Investments and financial structure with imperfect financial markets: an intertemporal discrete-time framework

by

Marco Mazzoli

March 2000

Università degli Studi di Modena e Reggio Emilia
Dipartimento di Economia Politica
Via Berengario, 51
41100 Modena (Italia)
e-mail: me-ma@pt.tizeta.it
Abstract  This paper deals with the problem of simultaneity between the firm’s investments and financial structure, in a context of dynamic optimization, characterised by two main assumptions: first of all, diverging incentives for managers and shareholders, secondly, financial markets imperfections generating a risk premium on the borrowed finance. A "discrete-time" framework has been introduced in order to better model the relevance of timing in the co-ordination process between financial and investment decisions, assumed to take place simultaneously. The simple model proposed here may provide some intuitive interpretation for a number of phenomena such as the propagation of financial shocks into the real economy and the countercyclical mark-ups.

1 Introduction

The relevance of the firm’s financial structure for investments has been the object of a great deal of contributions both in finance and industrial economics, although the problem of simultaneity between investment and financial structure decisions is raised very rarely. In industrial economics, for instance, this issue has been analysed - within the context of predatory pricing models - by the literature on the "deep pocket argument" (Telser, 1966, Benoit, 1984, Poitervin, 1989a) and by the literature on the assumption of "limited liability effect" (Brander

\[\text{I am very grateful to Giuseppe Marotta, Sergio Pastorello, Daniele Ritelli and Kumareswamy Velupillai for their very helpful comments to a previous draft of this paper. All mistakes are mine.}\]
and Lewis, 1985, Poitervin 1989b). In the former, dominant firms can afford a long-lasting price war because they can rely on large financial resources. In the latter, a high level of debt is regarded as a pre-commitment for an aggressive policy: a signalling game of entry deterrence yields, as a result, the optimal financial structure for both the incumbent and the entrant.

In many New-Keynesian literature focusing on the macroeconomic implications of information asymmetries in financial markets and agency problems, the firm's financial structure is often seen as a potential vehicle of diffusion of instability and business cycle, but the attempts to model simultaneous decisions of investment and financial structure are extremely rare in an intertemporal context, if one excludes very simplified two-period models.

In intertemporal investment models the use of a constant and exogenous discount rate in investment models with dynamic optimization is, of course, only a convenient algebraic simplification of reality. In fact, recent contributions have introduced assumptions of stochastic behaviour in the interest (see for instance Saltari and Caiagnini, 1995 for a formalization where the discount rate is assumed to follow a Brownian stochastic process) and in financial economics models (due to uncertainty and information asymmetries) it is very common to "correct" the firm's cost of financial capital for a risk parameter, usually assumed to be a monotonic function of the gearing ratio or the leverage ratio.

However the assumption of simultaneity of investment and choice of financial structure generates a number of problems.

First of all, as pointed out by Aoki and Lejonhufvud (1988), the value of the endowment of the physical capital of a firm is not independent of the endowment of physical capital of the competitor firms. This also means that the concept of market value of physical capital may become somehow ambiguous, and even more so if we think of oligopolistic sectors where the behaviour of the competitors may be modelled by bayesian games. A similar problem could arise even in a perfectly competitive environment if we admit that perfectly rational individuals optimizing the use of information, may have different expectations of future profits and asset prices, due to their different theoretical priors, as postulated, for instance by Kurz rational beliefs theory (Kurz, 1994a, 1994b).

Secondly, a great deal of complexity is associated with the presence of agency problems between managers and shareholders.

If we assume that the management has some discretionary power in allocating the internally generated cash flow and we further admit that in an imperfectly competitive framework a firm might have incentive not to reveal the amount of output and profits associated to a certain level of physical capital or, more generally, we admit that the value of its physical capital might be perceived differently by the firm itself and by its competitors, we might turn out to have some difficulty in defining the concept of expected profits generated by the newly installed capital. In other words, even if the expectations concerning the profits generated by the newly installed capital diverge only in the short run for the firm and its competitors, exogenous causes affecting the capital gain on the shares (and therefore the cost of internally generated finance with respect to the borrowed finance) may affect the firm's financial structure, its
risk (such as perceived by the external financial investors) for long enough to
generate an impact on the (adjusted-for-risk) firm's rate of discount and level
of investments. Furthermore, the length this impact might be associated with
rate of capital depreciation, due to the balance-sheet constraint connecting the
existing stock of capital and the various sources of finance.

In a world of financial market imperfections, where the internally generated
cash flow constitutes a cheaper source of finance than borrowing and issuing
new shares, the behaviour of the share price and capital gains certainly affects
the dividend policy of the management which again affects the firm's financial
structure by determining the rate of profits retention, which, in its turn, determines
the volume of investments financed by internal finance. All that would not have any relevant or persistent effect on the real investments if the share
price rapidly adjusted to the net present value of the profits (possibly adjusted
to account for the presence of uncertainty and sunk costs) generated by the
firm. Our conjecture is, on the other hand, that this could have relevant and persistent effects if we accept that stock prices may diverge for long enough
(possibly because of a persistent bubble) from the net present value of future
profits. Even more relevant and persistent would be the effects if we accepted
that different groups of rational agents may have divergent expectations, due to
their beliefs (as suggested, as we said, by Kurz, 1994a, 1994b).

In Mazzoli (1998, ch. 7) the issue of simultaneity between financial structure
and investments decision has been informally analysed within a continuous time
optimal control framework where the simultaneity of the investments and finan-
cial structure decision generates a functional link among the state and control
variables and the rate of discount of future profits. This functional link boils
down into a time-dependent Hamiltonian, but the time dependency is removed
(although at the cost of some loss of generality) by postulating that the relation-
ship between the firm profits and the share price was affected by another time
dependent mechanism of information spreading consistent to the ones informally

Timing in the process of coordination between financial and investment de-
cisions is essential for the definition of flow variables and might not be properly
captured by the conventional continuous time models. In order to take into
account the relevance of timing we introduce here a recursive structure in the
intertemporal problem of the firm's investments characterized by the presence
of simultaneity between firms investments and financial structure. This models
constitute an attempt to formalise a causal link between the flow of profits, the
firm's optimal choice between own capital (defined as non-distributed profits)
and borrowing and the rate of discount of the future flows of profit. An attempt
is also made to remove the possible problems of time dependency by using the
implications of the rate of capital depreciation for the duration of each newly
installed unit of capital.

The next section contains the description of the model, section 3 contains a
proposed approach to solve the problems arising from the time structure of the
model, and the last section contains a few concluding remarks.
2 The model

The main assumptions of the present analysis may be summarized as follows:

i) The market for goods is assumed to be imperfectly competitive, although perfect competition can be a particular case;

ii) The management is composed of members of the controlling group of the firm and acts in the interest of the controlling group of the firm.

The capital is installed at time \( t - 1 \), and is financed with the financial sources raised by the firm at time \( t - 1 \) with a contract establishing also their remuneration which will be paid out at time \( t \).

At time \( t \) the production process takes place, generating the profits \( \pi_t \), and the investment decisions, as well as the payment of the interest on borrowed capital and the dividends on the own capital are taken. \( \Phi_{t-1}^* \) is the weighted average of the cost of own capital and borrower capital, established at time \( t - 1 \) and paid at time \( t \).

Before formalizing the maximand of the intertemporal problem we have to make here a few points. Defining the firm's intertemporal investment decision over an infinite horizon would create some difficulties, since one could not apply the conventional solution approach for discrete-time models, based on backward induction. For this reason we define the firm's investment decision problem over a finite horizon, and, in order to avoid an arbitrary choice of the ending time \( T \), we use the fact that the rate of capital depreciation \( \delta \) (if defined as \( 1/m \), i.e. the reciprocal of an integer \( m \)) also implicitly defines a time \( T = 1/\delta \) where all the persistent modifications on the equilibrium determined by the firm's investment and financial decisions exhaust their effects. In other words, each and every unit of newly installed capital is also characterized by a physical life implicitly defined by its rate of depreciation\(^2\).

On the basis of the above assumptions, the problem of the firm may be represented in the following way.

\[
V_t = \sum_{t=1}^{1/\delta} \left\{ \left[ \pi_t (k_{t-1} | \omega^t) - I_t \right] \cdot \frac{1}{(1 + \Phi_{t-1}^*)^{t-1}} \right\}
\]

where \( \pi_t \) are the profits at time \( t \), \( k_{t-1} \) is the capital installed at time \( t - 1 \), \( I_t \) is the amount of investments (decided at time \( t \) that will contribute to determine the stock of capital at time \( t + 1 \).

\( \pi_t (k_{t-1} | \omega^t) \) is assumed to be the profits maximum value function: in other words, the firm is at any given moment assumed to maximize profits conditional

\(^2\) Another possibility could have been defining the time horizon of the investment decision according to the decision horizon of the firm's management. However, this approach we have chosen here, by allowing to "save one assumption", reduces the degree of the subjectiveness of the model foundations.
on the parameter \( \nu \) (describing the market structure and the competitive environment), and on the labour costs \( \omega \), assumed to be given. In what follows we will omit both \( \nu \) and \( \omega \).

The maximand is subject to the following constraints:

\[
I_t = k_t - (1-\delta)k_{t-1} \tag{2}
\]

\[
\pi_t (k_{t-1}) - \Phi_{t-1} k_{t-1} + \Delta B_t + \Delta E_t = I_t \tag{3}
\]

where \( B_t \) represents the borrowed finance (and, of course, \( \Delta B_t = B_t - B_{t-1} \)), \( E_t \) represents the existing stock of the firm's shares, valued at their issue price; as we said, \( \delta \), the rate of capital depreciation, is expressed in the form of \( 1/m \), where \( m \) is an integer representing expected life of the newly acquired capital; the non-distributed profits can be defined as

\[
\pi_t (k_{t-1}) - \Phi_{t-1} k_{t-1} = \Delta R. \tag{4}
\]

3 is a flow-of-funds condition saying that the new investments \( I_t \) can be financed either by issuing new shares, or by borrowing money or with the residual cash flow which is left after remunerating the financial capital (employed to finance the physical capital \( k_t \)). The latter is defined as the weighted average of borrowing and non-distributed profits (i.e. profits in excess to the dividends that the firm pays out to remunerate the shareholders at the exogenously determined yield \( r^* \)). For simplicity, and since the purpose of this paper is to emphasize the choice between debt and internally generated cash flow, we will assume in what follows that the (exogenously given) time path and variation of the share price and dividends is such as \( \Delta E_0 = 0 \), i.e. the firm is not issuing new shares in the periods under consideration. Therefore, we will rewrite 3 as follows:

\[
\pi_t (k_{t-1}) - \Phi_{t-1} k_{t-1} + \Delta B_t = I_t \tag{4}
\]

To justify this sort of "ceteris paribus" assumption we may think of a labour market characterized by a simplified "efficiency wages" mechanism, where wages and employment are fixed in the short run and are mainly affected by macroeconomic factors.

Introducing the possibility for the firm to issue new shares would not have modified the qualitative meaning of this simple discussion. For instance, if we define \( r^* \) as the yield on the firm's shares at time \( t \), \( p_{s,t} \) the share price, \( \Delta p_{s,t} \) its variation with respect to time \( t-1 \), \( N_t \) the number of existing shares, we could interpret the situation where the firm issues new shares as the case where, given \( D = r^* \cdot p_{s,t} \cdot N_t - \Delta p_{s,t} \cdot N_t \), we have \( \Delta p_{s,t} \cdot N_t > r^* \cdot p_{s,t} \cdot N_t \), i.e. a case of "negative dividends" generated by an extremely favorable valuation of the firm by the market which makes extremely convenient for the management to raise risk capital.
Furthermore, $\Phi_t^*$ is defined as follows:

$$
\Phi_t^* = \min_{\mu_t} \left[ (1 - \mu_t) i_t + \mu_t \left( r^*_t + \phi(\mu_t) \right) \right] 
$$

(5)

where $\phi(\mu_t)$ is the risk premium on the interest rate on the firm’s borrowing and it is assumed to be a monotonically increasing function of the gearing ratio $\mu_t = B_t/k_t$. $\Phi_t^*$ represents here the minimum value function of the firm’s financial costs minimization problem: at every time $t$ the firm is optimizing its financial structure by choosing the optimal gearing ratio $\mu_t = B_t/k_t$ that minimizes the cost of financial capital, defined as the weighted average between the borrowed and the internally generated finance.

The optimized financial structure determines the rate of discount appearing in the intertemporal problem. However, the optimal rate of discount is conditional on the flow of non-distributed profits of the previous period. In this way the "firm-specific" rate of discount is recursively determined as a function of the lagged stock of physical capital and lagged cost of financial capital.

in 5, $i_t$ represents the cost of the internally generated own capital, defined as follows:

$$
i_t = \frac{\overline{D}}{E_0 + R_t}
$$

(6)

where

$$
E_0 = p_{s,0} \cdot N_t
$$

and

$$
\overline{D} = r^*_{s,t} \cdot p_{s,t} \cdot N_t - \Delta p_{s,t} \cdot N_t
$$

(7)

where $r^*_{s,t}$ is the yield on the firm’s shares at time $t$, $p_{s,t}$ is the share price, $\Delta p_{s,t}$ its variation with respect to time $t-1$, $N_t$ is the number of existing shares. Given the (exogenously determined) share price and its short run capital gain, the management of the firm choose a yield $r^*_{s,t}$ which determine the amount of dividends they want to pay to the shareholders. $r^*_{s,t}$ may be interpreted as a financial market constraint on the behaviour of the management: in other words it represents the remuneration that they have to grant to the shareholders in order to keep them happy. It could be interpreted as the result of an implicit negotiation between the shareholders and the management and reflects the typical principal-agent problem of a firm where there is asymmetric information between the shareholders and the management. In this regard, we could have two possible benchmarks:

1. the management only pays out to the shareholders the amount of dividends consistent with the market remuneration of the shares;
The first case was considered in Mazzoli (1998, ch. 7) and was meant to describe those kinds of "non-easily-removable" agency problems between shareholders and managers that may characterise some of the "bank oriented" financial system of continental Europe. The second case would obviously correspond to a situation of absence of agency problems and is consistent with the assumption that managers act in the interest of the shareholders. We will simply assume here that the scenario described by our model lies in between the two extreme frameworks.

Note that for the shareholder the yield on shares is given by 
\[ r_{s,t}^* = \frac{\Delta P_{s,t}}{P_{s,t}^* N_t} + \Delta P_{s,t}^* , \]
while for the management the cost of capital is affected by the issuing value \( P_{o,t} \cdot N_t \) of the shares. This may be better understood if we accept the idea that the stock price may diverge in the short run \(^5\) from the value of the newly installed physical capital and if we admit the possibility of diverging interests between the shareholders and the management.

As we said, the above assumptions are bound to generate not only a recursive structure in the problem but also a certain persistence of the past profits influence on the discount rate. The extent of this persistence is limited by the rate of capital depreciation \( \delta \). In fact, having defined an initial time \( t = 0 \) for the beginning of the firm's activity, by writing in dynamic terms the flow of funds condition 3, we get:

\[
\sum_{i=1}^{t} I_i = \sum_{i=1}^{t} \left[ \pi_i(k_{i-1}) - \Phi_{i-1}^* k_{i-1} \right] + \sum_{i=1}^{t} \Delta B_i
\]

or

\[
\sum_{i=1}^{t} (k_i - (1 - \delta)k_{i-1}) = \sum_{i=1}^{t} \left[ \pi_i(k_{i-1}) - \Phi_{i-1}^* k_{i-1} \right] + \sum_{i=1}^{t} \Delta B_i \tag{8}
\]

or again

\[
\sum_{i=1}^{t} (k_i - k_{i-1}) = \sum_{i=1}^{t} \Delta R_i + \sum_{i=1}^{t} \Delta B_i - \sum_{i=1}^{t} \delta k_{i-1} \tag{9}
\]

By looking at 8 and 9 it is clear that the flow of profits has a persistent effect on the amount of reserves (i.e. on the share of internally financed physically capital). In particular, 9 shows that the degree of persistence is determined by the reciprocal of the rate of capital depreciation \( 1/\delta \). In this regard we may identify two extreme cases: in the first one \( \delta = 1 \), and in the second one \( \delta = 0 \).

\(^5\) or even in the long run according to the "rational beliefs" theory (see, for instance Kurz (1994a), (1994b)).
When $\delta = 1$, the capital only lasts for one period and we have

$$k_t = [\pi_t(k_{t-1}) - \Phi_t^\ast k_{t-1}] + B_t,$$

for

$$t > 1.$$

This means that our problem is still recursive but the impact of the current profits on the firm's gearing ratio and discount rate is limited to one period ahead. In this case we would be back to a simpler case of model with intertemporally nonadditive preferences similar to the one summarized in Obstfeld and Rogoff (1996, pp. 722-724) on the basis of Uzawa (1968).

On the contrary, the assumption $\delta = 0$ correspond to the case of ever-lasting influence of the entire past history of borrowing and profits retention for the determination of the present and future firm's discount rate.

In this last case, $\delta$ would become the following:

$$k_t = \sum_{i=1}^{t} \Delta R_i + \sum_{i=1}^{t} \Delta B_i \quad (10)$$

or

$$k_t = \sum_{i=1}^{t} [\pi_t(k_{i-1}) - \Phi_t^\ast k_{i-1}] + \sum_{i=1}^{t} \Delta B_i = R_t + B_t$$

and, having substituted $7$ and $3$ into it, it becomes the following:

$$i_t = \frac{r^\ast_{t,t} \cdot p_{t,t} \cdot N_t - \Delta p_{t,t} \cdot N_t}{E_0 + \sum_{i=1}^{t} (\pi_t(k_{i-1}) - \Phi_t^\ast k_{i-1})} \quad (11)$$

However let us return now to the more general case where $0 < \delta < 1$.

Equation 5 is the minimum possible weighted average of the borrowed and internally generated finance. Since the internally generated finance is predetermined (by the non-distributed profits at time "t - 1"), by choosing the value $\Delta B_t$ the firms also delimits the maximum amount of feasible new investments at time "t" and the gearing ratio at time "t", which will be incorporated in the new debt contracts that the firm issues in order to finance part of its investments. In other word, as we are going to show later, $\Delta B_t$ assumes in this context the nature of control variable.

To better understand the nature of the problem, it may be useful to first develop the minimum value function $\Phi_t^\ast$. Assuming that the second order conditions be satisfied, the first order conditions are the following:

$$d\Phi_t^\ast/d\mu = r^\ast_{t} + \phi (\mu_t) + \mu_t \phi' (\mu_t) - i_t = 0$$
this equation (stating that in equilibrium the marginal cost of borrowing equals the marginal cost of the internally generated finance) can be simplified by assuming that \( \xi(\mu_t) = \phi(\mu_t) + \mu_t \phi'(\mu_t) \) can be rearranged into a monotonically increasing and invertible function of \( \mu_t \). One can easily verify that this would be always true if \( \phi(\mu_t) \) is convex \(^6\) in \( \mu_t \), which is what we are going to assume.

In this case we would get

\[
\mu_t = \xi^{-1}(i_t - r^f_t)
\]

This means, in other words, that the gearing ratio is an increasing function of the difference between the cost of own capital \( i_t \) and the interest rate on risk-free assets \( r^f_t \), since, for a given \( r^f_t \), the higher the cost of own capital, the higher the incentive for the firm to borrow by increasing the gearing ratio.

By looking at the constraints 2 and 4, one immediately sees that they both are dynamic equations putting into relation two flow variables \( I_t \) and \( \Delta B_t \) with the state variable \( k \) at two different moments in time, \((t - 1)\) and \( t \). In particular, while \( I_t \) relates the state variables \( k_{t-1} \) and \( k_t \) for a given rate of discount \( \Phi^*_t, \Delta B_t \) does the same job and in addition determines (together with \( k_t \) ) the optimal rate of discount. In other words, differently from the conventional neoclassical intertemporal investment models, it is not \( I_t \) but \( \Delta B_t \) that acts as a control variable in this context.

Since we know from 4 that \( \pi_t(k_{t-1}) - I_t = \Phi^*_{t-1} k_{t-1} - \Delta B_t \), then we may express 1 in terms of the control variable \( \Delta B_t \) and the state variable \( k_{t-1} \), while by putting together the two constraints 4 and 2 we can eliminate \( I_t \) and express the intertemporal constraints too in terms of \( \Delta B_t \): Therefore the firm’s problem can be redefined as follows:

\[
V_t = (\Phi^*_t \cdot k_{t-1} - \Delta B_t) + \sum_{t=1}^{\infty} \left\{ \frac{(\Phi^*_t \cdot k_t - \Delta B_{t+1})}{(1 + \Phi^*_t)^t} \right\}
\]

s.t.

\[
k_t = (1 - \delta) k_{t-1} + \pi_t(k_{t-1}) - \Phi^*_{t-1} k_{t-1} + \Delta B_t
\]

A brief look at the optimization problem described by 13 and its constraints allows to make a few comments.

On the basis of the assumptions we made about the co-ordination between financial and investments decisions and their timing, the stock of capital at time \( T \) can be defined as follows:

\(^6\)This would be true also if \( \phi(\mu_t) \) is concave but with a second derivative sufficiently small in absolute value, i.e. if its curvature is "relatively flat". However the assumption of convexity for \( \phi(\mu_t) \) is quite general, since it could "capture" the situation where highly indebted firms would have to pay an extremely high risk premium on borrowed capital. Furthermore, if \( \phi(\mu_t) \) has an asymptotic behaviour and tends to infinite when \( \mu_t \) approaches 1 , one could reproduce the case of credit rationing by choosing appropriate analytical form and parameters for the function \( \phi(\mu_t) \).
First of all, if one allows for stochastic shocks in the profit function \( \pi_t \) (possibly in the variable \( \nu \), associated to the competitive environment in which the firm operates, or in the labour market variables) would be transferred to the rate of discount of the future profits from the next period on.

Secondly (and obviously) the share price and its variations also affect the rate of discount (through the dividend policy decided by the management and the consequent cost of own capital).

Finally the very simple model formalisation proposed here may have relevant implications for the empirical phenomenon of countercyclical mark ups. If as suggested by the game-theoretic approach initially proposed by Rotemberg and Saloner (1986), the mark ups are affected by changes (over the business cycle) in the incentives to break the collusion, one of the obvious determinant of the expected discounted ratio of future profits is the "firm-specific" discount factor, represented here by \( \Phi_* \).

In other words, as we can see from 6, 8, 12 and 5, if we assume that \( \pi_t \) follows a cyclical trend and \( \delta \) is sufficiently small, even if the share price were strictly correlated to the firm's profits, the rate of discount would be slow and sluggish in adjusting to the trend of the firm's profits. This implies, in other words, that in phases of high profits and mark ups the firm might still have a high (and anti-collusive) discount rate determined by the previous negative phases, and, viceversa, at the beginning of the slump, the discount rate might still be low, under the influence of the cheap own capital accumulated in the phases of expansion.

Furthermore, as we can see again from 6, 8, 12 and 5, a change in the firm's discount rate (and therefore in the incentives to keep or break the collusion) could be caused by a purely random shock in the share price and in its variation (possibly a sufficiently persistent speculative bubble). In other words, a random financial shock modifying the optimal dividend policy of the firm's managers would also modify the cost of own capital, the optimal gearing ratio, and, as a consequence, the discount rate and the incentive of the firm to collude.

### 3 A proposed solution approach

Following TU (1991), who formalizes an extension of Pontryagin's maximum principle to the case of discrete time (and keeping in mind that we are in a case of intertemporally nonadditive preferences), we can define the discrete Hamiltonian as follows:

\[
H_t = (\Phi_{t-1}^* \cdot k_{t-1} - \Delta B_t) + \sum_{i=1}^{t+1/2} \left\{ \left( \Phi_{i}^* - k_i - \Delta B_{i+1} \right) \right\} + \\
+ \lambda_{t-1} (\Delta B_t - k_t + (1 - \delta)k_{t-1} + \pi_t(k_{t-1}) - \Phi_{t-1}^* \cdot k_{t-1})
\]
where \( \Phi_t^r = \Phi_t^r(i_t - r_t^f) \) and

\[
i_t = \frac{r_{n,t}^* \cdot p_{n,t} \cdot N_t - \Delta p_{n,t} \cdot N_t}{E_0 + \sum_{i=t}^{i-1} (\pi_i(k_{i-1}) - \Phi_i^r k_{i-1})}
\]

Assuming now that the regularity conditions for \( H_t \) are satisfied, we have:

\[
\frac{\partial H_t}{\partial(\Delta B_t)} = 0 \implies \lambda_t = 1
\]  \hspace{1cm} (14)

\[
\frac{\partial H_t}{\partial k_{t-1}} = \lambda_t
\]

which implies

\[
\frac{\partial \pi_t}{\partial k_{t-1} - \delta =}
\]

\[
= \left( \frac{\partial \Phi_t^r}{\partial R_t} \frac{\partial R_t}{\partial(\Delta R_t)} \right) \left( \frac{\partial \pi_t}{\partial k_{t-1} - \Phi_t^r - \Phi_t^r k_t - \Delta B_t}{(1 + \Phi_t^r)^2} - \right.
\]

\[
- \left( \frac{1}{1 + \Phi_t^r} \frac{\partial \Phi_t^r}{\partial R_t} \frac{\partial R_t}{\partial(\Delta R_t)} \right) \left( \frac{\partial \pi_t}{\partial k_{t-1} - \Phi_t^r - \Phi_t^r k_t - \Delta B_t}{1 + \Phi_t^r} \right) \frac{k_t}{+}
\]

\[
+ \sum_{i=t+1}^{t+1/\delta} \left\{ \frac{1}{1 + \Phi_t^r} \frac{\partial \Phi_t^r}{\partial R_t} \frac{\partial R_t}{\partial(\Delta R_t)} \left( \frac{\partial \pi_t}{\partial k_{i-1} - \Phi_t^r} \right) \left( \frac{\Phi_t^r k_t - \Delta B_t}{1 + \Phi_t^r} \right) \right\}
\]  \hspace{1cm} (15)

Where the three addends on the right-hand side of 15 could be interpreted as follows:

The addend \( \frac{\partial \Phi_t^r}{\partial R_t} \frac{\partial R_t}{\partial(\Delta R_t)} \left[ \frac{\partial \pi_t}{\partial k_{i-1} - \Phi_t^r} \right] \left( \frac{\Phi_t^r k_t - \Delta B_t}{1 + \Phi_t^r} \right) \) can be thought of as the effect of how the modifications in the discount rate generated by a change in the state variable affect the way the future values of \( (\Phi_t^r k_t - \Delta B_t) \) are discounted.

The addend \( -\frac{1}{1 + \Phi_t^r} \frac{\partial \Phi_t^r}{\partial R_t} \frac{\partial R_t}{\partial(\Delta R_t)} \left[ \frac{\partial \pi_t}{\partial k_{i-1} - \Phi_t^r} \right] \cdot k_t \) describes how again the modifications
in the discount rate modify the flow of dividends and interest rates that have to be paid on the future capital \( k_t \) (equal to the financial capital \( B_t + R_t \)).

The addend

$$
\sum_{i=t+1}^{t+1/\delta} \left\{ \left( \frac{1}{(1 + \Phi_i)} \frac{\partial \Phi_i}{\partial \Phi_i} \frac{\partial R_i}{\partial \Phi_i} \frac{\partial R_i}{\partial (\Delta R_i)} \right) \left( \frac{\partial \pi_t}{\partial k_{t-1}} - \Phi_{t-1}^* \right) \left( \frac{\Phi_t^* k_i - \Delta B_t}{1 + \Phi_t^*} - k_i \right) \right\}
$$

jointly represents the two above-mentioned effects for all the future periods in which the change in the rate of discount originated by a change in the state variable still manifest its influence. Equation 15 can be written more compactly in the following way:

$$
\frac{\partial \pi_t}{\partial k_{t-1}} - \delta = \left( \frac{\partial \pi_t}{\partial k_{t-1}} - \Phi_{t-1}^* \right) \sum_{i=t}^{t + 1/\delta} \left( \frac{1}{(1 + \Phi_i)^i} \frac{\partial \Phi_i}{\partial \Phi_i} \frac{\partial R_i}{\partial \Phi_i} \frac{\partial R_i}{\partial (\Delta R_i)} \right) \left( \frac{\Phi_t^* k_i - \Delta B_t}{1 + \Phi_t^*} - k_i \right)
$$

(16)

By choosing some specific analytical form for all the functions involved in the optimization problem, the result could be seeked recursively from period \( T \) backwards.

However a simple look at equation 16 allows to see that the situation of motionless equilibrium, where the marginal profitability of capital simply equals the rate of capital depreciation \( \delta \), requires the condition \( \frac{\partial \pi_t}{\partial k_{t-1}} - \Phi_{t-1}^* = 0 \), which also corresponds to the case of exhaustion of profits into dividends and interest rate payments, with no new investments nor increase in borrowing.

4 Conclusions and possible extensions

This paper provides an initial framework to model simultaneity between firm’s investment and financial decisions, assuming diverging incentives for managers and shareholders and financial markets imperfections that generate a risk premium on the borrowed finance.

In such a framework, timing in the process of coordination between financial and investment decisions is essential for the definition of flow variables and might not be properly captured by the conventional continuous time models. For this reason a discrete time recursive structure in the intertemporal problem
of the firm’s investments has been introduced. This determines a causal link between the flow of profits, the firm’s financial structure and the rate of discount of the future flows of profit. The resulting model implies the endogeneity of the rate of discount in the intertemporal investment problem and a certain lag in its adjustment to the trend followed by the profits of the firm. This particular feature - which might be called "staggering in the rate of discount" - can reproduce (according to our conjecture discussed here) the empirical phenomenon of countercyclical mark ups.

Further developments in this line of research can be pursued by assuming that the price share, instead of being exogenous, follows a stochastic path, which might or might not be related to the firm’s profits according to the assumptions one can make about financial markets efficiency and about the process of information spreading that takes place in financial markets.

Bibliography


Pindyck, R., (1991), 'Irreversibility, Uncertainty and Investment', Journal of Economic Literature, 29, pp. 1110-1148


112. Massimo Baldini [1995] "Aggregation Factors and Aggregation Bias in Consumer Demand", pp. 31
113. Costanza Torricelli [1995] "The information in the term structure of interest rates: Can stochastic models help in resolving the puzzle?", pp. 25
118. Mario Forni e Marco Lippi [1995] "Permanent income, heterogeneity and the error correction mechanism.", pp. 21
120. Mario Forni e Luca Rezelchin [1995] "Dynamic common factors in large cross-section", pp. 17
124. Barbara Pistoressi e Marcello D’Amato [1995] "La riservazione del banca centrale è un bene o un male? Effetti dell’informazione incompleta sul tasso di interesse in un modello di politica monetaria.", pp. 32
128. Carlo Alberto Magni [1996] "Repeatable and a tantum real options a dynamic programming approach.", pp. 23
130. Carlo Alberto Magni [1996] "Vaghezza e logica fuy in nella valutazione di un’opzione reale.", pp. 20
133. Carlo Alberto Magni [1996] "Un esempio di investimento industriale con interazione competitiva e avversione al rischio.", pp. 20
134. Margherita Russo, Peter Börkey, Emilio Cubel, François Lévéque, Francesco Mas [1996] "Local sustainability and competitiveness: the case of the ceramic tile industry.", pp. 66
136. David Avra Lane, Irene Poli, Michele Lalla, Alberto Rovereto [1996] "Lezioni di probabilità e inferenza statistica.", pp. 288
137. David Avra Lane, Irene Poli, Michele Lalla, Alberto Rovereto [1996] "Lezioni di probabilità e inferenza statistica - Esercizi svolti.", pp. 302
139. Luisa Malaguti e Costanza Torricelli [1995] "Monetary policy and the term structure of interest rates.", pp. 30
140. Mauro Dell’Amico, Martine Labbé, Francesco Maffioli [1996] "Exact solution of the SONET Ring Loading Problem.", pp. 20
141. Mauro Dell’Amico, R. M. Vaessen [1996] "Flow and open shop scheduling on two machines with transportation times and machine independent processing times in NP-hard.", pp. 10
146. Paolo Bertolini [1996] "La modernizzazione de l’agricolture italiane e il caso de l’Emilie Romagne.", pp. 20
148. Maria Elena Bontempi e Roberto Colinelli [1996] "Le determinanti del leverage delle imprese: una applicazione empirica ai settori industriali dell’economia italiana.", pp. 31
149. Paolo Bertolini [1996] "L’agricolture et la politique agricole italienne face aux recent scenarios.", pp. 20
151. Enrico Giovannetti [1996] "Il 1° ciclo del Diploma Universitario Economia e Amministrazione delle Imprese.", pp. 25
155. Paolo Silvestri, Giuseppe Catalano, Cristina Beltracchi [1996] "Le tasse universitarie e gli interventi per il diritto allo studio: la prima fase di applicazione di una nuova normativa.", pp. 159
158. Carlo Alberto Magni [1996] "Un semplice modello di opzione di differimento e di vendita in ambito discreto.", pp. 10
162. David Lane [1996] "Is what is good for each best for all? Learning from others in the information coevolution model.", pp. 18
167. Marcello D'Amato e Barbara Pistoresi [1996] “So many Italians: Statistical Evidence on Regional Economic Concentration” pp. 31
173. Mauro Dell'Amico [1997] “A Linear Time Algorithm for Scheduling Outforests with Communication Delays on Two or Three Processors” pp. 18
175. Paolo Bosi e Massimo Matteuzzi [1997] “Nuovi strumenti per l'assunzione sociale” pp. 31
176. Mauro Dell'Amico, Francesco Maffioli e Marco Trubian [1997] “New bounds for optimum traffic assignment in satellite communication” pp. 21
177. Carlo Alberto Magni [1997] “Paradosso, inverosimiglianze e contraddizioni del Van; operazionieree” pp. 9
185. Gias Paolo Caselli e Maurizio Battini [1997] “Following the tracks of atkinson and micklewright the changing distribution of income and earnings in poland from 1989 to 1995” pp. 21
186. Mauro Dell'Amico e Francesco Maffioli [1997] “Combining Linear and Non-Linear Objectives in Spanning Tree Problems” pp. 21
203. Stefano Bordoni [1997] “Supporti Informatici per la Ricerca delle soluzioni di Problemi Decisionali” pp. 30
212. Alberto Roverato [1997] “Asymptotic prior to posterior analysis for graphical gaussian models” pp. 8
Distretto ceramico, Engraved Agents/Artifacts Space in Tile Decoration: from Silk Dynamics in a Local Production System.

Insegnanti e Nuove "Comportamenti dynamics in Italy in the early nineties"

Consolato Pellegrino Statistica relativa alla metodologiche"

Statistica Gisella Facchinetti

Gisella Facchinetti, Giovanni Mastroleo e Sergio Paba "A Fuzzy Approach to the Empirical Identification of Industrial Districts"

Tommaso Minerva "Poli e Sebastiano Brusco "A Cellular Automaton as a Model to Study the Dynamics of an Industrial District"

Gisella Facchinetti "Il problema della misurazione del rischio di credito: una rassegna critica di metodologie"