The Real Effects of Monetary Policy: a New VAR Identification Procedure

by

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Abstract

This paper proposes a new identification criterion to analyse the real effects of monetary policy within a structural VAR approach. The monetary policy shock is the one having (i) zero impact effect on real GDP and prices; (ii) a large impact effect of opposite sign on non-borrowed reserves and federal funds rate - the liquidity effect. This definition provides (a) a set of partial identifying conditions and (b) a set of quasi-identifying conditions applied to US monthly data relative to the period 1965:1-1994:3. Results show that a contractionary monetary policy shock produces a large negative effect on the real GDP which reduces but does not vanish in the long-run. We find strong evidence against money-neutrality in the short-term and a suspicion of non-neutrality even in the long-run.

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1 Introduction

The purpose of this paper is to establish what effects monetary policy produces on real variables, in particular on real GDP. The neutrality of money and the effectiveness of monetary policy have been one of the most discussed topics in economic theory for half a century. We can distinguish three theoretical positions in the money neutrality debate. Neutralists, (see e.g. Sargent and Wallace, 1977), by assuming a theoretical model where agents have rational expectations, assert that monetary policy cannot be a useful instrument for influencing economic activity; this because systematic monetary policy actions are always anticipated by the agents and the only effect they produce is a variation of the price level, leaving the real GDP unchanged. Only when monetary authorities succeed in surprising agents' expectations - that is only when changes in monetary conditions are unanticipated - may monetary policy affect real GDP in the short-run (Lucas, 1972). Monetarists and New-Keynesians, although with different recommendations of political economy, assert that monetary policy produces real short-term effects; the former refer to the adaptive expectation hypothesis (Phelps, 1967 and Freedman, 1968) the latter to the presence of some rigidities in the labour (e.g. Fisher, 1977 and Taylor, 1980, see Taylor, 1998 for a survey) or goods (Blanchard, 1984) market that cause the price level to move with some period of lag. These effects are always short term effects because in the long run the price level will adjust to the new quantity of money and the real GDP will return to its steady level. Abandoning the vertical long-run supply curve hypothesis, the third theoretical position asserts that monetary policy may produce both short and long-run real effects. Those who support this thesis refer to the presence of imperfections in the labour market that permanently affect economic activity (Blanchard and Summers, 1986), or to the mechanisms that determine international demand (Ginzburg and Simonazzi, 1997), or to the complexity of the financial structure through which monetary policy impulses are transmitted to the economy (Freedman and Kuttner, 1992).

Starting from the contribution by Sims (1980), VAR analysis has been widely used in empirical macroeconomics. VAR models enable the dynamic effects of
exogenous shocks of various nature to be determined over a definite time horizon. The basic idea of VAR models is the propagation impulse mechanisms of Slutsky (1937) and Frisch (1933) and the corresponding Wold theorem. The economic cycle is seen as the sum of white noise shocks of different nature that, through complex propagation mechanisms, cause booms and recessions. By means of VAR analysis it is possible to separate the effects of the single shocks and study their relative weight over the cycle. In the last decade there have been numerous contributions concerning the real effects of monetary policy shock by means of structural VAR analysis; the research has been directed toward both more refined econometric techniques and new identification methods. See Bernanke (1986), Blanchard and Watson (1986), Blanchard and Quah (1989), Bernanke and Blinder (1992), Christiano and Eichenbaum (1992), Gali (1992), Gordon and Leeper (1994), Strongin (1995), Leeper, Sims and Zha (1996), Christiano, Eichenbaum and Evans (1996), Sims and Zha (1998).

This paper, resuming the recent contributions of Bernanke and Mihov (1998a, 1998b) and Uhlig (1998), proposes a new criterion for identifying the effects of a contractionary monetary policy shock. Such a criterion involves a partial identification - only the effects of the monetary policy shock are identified - thus reducing the a priori information to be imposed. The identification is based on hypotheses concerning the shock impact effect on real GDP, prices, non-borrowed reserves and federal funds rate. No restrictions are imposed on the long run behaviour. The monetary policy shock is the one having (i) a zero impact effect on real GDP and prices and (ii) a large effect of opposite sign on non-borrowed reserves and federal funds rate, according to the theory of the liquidity effect. While the first hypothesis involves standard restrictions of zero impact coefficient for the impulses of real GDP and prices, the second involves non-standard restrictions that are obtained by jointly maximising the impact effects on non-borrowed reserves and federal funds rate. Once the shock is identified, we develop a procedure of quasi-identification that allows us to relax the identification hypothesis, in particular the one relative to the maximisation. This procedure enable us to recover the sign and the shape of the impulse of the GDP, confirming the identification results and strengthening the identification criterion.
The results are as follows. The responses of non-borrowed reserves and federal funds rate exhibit a large and significant impact effect of opposite sign which reduces after the first year losing significance. The response of total reserves presents a negative effect smaller than that of non-borrowed reserves. This result suggests the existence of a substitution effect between non-borrowed and borrowed reserves in accordance with a short-term demand curve inelastic to the interest rate (see Strongin, 1995). GDP deflator moves sluggishly, showing a high degree of price stickiness. The commodity price index moves more rapidly than the deflator and the effect persists in the long run. The most interesting result is the one regarding real GDP. For the first semester after the shock, the response of real GDP is close to zero: the GDP reacts sluggishly. After the first semester the effect becomes significant and negative. From the second year after the shock the effect reduces but does not vanish in the long-run. We find evidence against money neutrality in the short run and a suspicion of non-neutrality in the long run.

This paper is organized as follows. Section 2 presents fundamentals of VAR econometrics. Section 3 discusses the theoretical identification hypothesis. Section 4 describes the identification method. Section 5 illustrates the results. Section 6 describes the quasi-identification method.

We use US monthly data relative to the period 1965:1-1994:3. The data set includes real GDP (GDP), the GDP deflator (GDPD), a commodity price index (CP), total reserves (TR), non-borrowed reserves (NBR) and the federal funds rate (FFR).

All the data elaboration programs were constructed by the author in MATLAB.

2 Structural VAR Econometrics

Representations

The building block of VAR econometrics is the Wold Representation Theorem that states that any stationary stochastic process can be decomposed in two
orthogonal components in the following manner:

\[ X_t = \sum_{j=0}^{\infty} \alpha_j \varepsilon_{t-j} + d_t \]  

(1)

where \( \sum_{j=0}^{\infty} \alpha_j \varepsilon_{t-j} \) represents the stochastic component, with \( \sum_{j=0}^{\infty} \alpha_j^2 < \infty \), \( \{\varepsilon_{t-j}\}_{j=0}^{\infty} \) is a zero mean white noise process, that is (a) \( E\varepsilon_{t}\varepsilon'_{t} = \sum_{k} E\varepsilon_{t}\varepsilon'_{t-k} = 0 \) for \( k \neq 0 \), and \( d_t \) represents the purely deterministic component, the one perfectly predictable by using past information. As usual in VAR literature, in the remainder of this paper we will consider only regular processes, that is those processes for which \( d_t = 0 \). Rewriting (1) in lag operator terms and assuming \( k_t = 0 \) we have

\[ X_t = A(L)\varepsilon_t \]  

(2)

where \( A(L) = I + A_1 L + A_2 L^2 + \ldots \) is a polynomial matrix of dimension \((n \times n)\) and \( \varepsilon_t \) is a vector of dimension \((n \times 1)\). Equation (2) is the Wold representation of the process \( X_t \) and the following conditions hold: (b) all the roots of the determinant of \( A(L) \) are outside the unit circle in the complex field, (c) \( A(0) = I \). Conditions (a), (b) and (c) guarantee the uniqueness of the representation.

From the Wold representation is possible to derive the class of fundamental representations of the process \( X_t \). Given any non-singular matrix of constants \( R \) it is possible to rewrite (2) as follows

\[ X_t = A(L)RR^{-1}\varepsilon_t = B(L)u_t \]  

(3)

where \( B(L) = A(L)R \) and \( u_t = R^{-1}\varepsilon_t \). Since \( R \) can be any non-singular matrix of constants, it follows that the class of fundamental representations defined by (3) has infinite representations that differ from each other for a particular \( R \). From the class of fundamental representations we may define a subclass, that of orthonormal representations. Let \( S \) be the Choleski factor of \( \Sigma \varepsilon \) such that \( SS' = \Sigma \varepsilon \). Postmultiplying \( A(L) \) for \( S \) and premultiplying \( \varepsilon_t \) for \( S^{-1} \) in (2) we obtain

\[ X_t = D(L)\eta_t \]  

(4)

\[ \text{The representations for which condition (b) holds; for non-fundamental representations see Lippi and Reichlin (1993).} \]
where \( D(L) = A(L)S \) and \( \eta_t = S^{-1} \varepsilon_t \). Equation (4) is the Choleski representation of \( X_t \) and has the following properties: \( D(0) = S, D(L) = S + D_1 L + D_2 L + \ldots, \sum \eta = E \eta \eta' = S^{-1} \sum \varepsilon \varepsilon' = I \). As for the class of fundamental representations, even in this case it is possible to generalize to the whole class of orthonormal representation. For any matrix \( H \) such that \( H H' = I \), by post-multiplying \( D(L) \) for \( H \) and premultiplying \( \eta_t \) for \( H' \) we obtain the following representation:

\[
X_t = D(L)HH' \eta_t = C(L)e_t
\]

where \( C(L) = D(L)H \) and \( e_t = H' \eta_t \); representation (5) has the same properties of representation (5) and differs from (4) for \( H \). The class of orthonormal representations, as subclass of that of fundamental, contains infinite representations which differ from each others for a particular \( H \) matrix.

Given a matrix \( H \), equation (2) and (5) set up our model: the first is the reduced form and the second the structural form of the VAR. The following relations hold: \( C(L) = A(L)S H \) and \( e_t = H' S^{-1} \varepsilon_t \).

**Partial Identification**

In VAR literature, the classical procedure for identifying consists in making the number of parameters of the structural form equal to the number of parameters estimated in the reduced form by setting some parameters of \( C(L) \) equal to zero\(^2\). Such restrictions can be short-term restrictions, imposed on \( C(0) \), or long-run restrictions, imposed on \( C(1) \), and result from specific assumptions on the effect of the shocks over the variables included in the model. The orthonormality condition provides us with \( (n^2 + n)/2 \) restrictions; since the number of free parameters is \( n^2 \), we need \( (n^2 - n)/2 \) further restrictions. Such restrictions can be obtained by two different methods. The first method implies a recursive model, that is \( C(0) \) (Sims, 1980) or \( C(1) \) (Blanchard and Quah, 1989) must be triangular. For the case \( C(0) \) triangular, \( C(0) = S \) and \( H = I \); for the case \( C(1) \) triangular, we must choose \( H \) such that \( A(1)S H \) be triangular. The second method imply a number of coefficients equal to zero

\(^2\)The classical procedure is not the only usable one; Uhlig (1998) proposes a procedure based on the minimisation of a penalty function that gives increasing weight to the coefficients of the impulses that do not respect sign restrictions.
on the basis of theoretical considerations about the effects of the shocks on the variables included in the VAR (Blanchard and Watson, 1986 and Bernanke, 1986).

Let us suppose we are not interested in completely identifying — in determining the effects of all the $n$ shocks — but are interested in a determinate subclass of shocks of dimension $k$ with $k = 1, \ldots, n - 1$. In this case, we need only the first $k$ columns of the matrix $C(L)$. In order to identify the first $k$ columns of $C(L)$ it is sufficient to choose the first $k$ columns of $H$. Partitioning $H$, $D(L)$ and $\eta_t$ as follows, $D(L) = (D_1(L)|D_2(L))$, $H = (H_1|H_2)$ and $\eta_t = (\eta_{1t}|\eta_{2t})$ our model will be the follows:

$$X_t = D_1(L)H_1\eta_{1t} + D_2(L)H_2\eta_{2t} = C_1(L)e_{1t} + C_2(L)e_{2t}$$ (6)

where $\eta_{1t}$ is the vector of $k$ shocks to be identified and $\eta_{2t}$ the vector of the other $(n - k)$ shocks; $H_1$ is the matrix we must choose for identifying; $D_1(L)$ and $D_2(L)$ are two polynomial matrices in $L$ and $C_1(L)$ is the matrix of the impulses relative to the $k$ shocks that we have identified. The partial identification allows us to choose only part of $H$ thus reducing the number of $a$ priori restrictions. The orthogonality condition provides us $(k - 1)/2$ restrictions and the orthonormality condition $k$ restrictions; the number of free parameters is $nk$, so we need $k(2n - k - 1)/2$ further restrictions. As with the exact identification, such restrictions can be short-term or long-term restrictions imposed on the first $k$ columns of $C_1(0)$ or $C_1(1)$.

3 The hypothesis

In VAR literature the monetary policy shock can be identified by two different kind of assumptions. Some authors (e.g. Christiano and Eichenbaum, 1992, Bernanke and Blinder, 1992, Strongin, 1995, Bernanke and Mihov, 1996) assume that monetary policy affects real activity with a period of lag, in other words, they assume that real variables are predetermined relative to the reaction function of monetary authorities 3. Such an assumption derives from the

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3By using a recursive model, real variables are ordered first and non-policy variables second.
consideration that actual production depends on past decisions, so that variations in monetary conditions are unable to produce variations in real GDP. Other authors (e.g. Gordon and Leeper, 1994, Sims and Zha, 1995, Leeper, Sims and Zha, 1996) assume that monetary authorities observe GDP realizations with a period of lag — that is real variables are not predetermined — and that monetary policy affects real activity contemporaneously; this second theoretical position refers to the presence of impediments in monitoring the real economy and in the flows of information.

The identification proposed in this work is based on two different groups of theoretical hypothesis. On the one hand we refer to macroeconomic assumptions, on the other hand we refer to the working mechanism of the market for bank reserves.

*Macroeconomic Hypothesis*

In this work we adopt a theoretical framework where monetary policy produces real effects with at least one period of lag and monetary policy has no simultaneous effects on real GDP (resuming the first theoretical position we referred to). We assume that (i) prices move sluggishly, imposing one period of lag. Such an assumption is derived from sticky price models (see e.g. Fisher, 1977, Blanchard, 1984 and Taylor, 1998, for a survey) and it appears reasonable because even a low degree of rigidity (as in our case) makes the simultaneous adjustment of price level to the new quantity of money impossible. Moreover, we assume that monetary policy produces no simultaneous effects on commodity prices because exchange rate variations induced by monetary policy actions *via* capital flows produce effects in commodity prices over longer horizons. We assume (ii) that real GDP reacts to monetary policy change with a period of lag; that assumption too appears reasonable because actual production depends on past decision so that interest rate variations have real effects after more than one month.

*Reserves Market*

The second group of hypothesis is based on the operating mechanisms of the market for bank reserves. We assume that a contractionary monetary policy shock has (iii) a negative impact effect on non-borrowed reserves, (iv) a positive
impact effect on federal funds rate, and that (v) these effects are jointly large. By considering both the effects we exclude endogenous effects produced by reserve demand\textsuperscript{4}. With an interest rate targeting operating procedure, variations in the supply of reserves may be due to an accommodating behaviour of the central bank in consequence of demand variation in order to keep the interest rate constant. On the other hand, with a non-borrowed reserves targeting operating procedure, changes in the interest rate may be due to variations in reserve demand in order to keep the supply constant. Hypothesis (iii) and (iv) derive from the theory of the liquidity effect: for a given demand, a change in non-borrowed reserves produces a change of opposite sign in the interest rate in the short term (see e.g. Cagan and Gandolfi, 1968, Leeper and Gordon, 1992, Strongin, 1995, Bernanke and Mihov, 1998b). Hypothesis (v) derives from the consideration that since non-borrowed reserves and federal funds rate are under the control of monetary authorities, and monetary changes are highly explicative of their variances, an exogenous monetary policy shock will have a large impact effect on these variables.

4 The Identification Criterion

Let us consider equation (6). Be $X_t$ the vector of variables GDP, GDPD, CP, NBR, FFR, TR and be $e_1$ the monetary policy shock. Our purpose is to determine $C_1(L)$; since $k = 1$, $C_1(L)$ is the first column of $C(L)$ and $H_1$ is the first column of $H$. Our criterion focuses on the choice of $H_1$ because $D(L)$ is obtained by postmultiplying $A(L)$ for $S$ and is given. In order to determine the effects of the contractionary monetary policy shock, $e_1$, let us proceed as follows. From the hypothesis of orthonormality of the matrix $H$, the following conditions must hold, $\sum_{i=1}^{n} h_{i1}^2 = 1$ where $h_{i1}$ are the elements $H_1$ with $i = 1, \ldots, n$. From hypothesis (i) and (ii), $c_1 = 0$, $c_2 = 0$ and $c_3 = 0$, where $c_1$, $c_2$ and $c_3$ are respectively the first, second and third elements of

\textsuperscript{4}As suggested by several authors, see e.g. Gordon and Leeper, 1994, Strongin, 1995 and Bernanke and Mihov, 1998a
c^5. Hypothesis (iii) involves c_4 < 0 and hypothesis (iv) involves c_5 > 0. From hypothesis (v), c_4 and c_5 must be jointly large. In order that c_4 and c_5 be jointly large, we need, first, an idea of their size and, second, a technical procedure to keep them jointly large. Since E\epsilon_t\epsilon'_{t-k} = 0 for k \neq 0, the variances of the series NBR and FFR are

\[
\text{var}(X_4) = \sigma^2_{NBR} = \sum_{j=1}^{6} \sum_{k=0}^{\infty} \text{var}(e_j)c^2_{4jk} = \sum_{j=1}^{6} \sum_{k=0}^{\infty} c^2_{4jk}
\]

and

\[
\text{var}(X_5) = \sigma^2_{FFR} = \sum_{j=1}^{6} \sum_{k=0}^{\infty} \text{var}(e_j)c^2_{5jk} = \sum_{j=1}^{6} \sum_{k=0}^{\infty} c^2_{5jk}
\]

with j = 1, ..., 6 the number of shocks, k the periods after the shock and \text{var}(e_j) = 1. In order to have an idea of the size of the impact effect of the shock on FFR and NBR let us maximize separately \(\theta_1 = \frac{c^2_{4}}{\sigma^2_{NBR}}\) and \(\theta_2 = \frac{c^2_{5}}{\sigma^2_{FFR}}\). These two quantities, \(\Theta_1 = \max(\theta_1)\) and \(\Theta_2 = \max(\theta_2)\) signify the largest contribution of the monetary policy shock in the first month to the variances of the two series (NBR and FFR). In order to keep the impact effects jointly large, we compute the ratio \(r = \theta_1/\theta_2\) and maximize \(\theta_2\) under the constraint \(\theta_1/\theta_2 = r\). In such a way we obtain \(\tilde{\Theta}_1\) and \(\tilde{\Theta}_2\). From \(\tilde{\Theta}_1\) and \(\tilde{\Theta}_2\) it is possible to derive the impact coefficients \(\tilde{c}_4 = \sqrt{\sigma^2_{NBR}\tilde{\Theta}_1}\) and \(\tilde{c}_5 = \sqrt{\sigma^2_{FFR}\tilde{\Theta}_2}\). This passage completes the characterization of the vector \(H_1\) and the identification (see Appendix B).

### 5 Results

The results are shown in Figure 1; the impulse response functions are plotted with 90\% confidence bands\(^6\).

\(^5\)The variables are ordered as follows: GDP, GDPD, CP, NBR, FFR, TR.

\(^6\)The confidence bands are constructed with the bootstrapping method; let \(\{\hat{\epsilon}_t\}_{t=1}^{T}\) be the vector of residuals of the VAR with \(T\) the number of observations. By extracting with introduction \(T\) times from \(\{\hat{\epsilon}_t\}_{t=1}^{T}\) we construct 1000 new residual matrices \(\{\hat{\epsilon}_t(j)\}_{t=1}^{T}\) with \(j = 1, ..., 1000\). For each \(j\) matrix, given initial conditions, we construct a new set of series. For each set we estimate the VAR, and collect the impulse functions. For each variable at any lag we extract the 50\(^{th}\) lower value and the 950\(^{th}\) higher value. In so doing, for each original impulse function we obtain two 90\% confidence
The response to the shock of non-borrowed reserves shows a negative and significant impact effect and reaches its minimum during the second month. Beginning from the third month the effect reduces and after three years converges at a negative but not significant value. The behaviour of non-borrowed reserves is consistent with the theory: after the initial contraction the effect of the shock begins to reduce, losing significance. This phenomenon has two explanations: from an econometric point of view it depends on the presence of feedback effects due to the reaction of the other variable included in the VAR; from a theoretical point of view it depends on the decrease of the degree of monetary policy tightness (in line with a countercyclical policy view).

The response of the federal funds rate is similar to that one of non-borrowed reserves. After a strong positive initial effect — the liquidity effect — the effect reduces losing significance. The behaviour of the interest rate too is consistent with the theory: while in the very short term the liquidity effect holds, over longer horizons other phenomena begin to play a determinant role, such as price expectations adjustment, which has an effect of opposite sign on the interest rate. The effect of the shock, as in the case of non-borrowed reserves, is transitory and vanishes in the long run. This is an interesting point because our result is in contrast with the Fisherian theory concerning the long-run behaviour of the interest rate: that theory asserts that, in the long run, prices and interest rate move in the same direction. This means that we should expect the interest rate become negative before converging; but this is not the case.

The response of total reserves follows that of non-borrowed reserves but the impact effect is smaller. This may sound strange, but in fact it is not. Let us look at a possible explanation: total reserves are the sum of non-borrowed and borrowed reserves. If the reserves demand is perfectly inelastic, a reduction in non-borrowed reserves will be offset by an increase in borrowing by commercial banks leaving total reserves unchanged (Strongin, 1995). With a very (but not perfectly) inelastic demand, total reserves will reduce, but less than non-borrowed reserves, as in our case.

GDP deflator reacts with lag to the monetary policy shock. Until the fourth
month it remains very close to zero, suggesting a degree of stickiness higher than the one imposed *a priori*. However for a very long period, four years, the effect is not significant. How to interpret the non-significance of the GDP deflator? On the one hand we could point to the bad properties of the series or toward other technicalities. On the other hand, wishing to find an economic explanation, since the deflator is a net-import index, we could consider the inflation process as a cost-determined process rather than demand-determined. If so, in excluding the cost of imports we would exclude one of factors which best account for the price variation.

The response of the commodity price index is different from that of the deflator; starting from the second month, it shows a negative effect which increases for two years and thereafter converges at a significant value: the monetary policy shock produces a permanent negative effect. The reduction of the commodity price operates presumably via dollar appreciation; a contractionary monetary shock, by increasing interest rate, causes external capital inflow, making the dollar appreciate. Such appreciation sets off a disinflation process which reduces the price of commodities.

The most interesting result is that regarding the response of real GDP. For the first months the response is very small and not significant. After six months the shock begins to produce effects: during the second semester the response shows strong negative effects and reaches its minimum value at the end of the first year. During the second year the response oscillates around his minimum value and then, after the second year, begins to reduce. In the long run however, the effect does not vanish but converges at a negative and significant value.

A contractionary monetary policy shock produces a permanent and significant reduction of the real GDP. This reduction is very strong in the short term but although diminishing persists in the long run. Such a result provides us with new evidence against money neutrality in the short and a strong suspicion of non-neutrality in the long run.

In conclusion, the results present three interesting elements. 1) A contractionary monetary policy shock produces a permanent effect on real GDP. 2) The behaviour of non-borrowed reserves and federal funds rate after the shock is consistent with the theory: a strong short period effect which vanishes in the
3) The response of prices may induce one to consider the inflation process as cost-determined rather than demand-determined.

6 A Quasi-Identification Criterion

In Section 4 we identified the VAR by jointly maximizing the impact effects of non-borrowed reserves and federal funds rate. It could be objected that our methodology has an arbitrary aspect: though the effects are large, they may not be the largest. If that were the case (but one should reflect on what kind of shock could generate larger impact effects while leaving income unchanged) our criterion would not be correct.

In this Section we propose a method that, by relaxing the identification hypothesis, confirms the results concerning the GDP response to the shock — which strengthens the identification procedure. We call this method Quasi-Identification Criterion since it does not identify but may (such in this case) enable them to recover the sign and the shape of the impulse functions. This kind of analysis reduces the number of the a priori restrictions; exact identification requires $n(n-1)/n$ restrictions, partial identification $k(2n-k-1)/2$, quasi identification one at least. Quasi identification criterion involves the study of all impact vectors for which restrictions are respected. Although there are several vectors generating impulse functions in conformity with the restrictions, let us suppose that all the impulses have the same shape or, more simply, that they all present the same property. In this case we can say nothing about the exact size of the effect but we may recover both the sign and the shape of the effect. On the other hand if the impulses had very different shapes, we could say nothing not only about the size but also about the sign and the shape of the effect.

The hypotheses of quasi-identification are the following; a contractionary monetary policy shock has (i) zero impact effect on real GDP, (ii) a zero impact effect on prices, (iii) a positive impact effect on federal funds rate, (iv) a negative impact effect on non-borrowed reserves and (v) an impact effect on federal funds rate and non-borrowed reserves such that the contribution of the shock
to the variance of the two series in the first month is at least equal to one third of the largest contribution possible, \( \Theta_1, \Theta_2 \), calculated under the constraint \( r = \theta_1/\theta_2 \). Hypothesis (v) entails that any impact vector generating an impact effect on non-borrowed reserves and federal funds rate such that \( \theta_1 \leq 0.5(\Theta_1) \) and \( \theta_2 \leq 0.5(\Theta_2) \) conforms to the definition of monetary policy impact vector. This threshold appears reasonable and not particularly demanding, since the federal funds rate and non-borrowed reserves are under the Fed’s control, and the variances of the two variables are largely explained by monetary policy actions.

From the orthonormality hypothesis the following condition holds \( \sum_{i=1}^{n} h_{1i}^2 = 1 \). From hypotheses (i) and (ii) \( c_1 = 0, c_2 = 0, \) and \( c_3 = 0; \) in order to have zero impact effect on real GDP and prices, \( S \) being a lower triangular matrix, we put the first three elements of \( H_1 \) at zero (see Appendix B). On the basis of these considerations we parameterize vector \( H_1 \) as follows

\[
H_1 = \begin{pmatrix}
0 \\
0 \\
0 \\
\cos(\delta_1)\cos(\delta_2) \\
\cos(\delta_1)\sin(\delta_2) \\
\sin(\delta_1)
\end{pmatrix} \delta \in \mathbb{R}^2.
\]

(9)

Varying \( \delta = (\delta_1, \delta_2) \) in the interval \([0, 2\pi] \) by a discrete grid, \( g = 2\pi/20 \), \( 20^2 \) vectors \( \delta \) will result, any of which generates six impulse functions. For some of those the restrictions will be respected, for some others it will not. Let us consider the first vector \( \delta^1 \) and let \( C^1(L) \) be the impulse response functions relative to that vector. Let \( x^1_i = 1 \) if \( \theta_{1,\delta^1} \leq 0.5(\Theta_1) \) and \( x^2_i = 1 \) if \( \theta_{2,\delta^1} \geq 0.5(\Theta_2) \). We consider the following function

\[
f(x) = \begin{cases}
1, & \text{if } \sum_{i=1}^{n} x^1_i = 2 \\
0, & \text{otherwise}
\end{cases}
\]

(10)

For any vector \( \delta \) \( f(x) \) will be 1 or 0. Such a function identifies all vectors \( \delta \) for which those restrictions are respected\(^7\). All points for which \( f(x) = 1 -

\(^7\)The quasi identification procedure requires a minimum of one restriction up to a maximum of \((n - 2)\). Such restrictions can be of different type: zero restrictions, sign restrictions or value
i.e. the restrictions are respected – form a subspace in $\mathbb{R}^2$. The aim of this procedure is to explore this subspace and study the behaviour of the impulse response functions in order to recover the sign and the shape of the effect. Results of quasi-identification are shown in Figure 2. The impulses of the real GDP present a shape that is almost identical both in the short and in the long run for any vector $\delta$ for which $f(x) = 1 - c_1(1)$ varies between $-0.1$ and $-0.2$. The responses of federal funds rate and non-borrowed reserves are also similar. We conclude that the quasi identification analysis strengthens the identification criterion shown in Section 4 confirming the results on the reaction of GDP to the monetary policy shock. By relaxing the identification hypothesis concerning the size of the impact effect on non-borrowed reserves and federal funds rate, we obtain results in line with those obtained in Section 5; in so doing, the suspicion that maximisation could substantially affect results is dispelled.

7 Conclusions

In this paper we find evidence against money neutrality: the main conclusion is that monetary policy is non-neutral in the short term and, in contrast with the mainstream theories, may be so even in the long run. This conclusion is achieved by means of a new partial identification criterion based on restrictions on the sign and the size of the impact effect on GDP, prices federal funds rate and non-borrowed reserves. In addition, we propose a quasi-identification criterion that, by relaxing the identification hypothesis, enable the sign and the shape of the GDP impulse to be recovered confirming the conclusion of non-neutrality.

restrictions (in our case we have all three types). Zero impact effect restrictions are particularly useful because from the relations $SH_1 = C_1(0)$ and using a partial recursive scheme where the variables with zero impact effect are ordered first - $S$ being lower triangular - the first $k$ elements of $H_1$ must be zero, thus reducing the dimension of the space $\delta$. 

15
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Appendix A

The data set contains US monthly data for the following variables: real GDP (GDP), GDP deflator (GDPD), commodity price index (CP), total reserves (TR), non-borrowed reserves (NBR) and federal funds rate (FFR). All variables are taken in logarithms, except for the federal funds rate, and in first differences, except for the GDP deflator that is taken in second difference (being $I(2)$). Non-stationarity has been tested with the Dickey-Fuller test. We estimated the VAR with 12 lags with OLS estimator equation by equation. In order to have the impulse response functions we transformed the VAR model into an $AR(1)$ process, then inverted the $AR(1)$ in a $MA(\infty)$ process. $MA(\infty)$ being a finite variance process, we truncated it at $t = 100$. 
Appendix B

Here we show technical aspects of the identification criterion and how to choose the matrix $H_1$. The first step consists in transforming the model expressed by equation (2) into the orthonormal model expressed by equation (4) by postmultiplying $A(L)$ by the Choleski factor $S$ of the variance-covariance matrix $\Sigma_{\epsilon}$. The second step consists in choosing the matrix $H_1$ in order to determine $C_1(L)$ by postmultiplying $D(L)$ for $H_1$. $H_1$ is obtained as follows. Hypotheses (i) and (ii) entail $s_{11}h_1 = 0$, $s_{21}h_1 + s_{22}h_2 = 0$ and $s_{31}h_1 + s_{32}h_2 + s_{33}h_3 = 0$, $S$ being lower triangular. Such a conditions imply three further restrictions on the first three coefficients of $H_1$: $h_1 = 0$, $h_2 = 0$ and $h_3 = 0$. Hypotheses (iii) and (iv) involve $s_{44}h_4 < 0$ and $s_{54}h_4 + s_{55}h_5 > 0$. From hypothesis (v) $s_{44}h_4$ and $s_{54}h_4 + s_{55}h_5$ must be jointly large. We then calculate the ratio, $r = \Theta_1/\Theta_2 = \frac{s_{44}h_4^*}{s_{54}h_4^* + s_{55}h_5^*}$, where $s_{44}h_4^*$ and $s_{54}h_4^* + s_{55}h_5^*$ are the unconstrained maximum values for the impact effect of non-borrowed reserves and federal funds rate. Easy arithmetic passages lead to

$$h_5 = h_4 \frac{s_{44} - rs_{54}}{rs_{55}} \quad (11)$$

$H$ being an orthonormal matrix the following condition must hold

$$h_6 = \pm \sqrt{1 - h_4^2 - h_5^2} = \pm \sqrt{1 - h_4^2 - h_5^2 \left( \frac{s_{44} - rs_{54}}{rs_{55}} \right)^2} \quad (12)$$

From equation (18) we have

$$h_4^2 + h_4^2 \left( \frac{s_{44} - rs_{54}}{rs_{55}} \right)^2 \leq 1 \quad (13)$$

Since $s_{44}$ is constant, the impact effect on non-borrowed reserves will be maximum when $h_4$ is maximum, hence when $h_4^2$ is maximum, and we will have

$$h_4 = \pm \frac{1}{\sqrt{1 + \left( \frac{s_{44} - rs_{54}}{rs_{55}} \right)^2}} \quad (14)$$

In particular $h_4$ will be negative if $s_{44} > 0$ and positive otherwise. From equations (19) and (20) we have $h_6 = 0$; this last passage completes the identification.
and $H_1$ will result

$$H_1 = \begin{pmatrix}
0 \\
0 \\
0 \\
\frac{1}{\sqrt{1 + \left(\frac{s_{44} - r_{55}}{r_{55}}\right)^2}}.
\end{pmatrix}$$

(15)
Tables

**Table 1**: Percentage of the variance of NBR and FFR explained by the monetary policy shock after one month.

<table>
<thead>
<tr>
<th>$\Theta_1$</th>
<th>$\Theta_2$</th>
<th>$\Theta_1$</th>
<th>$\Theta_2$</th>
<th>$0.5\hat{\Theta}_1$</th>
<th>$0.5\hat{\Theta}_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.79</td>
<td>0.41</td>
<td>0.42</td>
<td>0.29</td>
<td>0.21</td>
<td>0.14</td>
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</tbody>
</table>

**Table 2**: Matrix $S$

<table>
<thead>
<tr>
<th></th>
<th>GDP</th>
<th>GDPD</th>
<th>CP</th>
<th>NBR</th>
<th>FFR</th>
<th>TR</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>0.0071</td>
<td>0</td>
<td>0</td>
<td>-0.0010</td>
<td>0.1042</td>
<td>0.0001</td>
</tr>
<tr>
<td>GDPD</td>
<td>0.0001</td>
<td>0.0018</td>
<td>0</td>
<td>-0.0006</td>
<td>0.0029</td>
<td>-0.0004</td>
</tr>
<tr>
<td>CP</td>
<td>0.0017</td>
<td>-0.0000</td>
<td>0.0172</td>
<td>-0.0027</td>
<td>0.0662</td>
<td>-0.0002</td>
</tr>
<tr>
<td>NBR</td>
<td>-0.0010</td>
<td>0.0134</td>
<td>0</td>
<td>0.0134</td>
<td>-0.1321</td>
<td>0.0051</td>
</tr>
<tr>
<td>FFR</td>
<td>0.1042</td>
<td>0.4495</td>
<td>0</td>
<td>0.4495</td>
<td>0</td>
<td>0.0033</td>
</tr>
<tr>
<td>TR</td>
<td>0.0001</td>
<td>0</td>
<td>0</td>
<td>0.0001</td>
<td>0</td>
<td>0.0069</td>
</tr>
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</table>

**Table 3**: Vectors $H_1$ and $C_1(0)$

<table>
<thead>
<tr>
<th>$H_1$</th>
<th>$C_1(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.7807</td>
<td>0.6515</td>
</tr>
<tr>
<td>-0.6249</td>
<td>-0.5408</td>
</tr>
<tr>
<td>0</td>
<td>0.1747</td>
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</table>
Figure 1: impulse response functions of GDP, GDPD, CP, NBR, FFR and TR with 90% bootstrapping confidence band.
Figure 2: quasi-identification; variables are GDP, GDPD, CP, TR, NBR and FFR.


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