), is increasing, strictly concave and continuously differentiable. The fixed costs of the monopolist are publicly observable so that, for

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\( ^2 \) For more on the comparison between different organizational structures with complementary products see Baron and Besanko (1992) and Gilbert and Riordan (1995).
convenience, are normalized to zero; the cost function has the following additive form

\[ C(q; \theta_a, \theta_b) = \theta_a c_a(q_a) + \theta_b c_b(q_b) \]

where the cost parameters \( \theta_k \), can take their values in the set \( \{ \theta_k, \bar{\theta}_k \} \) with \( \bar{\theta}_k > \theta_k > 0 \) and the functions \( c_k(\cdot) \) are increasing, convex and continuously differentiable. The cost parameters are private information of the monopolist and their joint probability, \( \Pr(\theta_a, \theta_b) \), is common knowledge.

From the regulator's point of view there are four types of monopolists. Type 1, the 'low monopolist', has low costs parameters in both goods and his cost function will be denoted by \( C_1(q) = C(q; \theta_a, \theta_b) \). Types 2 and 3 are the 'mixed monopolists' and their costs functions are respectively \( C_2(q) = C(q; \theta_a, \bar{\theta}_b) \) and \( C_3(q) = C(q; \bar{\theta}_a, \theta_b) \). Finally, type 4 is the 'high monopolist' with \( C_4(q) = C(q; \bar{\theta}_a, \bar{\theta}_b) \). The probability of each type will be denoted accordingly by

\[ \alpha_1 = \Pr(\theta_a, \theta_b) \quad \alpha_2 = \Pr(\theta_a, \bar{\theta}_b) \quad \alpha_3 = \Pr(\bar{\theta}_a, \theta_b) \quad \alpha_4 = \Pr(\bar{\theta}_a, \bar{\theta}_b) \]

The probability of monopolist having a low cost parameter in good \( a \) is \( p_a = \alpha_1 + \alpha_2 \), that is \( p_a \) represents the marginal distribution of \( \theta_a \); similarly, for good \( b \), we have \( p_b = \alpha_1 + \alpha_3 \). For future reference we also give the formula of the coefficient of correlation between the cost parameters, which is

\[ \rho = \frac{\alpha_1 \alpha_4 - \alpha_2 \alpha_3}{\sqrt{p_a p_b (1-p_a)(1-p_b)}} \]

A procurement contract consists of a set of monetary transfers to the monopolist and the corresponding quantities of the two goods that the monopolist is required to produce. In accordance with the revelation principle we consider only contracts which are truthful direct revelation mechanisms; therefore, a contract specifies, for each type of monopolist, a transfer, \( t_i \), and quantities \( q_i = (q_{ai}, q_{bi}) \) to be produced. Under the class of contracts \([t_i, q_i]\) the profit of type \( i \) monopolist, when he tells the truth, is \( t_i - C_i(q_i) \) and when he reports to be type \( j \neq i \) is \( t_j - C_i(q_j) \). Truthfully implementable contracts satisfy (ex
individual rationality (IR) and incentive compatibility (IC) constraints, i.e.

\[ t_i - C_i(q_i) \geq 0 \quad \text{and} \quad t_i - C_i(q_i) \geq t_j - C_i(q_j) \]

for any \( i \) and \( j \).

The problem of the regulator is to find, within the set of implementable contracts, those maximizing the expected social welfare. Social welfare is measured by the sum of consumer surplus and profits and also includes the cost of public funding. The expected welfare, under truthful revelation, is given by

\[ W = \sum_{i=1}^{4} \alpha_i [U(q_i) - C_i(q_i) - \lambda t_i] \]

where the fixed parameter \( \lambda \geq 0 \) is to include the distortionary effects on welfare of raising public funds. Expected social welfare is maximized subject to IR and IC constraints so that the regulator's problem can be stated as follows

\[ \max_{t_1, \ldots, q_1, \ldots} W \quad \text{s. to} \quad t_i - C_i(q_i) \geq t_j - C_i(q_j) \]  \quad (1)

for \( i = 1, \ldots, 4 \) and \( j = 0, 1, \ldots, 4 \) with \( t_0 = 0 \), \( q_0 = (0, 0) \) to include IR constraints.

The crucial point to the solution of screening problems is to derive the set of binding constraints. When private information is one dimensional the agent's types can be completely ranked and standard results shows that the IC binding constraints are only those between adjacent types and this constraints bind only in one direction. In the multidimensional case the problem is more complex since, in general, the ordering of agent's types is partial; this implies that IC constraints in all directions and with respect to all types have to be explicitly taken into account. For example, in our model, we have to explicitly consider not only the incentive of type 1 to report to be of type 2 but also the incentive of type 1 to report to be of type 3 and 4.

Spence (1980) suggests a general procedure to identify the subset of potentially binding problems in multidimensional screening problems. Let us define
\[ m_{ij} = C_i(q_i) - C_i(q_j) \]

for given quantities \( q_i \) and \( q_j \) and rewrite IR and IC constraints as \( t_i \geq m_{ij} + t_j \). For given quantities the \( t_i \)'s that minimize expected transfers and satisfy all the constraints are the solution to a linear programming problem

\[
\min_{t_1, \ldots, t_4} \sum_i \alpha_i t_i \quad \text{s. to} \quad t_i \geq m_{ij} + t_j
\]

Spence shows that the solution is unique, independent from the distribution of types and satisfies the following condition

\[
t_i = \min_j \{ m_{ij} + t_j \}
\]

for all \( i \) and \( j \). This procedure works for given quantities, but it does not give yet the binding constraints of problem (1) since optimal transfers and quantities have to be determined simultaneously. However, if we introduce sensible assumptions about quantities in the optimal contract this procedure turns out to be very useful. Let us assume that in any optimal procurement contract the quantity of good \( k \) produced by a monopolist with low costs on good \( k \) is greater than the quantity produced by the monopolist with high cost on the same good. More formally we make the following

Assumption 1. The procurement contracts satisfy the following conditions:

\[
\min\{q_{1a}, q_{2a}\} > \max\{q_{3a}, q_{4a}\} \quad \text{and} \quad \min\{q_{1b}, q_{3b}\} > \max\{q_{2b}, q_{4b}\}
\]

As we shall see in the Appendix, the procedure of Spence, given Assumption 1, allows us to select a subset of 6 potentially binding constraints out of 16 and therefore it helps to simplify the analysis of the regulator's problem. In the following two sections we illustrate this procedure in the simplified setting of a symmetric model for which we also provide a complete characterization of the optimal procurement contract. The main results also apply to the general model considered in this section.

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3. The binding constraints

We assume that the functional forms of social benefits and costs do not depend on the goods, that is we set \( u_k(\cdot) = u(\cdot) \), \( c_k(\cdot) = c(\cdot) \), \( \theta_k = \theta \) and \( \bar{\theta}_k = \bar{\theta} \) for \( k = a, b \). To simplify notation we normalize the difference between cost parameters to one, therefore we set \( \bar{\theta} - \theta = 1 \). We also assume that the marginal distributions of cost parameters are the same, i.e. \( p_k = p \), which in turn implies that the mixed monopolists are equally likely, i.e. \( \alpha_2 = \alpha_3 \).

Given the symmetric nature of the model we can restrict the analysis to the following class of symmetric contracts:

\[
\begin{align*}
&[t_1, q_1 = (e, e)] \quad \text{for type 1} \\
&[t_2, q_2 = (x, y)] \quad \text{for type 2} \\
&[t_3 = t_2, q_3 = (y, x)] \quad \text{for type 3} \\
&[t_4, q_4 = (z, z)] \quad \text{for type 4}
\end{align*}
\]

Under a symmetric contract the low monopolist produces the same quantities of both goods and so does the high type, whereas the mixed monopolists receive the same transfer and produce symmetric quantities of the goods. Moreover, since the cost functions of types 2 and 3 have the property that \( C_2(q_a, q_b) = C_3(q_b, q_a) \), it is easy to verify that when IR and IC constraints involving type 2 are satisfied, then any constraint involving type 3 will hold as well. Therefore, in the following analysis, we need not consider explicitly type 3.

As a benchmark for future comparison let us recall the solution to the regulator problem under complete information. In this case only IR constraints are to be considered, therefore, in the socially optimal contract, transfers are equal to costs and quantities, if positive, must equate marginal utility and marginal social costs; for example, the socially efficient quantity produced by the low monopolist, \( e \), satisfies the condition \( u'(e) = (1 + \lambda)\bar{\theta}c'(e) \). To avoid trivial cases we assume that it is socially optimal to produce positive quantities of each good even when the monopolist has high costs; therefore, we shall assume that \( c, u, \bar{\theta} \) and \( \lambda \) satisfy the following condition: \( u'(0) > (1 + \lambda)\bar{\theta}c'(0) \).

\footnote{In the Appendix it is shown that the optimal contract is indeed symmetric.}
Turning back to the case of private information the 9 constraints facing the regulator in the symmetric setting are:

Type 1:

\[ t_1 - 2\theta c(e) \geq 0 \quad \text{ir}(1) \]
\[ t_1 - 2\theta c(e) \geq t_2 - \theta[c(x) + c(y)] \quad \text{ic}(1, 2) \]
\[ t_1 - 2\theta c(e) \geq t_4 - 2\theta c(z) \quad \text{ic}(1, 4) \]

Type 2:

\[ t_2 - \theta c(x) - \bar{\theta} c(y) \geq 0 \quad \text{ir}(2) \]
\[ t_2 - \theta c(x) - \bar{\theta} c(y) \geq t_1 - (\theta + \bar{\theta})c(e) \quad \text{ic}(2, 1) \]
\[ t_2 - \theta c(x) - \bar{\theta} c(y) \geq t_4 - (\theta + \bar{\theta})c(z) \quad \text{ic}(2, 4) \]

Type 4:

\[ t_4 - 2\bar{\theta} c(z) \geq 0 \quad \text{ir}(4) \]
\[ t_4 - 2\bar{\theta} c(z) \geq t_1 - 2\bar{\theta} c(e) \quad \text{ic}(4, 1) \]
\[ t_4 - 2\bar{\theta} c(z) \geq t_2 - \bar{\theta}[c(x) + c(y)] \quad \text{ic}(4, 2) \]

where \( \text{ir}(i) \) refers to the IR constraint of type \( i \) and \( \text{ic}(i, j) \) is the IC constraint of type \( i \) with respect to type \( j \). To determine the binding constraints at the optimum we follow the procedure of Spence (1980) and derive recursively the minimizing transfers starting from type 4. Let us set the transfers \( t_1 \) and \( t_2 \) at the values equating the respective IR constraints, i.e. \( \text{ir}(1) \) and \( \text{ir}(2) \); substituting these values into the constraints of type 4 we obtain

\[ t_4 - 2\bar{\theta} c(z) \geq 0 \]
\[ t_4 - 2\bar{\theta} c(z) \geq -2c(e) \]
\[ t_4 - 2\bar{\theta} c(z) \geq -c(x) \]
therefore, the minimum transfer for type 4 is $t^*_4 = 2\hat{c}(z)$. The next step is the analysis of type 2 constraints by substituting $t_1$ and $t_4$, the latter as determined in the previous step:

\[
\begin{align*}
 t_2 - \hat{c}(x) - \hat{c}(y) & \geq 0 \\
 t_2 - \hat{c}(x) - \hat{c}(y) & \geq -c(e) \\
 t_2 - \hat{c}(x) - \hat{c}(y) & \geq c(z)
\end{align*}
\]

Clearly the minimum transfer $t_2$ must equate the last constraint therefore $t^*_2 = \left[ \hat{c}(x) + \hat{c}(y) \right] + c(z)$. Notice also that this value of the transfer still allows $t^*_4$ to satisfy the constraints of type 4 as can be easily checked. The last step is to substitute $t^*_4$ and $t^*_2$ into the constraints of type 1; this yields

\[
\begin{align*}
 t_1 - 2\hat{c}(e) & \geq 0 \\
 t_1 - 2\hat{c}(e) & \geq [c(y) + c(z)] \quad (2) \\
 t_1 - 2\hat{c}(e) & \geq 2c(z) \quad (3)
\end{align*}
\]

In this case it cannot be said in advance which one of the two IC constraints is binding since this depends on the values of $z$ and $y$ in the optimal contract. In both cases, however, it can be easily verified that $t_1$ and the transfers $t^*_2$, $t^*_4$ satisfy the constraints of all types.\(^5\)

To summarize the discussion, we obtained the following result: in any optimal contract the IR constraint of type 4 and the IC constraint of type 2 with respect to type 4 are binding. The optimal transfers for types 2 and 4 are therefore given by $t^*_4$ and $t^*_2$ and can be computed once the optimal quantities are determined. As for type 1, either (2) or (3) or both are potentially binding constraints, therefore they must be explicitly considered in the regulator’s problem.

\(^5\) Recall that, by Assumption 1, we have $\min\{e, x\} > \max\{y, z\}$.
4. The optimal contract

From the analysis of the previous section we know that the optimal procurement contract can be obtained by substituting $t^*_4$ and $t^*_2$ into the welfare function and solving the following optimization problem

$$\max_{c,x,y,z,t_1} \alpha_1 [2u(e) - 2\theta c(e) - \lambda t_1] + 2\alpha_2 [u(x) + u(y) - (1 + \lambda)(\theta c(x) + \theta c(y))$$

$$- \lambda c(z)] + \alpha_4 [2u(z) - 2(1 + \lambda)\theta c(z)]$$

subject to

$$t_1 - 2\theta c(e) - [c(z) + c(y)] \geq 0 \quad IC(1,2)$$
$$t_1 - 2\theta c(e) - 2c(z) \geq 0 \quad IC(1,4)$$

The solution satisfies the following first order conditions:

$$u'(e) = \theta c'(e) + \frac{\mu_{12} + \mu_{14}}{\alpha_1} \theta c'(e) \quad (4)$$
$$u'(x) = (1 + \lambda)\theta c'(x) \quad (5)$$
$$u'(y) = (1 + \lambda)\theta c'(y) + \frac{\mu_{12}}{2\alpha_2} c'(y) \quad (6)$$
$$u'(z) = (1 + \lambda)\theta c'(z) + \frac{2\lambda\alpha_2 + \mu_{12} + 2\mu_{14}}{2\alpha_4} c'(z) \quad (7)$$
$$\mu_{12} + \mu_{14} = \lambda\alpha_1 \quad (8)$$

where $\mu_{12} \geq 0$ and $\mu_{14} \geq 0$ are the Lagrange multipliers of the constraints IC(1,2) and IC(1,4). In addition, there are complementary slackness conditions.

By condition (8) at least one of the multipliers must be different from zero; also, by substituting the multipliers in (4) and comparing with (5), we immediately see that $e = x$ and that these quantities equate marginal social benefits and social marginal costs. Therefore, the monopolists with low costs on good $k$ produce the same quantity of the good and this quantity is equal to the socially efficient level. Moreover, it can be noticed\footnote{Indeed, suppose that $\mu_{12} = 0$ and thus $\mu_{14} > 0$; from the first order conditions (6) and (7) we see that $y > z$ and therefore IC(1,4) cannot be binding, but this in turns implies that $\mu_{14} = 0$ contrary to the assumption.} that $\mu_{12} > 0$, that is,
the constraint IC(1,2) is always binding. Therefore, the optimal quantities of the high cost good produced by the mixed and high monopolists, respectively \( y \) and \( z \), satisfy the inequality \( y \geq z \).

Since the multiplier \( \mu_{12} \) is always positive the characterization of the solution depends on the value of the multiplier \( \mu_{14} \). When \( \mu_{14} \) is positive both the IC constraints are binding and \( y = z \). By equating the righ-hand side of (6) and (7) and using (8) we obtain the values of the multipliers

\[
\mu_{12} = 2\lambda \alpha_2 \frac{\alpha_1 + \alpha_2}{\alpha_2 + \alpha_4} \tag{9}
\]

\[
\mu_{14} = \lambda \frac{\alpha_1 \alpha_4 - \alpha_1 \alpha_2 - 2\alpha_2^2}{\alpha_2 + \alpha_4} \tag{10}
\]

Since \( \mu_{14} \) is positive it must hold that

\[
\alpha_2 < \frac{\alpha_1 \alpha_4}{1 - \alpha_4} \tag{11}
\]

Condition (11) is also sufficient\(^7\) for \( \mu_{14} > 0 \).

Condition (11) turns out to be the crucial point for the characterization of the optimal procurement contract. If (11) holds then \( y = z \) and the optimal quantity is obtained from one of the first order conditions by substituting the multipliers (9) and (10). On the other hand when (11) does not hold we have \( y > z \) and the optimal quantities are computed for \( \mu_{12} = \lambda \alpha_1 \) and \( \mu_{14} = 0 \).

In loose terms, condition (11) means that the mixed monopolists are relatively unlikely or, in other words, that the costs of the two goods are highly and positively correlated. Indeed, the above result can be stated in terms of the coefficient of correlation between cost parameters, \( \varrho \), that is \( \mu_{14} > 0 \) if and only if\(^8\)

\[
\varrho > \hat{\varrho} \equiv \frac{p}{1 + p}
\]

Note that the threshold \( \hat{\varrho} \) depends only on the marginal distribution and lies between zero and 1/2. Therefore, if costs are positively and highly correlated

\(^7\) Indeed, suppose that \( \mu_{14} = 0 \) thus \( \mu_{12} = \lambda \alpha_1 \). From the first order conditions and \( y \geq z \) we have \( \alpha_1/2\alpha_2 \leq (2\alpha_2 + \alpha_1)/2\alpha_4 \), which contradicts (11).

\(^8\) Recall that (11) can be written as \( \alpha_2 < p(1 - p)/(1 + p) \) and \( \varrho = 1 - \alpha_2/p(1 - p) \).
the optimal contract sets \( y = z \), otherwise \( y > z \). Let us summarize the above discussion in the following.

**Proposition 1.** Let \( \hat{\theta} = p/(1 + p) \). The optimal procurement contract in the symmetric case is characterized as follows:

i) \( \theta = x \) and \( y \geq z \). Moreover

\[
\begin{align*}
    u'(e) &= (1 + \lambda)\theta c'(e) \quad \text{and} \quad u'(y) > (1 + \lambda)\theta c'(y),
\end{align*}
\]

\( t_1 = 2\theta c(e) + 2c(z) \)

ii) if \( \theta > \hat{\theta} \) then \( y = z \) and

\[
\begin{align*}
    u'(z) &= (1 + \lambda)\theta c'(z) + \lambda \frac{p}{1 - p} c'(z),
    \\
    t_1 &= 2\theta c(e) + 2c(z)
\end{align*}
\]

iii) If \( \theta < \hat{\theta} \) then \( y > z \) and

\[
\begin{align*}
    u'(y) &= (1 + \lambda)\theta c'(y) + \lambda \frac{\alpha_1}{2\alpha_2} c'(y),
    \\
    u'(z) &= (1 + \lambda)\theta c'(z) + \lambda \frac{1 - \alpha_4}{2\alpha_4} c'(z),
    \\
    t_1 &= 2\theta c(e) + c(y) + c(z)
\end{align*}
\]

iv) The optimal transfers \( t_2 \) and \( t_4 \) are given by

\[
\begin{align*}
    t_2 &= [\theta c(x) + \theta c(y)] + c(z), \quad \text{and} \quad t_4 = 2\theta c(z)
\end{align*}
\]

Under the optimal procurement contract the monopolist with low cost on some good produces the socially efficient quantity of that good. Each type of monopolist, except type 4, earns positive informational rents which depend on the optimal quantities of the high cost good produced by the mixed and the high type, respectively \( y \) and \( z \). These quantities, which are in any case below the socially efficient level, depend on the degree of correlation between cost parameters. If correlation is positive and sufficiently high then both types of monopolist produce the same quantity of the high cost good. Otherwise, the
mixed type produces a larger quantity than the high type; this is the case, for example, when there is no correlation between costs.

The main features of the optimal contract extends to the more general model of section 2.

**Proposition 2.** Let \( \hat{\varrho} = \sqrt{p_a p_b (1 - p_a) (1 - p_b) / (1 - p_a p_b)} \). The optimal procurement contract for the model of section 1 has the following properties:

i) \( q_{a1} = q_{a2} \) and \( q_{b1} = q_{b3} \) and these are the socially efficient quantities.

ii) If \( \varrho > \hat{\varrho} \) we have \( q_{a3} = q_{a4} \) and \( q_{b2} = q_{b4} \).

iii) If \( \varrho < \hat{\varrho} \) we have \( q_{a3} > q_{a4} \) and \( q_{b2} > q_{b4} \).

iv) In any case, \( q_{a3}, q_{a4}, q_{b2} \) and \( q_{b4} \) are below the socially efficient level.

The proof of Proposition 2 is in the Appendix.9

5. **Single-product contracts**

In this section we compare the optimal procurement contract derived in the previous section (which for the sake of brevity we define multiproduct contract) to the case where the regulatory authority builds on a scheme for the optimal provision of good \( a \) and good \( b \) by means of separate contracts (single product contracts). Unlike the multiproduct contract, where the transfer to the firm depends on the production level of both goods, in a single-product contract the firm receives two transfers and each of them depends on the quantity produced of only one good.10 Since multiproduct include as a special case the single-product contract we expect the latter to be weakly dominated in terms of social welfare

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9 This proposition is similar to Proposition 1 in Dana (1993), however our result is more general since Dana assumes constant marginal costs.

10 If the regulator has the option of choosing the organizational structure of the industry the single-product contract could be interpreted as the choice of a decentralized organization with two firms producing different goods, and the multiproduct contract as an integrated organization with just one firm. In this regulatory environment, however, there are also other regulatory schemes. For example, Dana (1993) considers a kind of contract proposed by Demski and Sappington (1984) and shows that decentralization is better than integration only when correlation of costs is positive and sufficiently high.
by the former. In what follows we show that, under certain conditions, the optimal multiproduct contract is actually strictly welfare improving.

Let us denote by $t_k$ the transfer to the low cost producer of good $k$ when he supplies the quantity $e_k$ and similarly denote by $\tilde{t}_k$ and $\tilde{z}_k$ the transfer and the quantity of the high cost producer. A single-product contract is formally described by $[(t_k, e_k), (\tilde{t}_k, \tilde{z}_k)]$. In the symmetric model the optimal single-product contract is the same for each good, therefore we shall drop the subscripts and assume that the producers of each good face the same contract $[(t, e), (\tilde{t}, \tilde{z})]$. The regulator's problem is the following

$$\max_{t, \tilde{t}, e, \tilde{z}} \quad p[2u(e) - 2\theta c(e) - 2\lambda \tilde{t}] + (1 - p)[2u(\tilde{z}) - 2\theta c(\tilde{z}) - 2\lambda \tilde{t}]$$

subject to the binding IR and IC constraints,\(^\dagger\)

$$\tilde{t} = \tilde{\theta} c(\tilde{z}) \quad \text{and} \quad t = \theta c(e) + c(\tilde{z})$$

The optimal quantities derived from the first-order conditions are

$$u'(e) = (1 + \lambda)\theta c'(e)$$

$$u'(\tilde{z}) = (1 + \lambda)\tilde{\theta} c'(\tilde{z}) + \lambda \frac{p}{1 - p} c'(\tilde{z})$$

By comparing this result with Proposition 1 we notice immediately that the optimal single-product and multiproduct contract are identical when the correlation between costs is positive and sufficiently large. In this case the two regulatory schemes are equivalent in terms of expected welfare and there is no way for the regulator to improve upon by tying the transfer to both goods. However, when $\varrho < \tilde{\varrho}$ the two procurement policies differ and therefore the multiproduct contract is strictly welfare improving. In particular, we have that $\tilde{z}$, the optimal quantity of the high cost producer under the single-product contract, lies in between $y$ and $z$ as determined under the optimal multiproduct contract.

\(^\dagger\) Under the single-product contract the asymmetric information problem with two parameters reduces to the single parameter case of Baron and Myerson (1982); therefore, the binding constraints are the IR constraint of the high cost producer and the IC constraint of the low cost with respect to the high cost producer.
Proposition 3. When \( q < \hat{q} \) the optimal multiproduct and single-product contracts differ and we have \( y > \bar{z} > z \).

Proof. From Proposition 1, point (iii), we see that to establish this result it is sufficient to show that

\[
\frac{\alpha_1}{2\alpha_2} < \left(\frac{\alpha_1 + \alpha_2}{\alpha_2 + \alpha_4}\right) < \frac{1 - \alpha_4}{2\alpha_4}
\]

First notice, from (11), that \( q < \hat{q} \) is equivalent to

\[
(1 - \alpha_4)\alpha_2 > \alpha_1\alpha_4
\]

Substituting \( 1 - \alpha_4 = \alpha_1 + 2\alpha_2 \) in (12) yields \( (\alpha_1 + 2\alpha_2)\alpha_2 > \alpha_1\alpha_4 \) and adding \( \alpha_1\alpha_2 \) to both sides gives the first inequality. Next, adding \( (1 - \alpha_4)\alpha_4 \) to both sides of (12) gives, after simple manipulations, the last inequality. Q.E.D.

The reason of this result rests on the greater flexibility of multiproduct compared to single-product contracts which allows the regulator to redistribute rents and quantities across types or states of nature. In fact, under a multiproduct contract the regulator can differentiate the quantities of the high cost good produced by the mixed and the high monopolist, respectively, \( y \) and \( z \). By decreasing \( z \) and raising \( y \) the regulator is able to trade off (i) an increase of social benefits in states 2 and 3 with a decrease in state 4 and (ii) a reduction of the mixed monopolists' rent with an increase of the low monopolist's rent. Clearly, when different costs of production across goods are more likely this reallocation of quantities in states 2 and 3 and of rents away from states 2 and 3 nets out an improvement in terms of expected social welfare.

On the other hand, when the correlation between costs is positive and relatively high the regulator has interest to reallocate quantities to state 4 and rents away from state 1. In this case, however, a multiproduct contract is not of much use. In fact, let say that we have a contract which sets \( y > z \); then the regulator has the opportunity of reducing the low monopolist's rent by decreasing \( y \). But, once \( y \) has been set equal to \( z \) the regulator does not
gain anything by further reducing \( y \), since the low monopolist's rent is now determined only by \( z \).

To sum up, the multiproduct is better than the single-product contract since it provides the regulator with more opportunities to redistribute quantities and rents across types. With this kind of contract when differences of costs between the two goods are relatively likely the regulator can increase the expected social welfare by reducing the rents of the mixed monopolists and increasing their production. This is exactly the same principle according to which a multiproduct monopolist may prefer to sell goods in bundles to maximize the surplus extracted from consumers with different private valuations across goods, as was shown by Adams and Yellen (1976) and McAfee, McMillan and Whinston (1989).

6. An application to quantity and quality regulation

In this section we apply the results obtained above to analyze the structure of the optimal contract when a quality index as well as a quantity index are included in a procurement contract. The provision of quality by a privately owned monopolist firm and the regulatory policy to be implemented in this case have been a very much debated issue since the contrasting results provided by Schmalensee (1970) and Swan (1970) (see Schmalensee (1979) for a survey). Schmalensee (1970) and other authors argued that a monopoly would produce goods of inferior quality than a competitive industry with equivalent cost conditions. On the other hand Swan (1970) argued that this conclusion was too strong and showed conditions under which the level of quality provided was the same, independently of the market structure.

In this section we analyze an example illustrating the optimal regulation of a multiproduct firm operating under demand and technological conditions such that Swan's independence result holds. The multiproduct firm produces two goods, \( a \) and \( b \), with a level of quality in each product line measured by quality.
indexes \(s_a\) and \(s_b\). The firm has private information on two cost parameters \(\theta_a\) and \(\theta_b\) which can take the values in \(\{\theta, \overline{\theta}\}\), with \(\overline{\theta} > \theta\). From the point of view of the regulator there are four types of monopolist, as in section 2, whose symmetric probability distribution is common knowledge. The inverse demand function is the same for each good and is given by \(P = (1-q)s\); the cost function for the production of the two goods \(^{12}\) is

\[
C(q, s; \theta_a, \theta_b) = \theta_a c(q_a, s_a) + \theta_b c(q_b, s_b)
\]

with \(c(q_k, s_k) = q_k s_k^2/2\) and \(k = a, b\).

An unregulated multiproduct monopolist would provide the following equilibrium quantities and qualities for each of the two goods: \(q^m = 1/3, s^m = 2/(3\theta)\). A social planner under complete information, just takes into account the IR constraint of the firm and will induce, for each good, the following mix of quantity and quality \(q^* = 2/3, s^* = 2/(3\theta)\). Since \(s^m = s^*\), Swan’s independence result holds in this example, that is the same quality level is provided in this industry both under monopoly and under perfect competition, i.e. the only consequence of a monopolistic structure is a positive price cost margin.

When the cost of public funding is included into the regulator’s problem the optimal solution will be: \(q^* = 2/3\), and \(s^{**} = 2/[3\theta(1 + \lambda)]\). We notice immediately that the presence of costs of public funding, raising the social marginal cost of quality provision, reduces the optimal quality, since \(s^* > s^{**}\), but leaves unaffected the optimal quantity.

Let us move now to the case of private information and consider first the optimal regulation based on single product contracts, that is a contract given by \([(t, q, \bar{s}), (\bar{t}, \bar{q}, \bar{s})]\). In this case the regulator will solve the following problem:

\[
\max_{t, q, s, \tilde{t}, \tilde{q}, \tilde{s}} \mathbb{P}[(q - q^2/2)\bar{s} - q\theta s^2/2 - \lambda t] + (1-p) [(\bar{q} - \bar{q}^2/2)\bar{s} - \bar{q}\theta s^2/2 - \lambda \bar{t}]
\]

subject to

\[
\tilde{t} - q\theta s^2/2 \geq 0, \quad \tilde{t} - q\theta s^2/2 \geq \tilde{t} - \bar{q}\theta \bar{s}^2/2
\]

\(^{12}\) This functional specification is a particular case of the general cost function used by the authors cited above and necessary to warrant Swan’s independence result, see Schmalensee (1979), p. 180.
In the optimal contract we have, for each good, $q = \bar{q} = 2/3$ and

$$ s = \frac{2}{3\theta(1 + \lambda)}, \quad \bar{s} = \frac{2(\alpha_2 + \alpha_4)}{3[(\alpha_1 + \alpha_2)\lambda + (\alpha_2 + \alpha_4)(1 + \lambda)\theta]} \quad (13) $$

Under single product contracts, therefore, the quantities produced by a regulated monopolist are the same as in the first best; however, to pursue rent extraction the regulator will distort the provision of quality. In particular it will stipulate the provision of the first best quality level to the low cost firm (no distortion at the top) and will reduce the quality provided by the high cost firm whose rent is set equal to the value of its outside option (no rent at the bottom). So private information introduces a trade-off in the regulator choice: in order to induce the low cost firm to produce the first best quality, it has to reduce the contracted quality to the high cost firm. Let us move now to analyze how a multiproduct contract may, in this model, relax the trade off faced by the regulator.

When the regulator exploits the bi-dimensionality of private information parameters it will offer a multiproduct contract to the firm given by the solution to the following problem:

$$ \max_{q_1, \ldots, q_4, s_1, \ldots, s_4} \sum_{i=1}^{4} \alpha_i \left[ (q_{ai} - q_{aii}^2/2)s_{ai} + (q_{bi} - q_{bii}^2/2)s_{bi} - C_i(q_{ai}, q_{bi}, s_{ai}, s_{bi}) \right] - \lambda t_i $$

subject to the constraints

$$ t_i - C_i(q_i, s_i) \geq 0, \quad t_i - C_i(q_i, s_i) \geq t_j - C_i(q_j, s_j) $$

From the first-order conditions of the problem it can be immediately noticed that any type of firm will be asked to provide the first best quantities of both goods, i.e. $q_{ai} = q_{bi} = 2/3$, for all $i$. Substituting these values into the objective function and the constraints, the above problem turns out to be formally equivalent to a particular case of the symmetric model of section 3, where the choice variables are now the qualities rather than the quantities. Therefore,
the results of section 4, i.e. Proposition 1, apply directly to this case yielding the following solution:

Case 1: when \( \theta < \hat{\theta} \) the qualities set by the optimal contract for types 1, 2 and 4 are respectively

\[
\begin{align*}
S_{a1} &= s_{b1} = s_{a2} = \bar{s} \\
S_{b2} &= 4\alpha_2/3[2\alpha_2(1+\lambda)\bar{\theta} + \lambda\alpha_1], \\
S_{a4} &= s_{b4} = 4\alpha_4/3[2\alpha_4(1+\lambda)\bar{\theta} + \lambda(1-\alpha_4)]
\end{align*}
\]

Case 2: when \( \theta > \hat{\theta} \) we obtain the same solution as in the case of single product contracts, that is the contract (13).

First of all we notice that Proposition 3 of section 5 holds; for example, for good 2 we have \( s_{b2} > \bar{s} > s_{b4} \) when \( \theta < \hat{\theta} \). In other words, when the correlation between costs is either negative or positive, but not too high, in the optimal multiproduct contract the quality provided by the mixed type in the high cost good will be larger than the quality produced by the high cost type; also, the quality provided by the high costs producers under a single product contract is set in between the qualities produced by the two types of high costs producers under a multiproduct contract. In this case it is also true that when \( \theta < \hat{\theta} \) the expected quality provided by the monopolist under a multiproduct contract is larger than the expected quality under the single-product contract; for instance, for good 2 we have

\[
\alpha_2(s_{b2} - \bar{s}) > \alpha_4(\bar{s} - s_{b4})
\]

if and only if \( \alpha_2 > \alpha_1\alpha_4/(1 - \alpha_4) \).

To summarize, in this section we have considered an example of regulatory policy for the provision of quality as well as quantities of two goods by a multiproduct firm with private information. Under demand and cost functions satisfying Swan's independence result the central authority will always induce the production of the first best quantity and, as expected, it will reduce the quality provided in order to minimize the informational rent of the regulated firm. We have shown that when the correlation between costs is either negative
or positive, but not too high, the reduction of the expected quality is lower when the central authority adopts a multiproduct rather than a single-product contract.

7. Conclusions

In this paper we have characterized the optimal procurement policy for a multiproduct monopoly with multidimensional private information about its costs. We have shown that, unless correlation between costs is too large, the central authority can increase expected social welfare by considering the regulated firm as a single unit rather than considering each line of product as an independent activity. Therefore the optimal procurement contract should regulate jointly the production of the various goods even when these goods are not linked by any technological or demand factor. The economic intuition behind this result is similar to the rent-extracting argument put forward by Adams and Yellen (1976) and McAfee et al. (1989) in order to justify the optimal selling strategy of a multiproduct monopolist. In both cases a bundling strategy allows the principal to reduce the informational rents of 'mixed type' agents when they are more likely.

From the analytical point of view we solved a mechanism design problem for an agent with two dimensional private information. Our solution generalizes a result of Dana (1993) (Proposition 1) since we consider a more general specification of agent's payoff. In addition we provided a further result concerning the quantities of the high cost good produced under a multiproduct and a single-product contract (Proposition 3). We have used this result to deal with the case of procurement of quantities and qualities to show that a multiproduct contract not only is welfare improving but also implements higher average qualities.

The results derived in the previous sections may have interesting implications in different contractual situations involving a public agency. As we have seen, in designing the optimal incentive for a regulated multiproduct firm, the government should not offer separate contracts for each activity, unless correlation between costs is large. Public utilities tend to be regulated by different
public agencies which often enter contractual relationships with the same private
multiproduct firm (say a conglomerate operating in different lines of business
like locomotives and biomedical equipment). In this case our result suggests
that there is an economic rationale for centralizing the public procurement ac-
tivities and therefore there should be a unique public agency delegated to sign
contracts for the provision of goods and services produced by the private firm.
The reason why this does not occur in practice may be due to political and
administrative constraints distinct from the purely incentive related aspects of
the problem analysed here, and could be the object of further research.

APPENDIX

Proof of Proposition 2.
In the first place we derive the set of binding constraints. By adopting the same
procedure of section 3, let us fix $t_1$, $t_2$ and $t_3$ so as to equate the corresponding
IR constraints, and verify that $t_4 = C_4(q_4)$ is the minimum transfer satisfying
type 4 constraints. Next, pass to type 3 and by substituting the $t_i$ of the
other types one notices immediately that $t_3$ has to be set so as to equate either
IC(3,2) or IC(3,4). If one tries the first route eventually gets to the requirement
$q_b2 > q_b3$, but this violate Assumption 1. Therefore, the binding constraint
must be IC(3,4). A similar argument for type 2 shows that IC(2,3) cannot be
binding either. So far, we have that the binding constraints are IR(4), IC(3,4)
and IC(2,4) and the corresponding transfers are given by

$$
\begin{align*}
t_4 &= C_4(q_4) \\
t_3 &= C_3(q_3) + c_b(q_b4) \\
t_2 &= C_2(q_2) + c_a(q_a4)
\end{align*}
$$

(1)
The next step consists in showing which of the constraints of type 1 is binding; using (1) we obtain

\[ t_1 - C_1(q_1) \geq c_a(q_{a4}) + c_b(q_{b2}) \quad IC(1,2) \]

\[ t_1 - C_1(q_1) \geq c_a(q_{a3}) + c_b(q_{b4}) \quad IC(1,3) \]

\[ t_1 - C_1(q_1) \geq c_a(q_{a4}) + c_b(q_{b4}) \quad IC(1,4) \]

Any of these constraints is potentially binding at the optimum; indeed, taking the transfers in (1) and \( t_1 \) as determined by any one of the equalities IC(1,2), IC(1,3) and IC(1,4) it is not difficult to show that there are quantities consistent with Assumption 1 and satisfying all the 16 IR and IC constraints.

In sum, we found that three constraints of types 2, 3 and 4 are always binding and that any of the IC constraints of type 1 are potentially binding. The regulator’s problem can then be simplified by substituting \( t_2 \), \( t_3 \) and \( t_4 \) into the social welfare function using (1) and then by considering only the three constraints IC(1,2), IC(1,3) and IC(1,4).

The first order conditions of the regulator’s problem are

\[ \lambda \alpha_1 = \mu_{12} + \mu_{13} + \mu_{14} \quad (2) \]

\[ u_a'(q_{a1}) = \theta_a c_a'(q_{a1}) + \frac{\mu_{12} + \mu_{13} + \mu_{14}}{\alpha_1} \theta_a c_a'(q_{a1}) \quad (3) \]

\[ u_a'(q_{b2}) = \theta_a c_a'(q_{a2}) \quad (5) \]

\[ u_a'(q_{a3}) = (1 + \lambda)\theta_a c_a'(q_{a3}) + \frac{\mu_{13}}{\alpha_3} c_a'(q_{a3}) \quad (7) \]

\[ u_a'(q_{a4}) = (1 + \lambda)\theta_a c_a'(q_{a4}) + \frac{\mu_{12} + \mu_{14} + \lambda \alpha_2}{\alpha_4} c_a'(q_{a4}) \quad (9) \]

\[ u_b'(q_{b4}) = (1 + \lambda)\theta_b c_b'(q_{b4}) + \frac{\mu_{13} + \mu_{14} + \lambda \alpha_3}{\alpha_4} c_b'(q_{b4}) \quad (10) \]

where \( \mu_{12} \geq 0, \mu_{13} \geq 0 \) and \( \mu_{14} \geq 0 \) are the Lagrange multipliers of the corresponding constraints. By substituting the multipliers in (3) and (4) from (2)
and comparing with (5) and (8) we obtain immediately point (i) of Proposition 2.

Let us consider now the values of the multiplier $\mu_{14}$. If $\mu_{14} > 0$ the constraint $IC(1,4)$ is binding and comparing it with $IC(1,2)$ and $IC(1,3)$ we notice that $q_{a4} \geq q_{a3}$ and $q_{b4} \geq q_{b2}$ must hold. By inspection of (7) and (9) we see that the first inequality cannot hold unless $\mu_{13} > 0$ and similarly, from (6) and (10), we see that the second inequality requires $\mu_{12} > 0$; hence, when $\mu_{14} > 0$ implies that all the multipliers are strictly positive. Since all the constraints are binding must hold $q_{a4} = q_{a3}$ and $q_{b4} = q_{b2}$ must hold. By using these equalities and equations (2), (6), (7), (9) and (10) we obtain the values of the multipliers

$$
\mu_{12} = \lambda \frac{p_b}{1-p_b} \alpha_2 \\
\mu_{13} = \lambda \frac{p_a}{1-p_a} \alpha_3 \\
\mu_{14} = \lambda \left[ \alpha_1 - \frac{p_b}{1-p_b} \alpha_2 - \frac{p_a}{1-p_a} \alpha_3 \right]
$$

therefore $\mu_{14} > 0$ if and only if$^{13}$

$$
\alpha_2 < \frac{p_a(1-p_b)^2}{1-p_a p_b}
$$

This condition can be stated in terms of the coefficient of correlation between the cost parameters$^{14}$ as follows

$$
\varrho > \hat{\varrho} \equiv \frac{\sqrt{p_a p_b (1-p_a)(1-p_b)}}{1-p_a p_b}
$$

This proves point (ii) of Proposition 2.

To prove point (iii) we notice that if $\varrho < \hat{\varrho}$ then $\mu_{14} = 0$ and from the IC constraints we have that at least one of the following inequalities must hold: $q_{a3} > q_{a4}$ or $q_{b2} > q_{b4}$. Let us suppose that $q_{a3} > q_{a4}$ and $q_{b2} \leq q_{b4}$. This means that the only binding constraint is $IC(1,3)$ so that we have $\mu_{13} > 0$ and

$^{13}$ To derive this formula recall that $\alpha_1 = p_a - \alpha_2$ and $\alpha_3 = p_b - p_a + \alpha_2$.

$^{14}$ Notice that $\varrho = [p_a(1-p_b) - \alpha_2]/\sqrt{p_a p_b (1-p_a)(1-p_b)}$. 

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\( \mu_{12} = 0 \); by using (6) and (10) this in turn implies that \( q_{b2} > q_{b4} \) and we get a contradiction. A similar contradiction obtains by starting from \( q_{b2} > q_{b4} \) and \( q_{a3} \leq q_{a4} \) and this completes the proof of point (iii).

Finally, to prove that the symmetric contract derived for the symmetric model of sections 2 and 3 is optimal notice that it satisfies all the first order conditions for \( \mu_{12} = \mu_{13} \).

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