Zelig and the Art of Measuring Residual Income

by

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ABSTRACT.

This paper tells the story of a student of economics and finance who meets a couple of alleged psychopaths, suffering from the 'syndrome of Zelig', so that they think of themselves to be concerned with economic and financial issues. While speaking, they come across the concept of residual income. The student explains them Stewart's (1991) EVA model and Peccati's (1987, 1991, 1992) decomposition model of a project's Net Present (Final) Value, showing that they are equivalent. The 'Zeligs' listen to him carefully, then try to apply the EVA model: Unfortunately, both She-Zelig and He-Zelig seem to feel uneasy with mathematics, so they make some mistakes. Consequently, each of them mis-calculates the residual income. Though making different mistakes, both reach, one way or another, the NFV of the project and their residual income coincides. Also, the model presented by the Zeligs, though alternative to the EVA model, seems to bear strong relations to the latter. The student is rather surprised.

I give my version of this event, arguing that the Zeligs are offering us a rational way of measuring excess profit, alternative to EVA but equally valuable. As I see it, they are only adopting a different cognitive interpretation of the concept of residual income.
Introduction

Residual income is an economic concept used both in theoretical economics and in finance and is strictly connected with the concept of Net Present Value (NPV) or Net Final Value (NFV): actually the problem of formally translating such a concept for evaluation purposes is equivalent to the problem of decomposing a NPV in periodic shares. In the former sense, Stewart (1991) introduces the Economic Value Added for valuing firms (or projects) and as a tool for rewarding managers; in the latter sense Peccati (1987, 1991, 1992) introduces a disaggregation model of a project's NPV (NFV) for valuing the periodic performance of the project itself.

In this paper I report the story of a student of economics and finance who meets two alleged psychopaths suffering from the 'syndrome of Zelig': 'Zeligs' are ill-minded people, inclined to take on the personality of the person they are dealing with. Although they are nearly always unacquainted with the matter they are disputing on, they have so intense an enthusiasm toward the matter they are coping with and so absolute a lack of cultural and social inhibitions, that their raving ideas often give new insights on the subject. Unfortunately, their way of communicating their ideas is not always clear. Not enough, they enjoy making fun of people so they conceal their ideas in seemingly irrational arguments.

Meeting the student, the two Zeligs of this story gradually take on his personality until they think of themselves to be students concerned with economic and financial issues. Coming across the concept of residual income and the related problem of evaluating the periodic performance of a project, the student presents the above mentioned models by Peccati and Stewart. After having listened carefully to him, the Zeligs try to repeat the EVA procedure. Due to their mental illness, they make some mistakes. I consider it rather interesting to report their mistakes, for they seem to offer us a new way of thinking of the residual income.

The paper is organized as follows. In section 1. the student explains Peccati's and Stewart's models, showing that they both formally translate the concept of residual income and are equivalent. In section 2. the two Zeligs try to repeat the procedure they have just learned but make irreparable mistakes. Their incorrect procedure is applied to a concrete case in section 3. In section 4. I give my version of the alleged psychopaths' lucubrations showing that they are offering us a rational tool for measuring residual income: Only, they are adopting a different cognitive perspective and shaping the evaluation process so that their procedure looks like an EVA model full of errors. I conclude the paper with some remarks.

2The terms "excess profit" and "supernormal profit" are more frequently used in economics (cfr. Begg, Fischer, and Dornbusch, 1984, p.121, for the second one).
1. Peccati’s and Stewart’s models

In this section I summarize the presentation of Peccati’s and Stewart’s models made by the student.

Suppose a decision maker aims at evaluating a project $P$ with equidistant cash flows $a_s \in \mathbb{R}$ at time $s$, $s=0,1,\ldots,n$. All flows are certain. The investor aims at evaluating the periodic performance of the project. The NFV (NPV) rule assumes that cash flows released by a project $P$ are withdrawn from (if negative) and invested in (if positive) an account, say $C$, whose value $C_s$ at time $s$ evolves according to the following recurrence equation:

$$C_s = C_{s-1}(1+i) + a_s \quad s = 1,\ldots,n$$

where $i$ is the so-called opportunity cost of capital. Following Peccati (1991) let us assume $C_0 = E_0$, where $E_0$ denotes the value of the evaluator’s net worth at time 0, $E_0 \in \mathbb{R}$. The decision maker faces two alternative courses of action: (i) to invest in project $P$, (ii) to keep her wealth in account $C$. Let us denote with $E_s'$ and $E_s''$, $s \geq 1$, the net worth at time $s$ for case (i) and (ii) respectively. We define the Net Final Value of $P$ as the difference

$$\text{NFV} = E_n - E^n,$$

which implies

$$\text{NFV} = (E_0 + a_0)(1+i)^n + \sum_{s=1}^{n} a_s (1+i)^{n-s} - E_0(1+i)^n = \sum_{s=0}^{n} a_s (1+i)^{n-s}. \quad (1.1)$$

The NPV is obtained by discounting (1.1) at present time. We aim then at decomposing (1.1) in $n$ shares $G_s$ so that $G_1 + \ldots + G_n = \text{NFV}$. The outstanding capital (or project balance) of $P$ at time $s$ at the rate $j$ is given by

$$w_0 = -a_0$$

$$w_s(j) = w_{s-1}(1+j) - a_s \quad s = 1,2,\ldots,n. \quad (1.2)$$

Assume that $P$ belongs to the class of Soper (1959), and let $x$ be the internal rate of return. Following Peccati’s argument, we focus on a generic period $s$: The investor places the sum $w_{s-1}(x)$ at the beginning of the period and receives $w_s(x)+a_s$ at the end of the period. The gain is $xw_{s-1}(x)$. So doing, she gives up the opportunity of investing $w_{s-1}(x)$ at the opportunity cost of capital $i$, that is she foregoes the gain $iw_{s-1}(x)$. The latter is then the opportunity cost (i.e. the foregone return), and the sum $w_{s-1}(x)(x-i)$ is then the residual income in period $s$, that is the difference between what the investor earns by choosing $P$ and what she would earn should she decide to keep funds in $C$. As each such share is
money referred to time $s$, we must compound to time $n$ before we can sum all shares. We have then

$$G_s = w_{s-1}(x)(x - i)(1 + i)^{n-s} \quad (1.3)$$

In such a way, the model meets both the requirements of finding periodic values for project $P$ being significant from an economic point of view (they measure the differential income of period $s$) and aggregating these values so as to obtain the NFV (which is the overall excess profit).

If the project is levered (i.e. it is partly financed with debt) an analogous argument is applied so that $G_s$ becomes

$$G_s = (w_{s-1}(x)(x - i) + D_{s-1}(\delta)(i - \delta))(1 + i)^{n-s} \quad (1.4)$$

where $\delta$ is the debt rate and

$$D_0 = f_0$$

$$D_s(j) = D_{s-1}(1 + j) + f_s \quad (1.5)$$

is the outstanding debt at the rate $j$, with $f_s \leq 0$, $s \geq 0$, denoting the debt’s cash flows.

Let us now turn to Stewart’s model. The Economic Value Added is a profitability index introduced by Stewart in order to provide a tool for evaluating (projects and) firms as well as for evaluating and compensating managers. The basic objective of EVA is to create a measure of periodic performance based on the concept of residual income: “Recognized by economists since the 1770s, residual income is based on the premise that, in order for a firm to create wealth for its owners, it must earn more on its total capital invested than the cost of that capital” (Biddle et al. (1999), p.70). To compute it, we calculate the firm’s (or project’s) total cost of capital, given by the product of the Weighted Cost of Capital (WACC) and the total capital invested (TC). Then the total cost of capital is subtracted from the Net Operating Profit After Taxes (NOPAT). Notationally, we have, for period $s$.

$$EVA_s = NOPAT - WACC \times TC \quad (1.6)$$

where subscripts are omitted for convenience. Summing for $s$ and discounting at time 0 (or compounding at time $n$) at a rate $i'$ we obtain the overall residual income, which Stewart calls Market Value Added (MVA)

$$MVA = \sum_{s=1}^{n} \frac{EVA_s}{(1 + i')^s}.$$
It is easy to show that (1.6) is equivalent to (1.4). In fact, (1.6) can be rewritten as

$$EVA_s = ROA \cdot TC - \frac{(ROD \cdot Debt + i \cdot Equity)}{Debt + Equity} \cdot TC$$  \hspace{1cm} (1.7)

whence

$$EVA_s = ROA \cdot TC - ROD \cdot Debt - i \cdot (TC - Debt)
= TC \cdot (ROA - i) + Debt \cdot (i - ROD)$$  \hspace{1cm} (1.8)

where ROA is the Return on Assets, ROD is the Return on Debt. Applying Stewart’s argument to project $P$, we have $TC = w_{s-1}(x)$, $ROA = x$, $Debt = D_{s-1}(\delta)$, $ROD = \delta$. The relation between (1.4) and (1.6) is straightforward:

$$G_s = EVA_s(1 + i)^{n-s}.$$  

If $i' = i$ the overall residual income in Stewart’s model (MVA), coincides with Peccati’s representation of the NPV:

$$NPV = \frac{NFV}{(1 + i)^n} = \frac{1}{(1 + i)^n} \sum_{s=1}^{n} G_s = \sum_{s=1}^{n} EVA_s(1 + i)^{-s} = MVA.$$  

The equivalence of the models vanishes only in discounting each $EVA_s$: Stewart uses the Weighted Average Cost of Capital ($i' = WACC$), whereas Peccati picks $i' = i$.  

2. The Zeligs’ mistakes

As we have seen, Peccati and Stewart (henceforth PS) offer equivalent translations of the notion of residual income, that is they offer different sides of the same medal. The Zeligs illustrate the procedure they have just learned. They firstly begin with the case $D_s = 0$ for all $s$, then generalize their arguments to include debt.

2.1 She-Zelig

Following PS’s model, the residual income of an unlevered project is given by

$$EVA_s = w_{s-1}(x)(x - i) = xw_{s-1}(x) - iw_{s-1}(x)$$  \hspace{1cm} (2.1.1)

where $iw_{s-1}(x)$ represents the foregone return, the well-known opportunity cost. But She-Zelig is so unacquainted with finance and mathematics that she misunderstands the concept.
of opportunity cost. She is convinced that the foregone return is $i w_{-1}(i)$, that is she replaces $x$ with $i$. She therefore computes project $P$’s differential income, unaware of such a mistake. Denoting with $EVA^S_\ast$ her vitiated Economic Value Added,\footnote{S=She-Zelig, $H$=He-Zelig.} she calculates

$$EVA^S_\ast = x w_{-1}(x) - i w^S_{-1}$$

(2.1.2)

where we let $w^S_{-1}:=w_{-1}(i)$. Afterwards, our evaluator assumes that the project is partly financed by a loan contract whose cash flows are $-f_\ast \in \mathbb{R}$ with $\delta$ being the contractual rate. In this case, we know that we should have

$$EVA_\ast = w_{-1}(x)(x-i)+D_{-1}(\delta)(i-\delta) = x w_{-1}(x) - \delta D_{-1}(\delta) - i(w_{-1}(x) - D_{-1}(\delta))$$

(2.1.3)

where $i(w_{-1}(x) - D_{-1}(\delta))$ is the opportunity cost. Again, She-Zelig seems to fall a prey to hallucinations and thinks the opportunity cost is $i(w_{-1}(i) - D_{-1}(i))$, that is she replaces both $x$ and $\delta$ with $i$, so that her ‘hallucinated’ $EVA$ is now

$$EVA^S_\ast = x w_{-1}(x) - \delta D_{-1}(\delta) - i(w^N_{-1} - D^S_{-1})$$

(2.1.4)

where we let $D^S_{-1}:=D_{-1}(i)$. She-Zelig is willing to verify whether such a result is consistent with the Net Final Value, as it should. Due to her liberty of conscience, typical of Zeligs, she refuses to follow basic rules of financial calculus, so she does not compound the shares so found, but sum all of them as such. Strangely enough, she finds that

$$\text{NFV} = \sum_{s=1}^{n} EVA^S_s.$$  

She is therefore satisfied and reinforced in her conviction that she is doing right.

### 2.2 He-Zelig

He-Zelig too tries to expound PS’s model. He states that he will measure the incremental income of $P$ using PS’s arguments but the student notices that He-Zelig does not correctly recognize $P$’s cash flows. Our alleged psychopath is actually convinced that $P$ consists of the sequence

$$p^H = (a^H_0, a^H_1, \ldots, a^H_n)$$

where $a^H_0 = a_0$ and

$$a^H_s = a_s + \sum_{k=0}^{s-1} a_k [\Phi_k(i) - \Phi_k(x)] \quad s \geq 1$$

(2.2.1)
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Not enough, he casts a sidelong glance at what his mate is doing with the same evaluation process. He realizes that she replaces $x$ with $i$ in expressing the opportunity cost and he finds it a commendable idea. So he takes $w_s(i)$ instead of $w_s(x)$. Letting $w^H_s := w_s(i)$ he accomplishes then (what the student thinks is) a correct argument with incorrect values:

"Let us focus on period $s$," he says. "The capital invested in the project at the beginning of the period is $w^H_{s-1}$, and at the end of the period I will receive the sum $w^H_s + a^H_s$." Denote with $x^H_s$ the internal rate of return of this uniperiodic project. This means, for all $s$,

$$w^H_{s-1}(1 + x^H_s) = w^H_s + a^H_s$$  \hfill (2.2.2)

or, which is the same,

$$w^H_s = w^H_{s-1}(1 + x^H_s) - a^H_s$$  \hfill (2.2.3)

so $w^H_s$ is the project balance at time $s$. "In period $s$," he argues. "I invest money at the rate $x^H_s$ and so doing I forego the return $iw^H_{s-1}$." Then the residual income he finds, i.e. his 'hallucinated' Economic Value Added, is

$$\text{EVA}^H_s = x^H_s w^H_{s-1} - iw^H_{s-1} = w^H_{s-1}(x^H_s - i).$$  \hfill (2.2.4)

Afterwards, he assumes that the project is partly financed by a loan contract whose cash flows are $f_s \in \mathbb{R}$ and with contractual rate $\delta$. Again, he falls a prey of his illness and does not recognize the debt's cash flows. He is convinced that the cash flows are $f^H_0 = f_0$ and

$$f^H_s = f_s + \sum_{k=0}^{s-1} f_k [\Phi_k(i) - \Phi_k(\delta)] \quad s \geq 1.$$  \hfill (2.2.5)

As before, he glances his mate's evaluation and considers it a good idea to replace $D_{s-1}(\delta)$ by $D_{s-1}(i)$. Letting $D^H_{s-1} := D_{s-1}$ the period rate of cost for the loan contract is then that rate $\delta^H_s$ such that

$$D^H_{s-1}(1 + \delta^H_s) = D^H_s - f^H_s$$  \hfill (2.2.6)

so that

$$D^H_s = D^H_{s-1}(1 + \delta^H_s) + f^H_s$$  \hfill (2.2.7)

where we let $D^H_s := D_s(i)$. Then he argues as before while taking into account that, at time $s-1$, he invests $w^H_{s-1} - D^H_{s-1}$ while receiving, at time $s$, the sum $w^H_s - D^H_s + a^H_s + f^H_s$.

"With such a levered project," he thinks. "I forego the return $i(w^H_{s-1} - D^H_{s-1})$." Hence, the residual income is now

$$\text{EVA}^H_s = x^H_s w^H_{s-1} - \delta^H_s D^H_{s-1} - i(w^H_{s-1} - D^H_{s-1}) = w^H_{s-1}(x^H_s - i) + D^H_{s-1}(i - \delta^H_s).$$  \hfill (2.2.8)
"This is a raving lunatic," the student thinks. He-Zelig is then willing to verify that such a residual income is consistent with the Net Final Value of the project. As his mate, he does not trust basic rules of financial calculus and he sums all residual incomes with no compounding. Strangely enough, he finds that

$$\sum_{s=1}^{n} \text{EVA}_s^H = \text{NFV}. $$

"Well done" he says "There is no mistake in my evaluation", so he is reinforced in his conviction that he is doing right.

3. Rational and irrational EVA’s

Our Zeligs decide to show the student the application of their own procedures to a levered project consisting of the sequence $(-100, 30, 38, 84)$ along with the debt’s cash flows $(40, -26, -3, -23)$ at time 0, 1, 2, 3 respectively. The internal rate of return is $\mu=0.2$ and the debt’s rate is $\delta=0.15$. The net cash flows are then $(-60, 4, 35, 61)$. Assuming $i=0.1$ the NFV of the levered project is

$$\text{NFV} = -60(1.1)^3 + 4(1.1)^2 + 35(1.1) + 61 = 24.48;$$

let us then follow the Zeligs in their calculations. The data collected by She-Zelig are:

$$w_0(x) = 100 \quad D_0(\delta) = 40$$
$$w_1(x) = 100(1.2) - 30 = 90 \quad D_1(\delta) = 40(1.15) - 26 = 20$$
$$w_2(x) = 90(1.2) - 38 = 70 \quad D_2(\delta) = 20(1.15) - 3 = 20$$
$$w_3(x) = 70(1.2) - 84 = 0 \quad D_3(\delta) = 20(1.15) - 23 = 0$$

$$w_0^S = 100 \quad D_0^S = 40$$
$$w_1^S = 100(1.1) - 30 = 80 \quad D_1^S = 40(1.1) - 26 = 18$$
$$w_2^S = 80(1.1) - 38 = 50 \quad D_2^S = 18(1.1) - 3 = 16.8$$
$$w_3^S = 50(1.1) - 84 = -29 \quad D_3^S = 16.8(1.1) - 23 = -4.52.$$

In the meanwhile, the diligent student computes the residual income by applying (2.1.3):

$$\text{EVA}_1 = 0.2 \times 100 - 0.15 \times 40 - 0.1(100 - 40) = 8$$
$$\text{EVA}_2 = 0.2 \times 90 - 0.15 \times 20 - 0.1(90 - 20) = 8$$
$$\text{EVA}_3 = 0.2 \times 70 - 0.15 \times 20 - 0.1(70 - 20) = 6$$
and finds back the NFV by just compounding each EVA, and then summing for s:

$$EVA_1(1.1)^2 + EVA_2(1.1) + EVA_3 = 8(1.1)^2 + 8(1.1) + 6 = 24.48 = NFV.$$ 

Conversely, She-Zelig applies her (2.1.4), so that

$$EVA_1^S = 0.2 \times 100 - 0.15 \times 40 - 0.1(100 - 40) = 8$$
$$EVA_2^S = 0.2 \times 90 - 0.15 \times 20 - 0.1(80 - 18) = 8.8$$
$$EVA_3^S = 0.2 \times 70 - 0.15 \times 20 - 0.1(50 - 16.8) = 7.68.$$

She is even more convinced of such a result when she sums the three shares with no compounding process:

$$EVA_1^S + EVA_2^S + EVA_3^S = 8 + 8.8 + 7.68 = 24.48 = NFV.$$

As for He-Zelig, he uses the following values:

$$w_0^H = 100 \quad (= w_0^S) \quad D_0^H = 40 \quad (= D_0^S)$$
$$w_1^H = 80 \quad (= w_1^S) \quad D_1^H = 18 \quad (= D_1^S)$$
$$w_2^H = 50 \quad (= w_2^S) \quad D_2^H = 16.8 \quad (= D_2^S)$$
$$w_3^H = -29 \quad (= w_3^S) \quad D_3^H = -4.52 \quad (= D_3^S).$$

He now needs find, for all s, the values of $x_s^H$ and $\delta_s^H$. To this end, he uses (2.2.3) and (2.2.6) so that

$$x_s^H = \frac{w_s^H + a_s^H}{w_{s-1}^H} - 1 \quad (3.1)$$
$$\delta_s^H = \frac{D_s^H - f_s^H}{D_{s-1}^H} - 1 \quad (3.2)$$

Using (2.2.1) and (2.2.5), (3.1) and (3.2) boil down to

$$x_s^H = \frac{w_s^H(1 + i) - a_s + a_s + \sum_{k=0}^{s-1} a_k[\Phi_k(i) - \Phi_k(x)]}{w_{s-1}^H} - 1$$
$$= i + \sum_{k=0}^{s-1} a_k[\Phi_k(i) - \Phi_k(x)] \quad (3.3)$$

and

$$\delta_s^H = \frac{D_s^H(1 + i) + f_s - f_s + \sum_{k=0}^{s-1} f_k[\Phi_k(i) - \Phi_k(x)]}{D_{s-1}^H} - 1$$
$$= i + \sum_{k=0}^{s-1} f_k[\Phi_k(i) - \Phi_k(x)] \quad (3.4).$$
respectively. Finally, using (3.3) and (3.4) he gets to

\[
\begin{align*}
\delta_1^H &= 0.1 + 0.025 = 0.15 \\
\delta_2^H &= 0.1 + 0.0125 = 0.16666 \\
\delta_3^H &= 0.1 + 0.0132 = 0.1785714.
\end{align*}
\]

He is now ready to apply (2.2.8):

\[
\begin{align*}
\text{EVA}_1^H &= 100 (0.2 - 0.1) + 40 (0.1 - 0.15) = 8 \\
\text{EVA}_2^H &= 80 (0.225 - 0.1) + 18 (0.1 - 0.1667) = 8.8 \\
\text{EVA}_3^H &= 50 (0.28 - 0.1) + 16.8 (0.1 - 0.1785714) = 7.68,
\end{align*}
\]

then sums the three shares with no compounding:

\[
\text{EVA}_1^S + \text{EVA}_2^S + \text{EVA}_3^S = 8 + 8.8 + 7.68 = 24.48 = \text{NFV},
\]

and rubs his hands. The student is rather surprised: “This is a very whimsical result,” he says. The Zeligs remark the student that, for all \( s \), their EVA’s coincide:

\[
\text{EVA}_s^S = \text{EVA}_s^H.
\]

They ask the student: “What about this?” and he answers: “You have applied the EVA model in an absurd way. Your residual incomes differ from the correct ones. Yet, I do not understand why your excess profits coincide and why you get to the NFV. All this is so irrational.” The Zeligs just reply: “No, this is just what you should have expected. There is no objective residual income at all.”

4. Systemic Value Added

In this section I would like to expose my opinion on this episode. I am convinced that the Zeligs are far from being irrational evaluators; they are only adopting a different interpretation of the notion of residual income. I will unmask the way of reasoning the two have followed and describe the perspective from which their seemingly irrational EVA’s have been drawn. We will be left with a sound index, which I shall name Systemic Value Added (SVA). I will show that the SVA is a rational tool for measuring residual income, with a significant economic meaning and capable of acting as an alternative to the EVA. As we shall see, the two Zeligs’ irrational EVA’s are just two ways of computing the SVA. Let us now begin:

The evaluation process starts at time 0, when two lines of action are compared:
(i) undertaking the project  
(ii) investing funds at the rate \(i\).

If we regard the investor's wealth as a dynamic system, we have that (i) and (ii) give rise to different paths of the system. As for (i) at time \(s\) the net worth \(E_s\) can be seen as structured in three accounts: The account \(C_s\), whose value is denoted by \(C_{s}\), the project, whose value is the outstanding capital \(w_s(x)\), and the loan contract, whose outstanding debt is \(D_s(\delta)\); if (ii) is instead chosen, the decision maker’s wealth \(E^*\) at time \(s\) will be composed of the only account \(C^*\), whose value I denote with \(C^*_s\), given by \(C^0\) plus the interest yielded at the rate \(i\). We have then that the following recurrence equations hold:

\[
C_0 = E_0 + w_0 - D_0
\]
\[
w_0 = -a_0
\]
\[
D_0 = f_0
\]
\[
C_s = C_{s-1}(1 + i) + a_s + f_s \quad s \geq 1
\]
\[
w_s(x) = w_{s-1}(x)(1 + x) - a_s \quad s \geq 1
\]
\[
D_s(\delta) = D_{s-1}(\delta)(1 + \delta) + f_s \quad s \geq 1
\]

for (i) and

\[
C^0 = E_0
\]
\[
C^*_s = C^{s-1}(1 + i) \quad s \geq 1
\]

for (ii). Graphically, we can conveniently depict the situations by means of double-entry sheets where uses and sources of funds are pointed out: At time \(s\) we have

\[
\begin{array}{l|l}
\text{Uses} & \text{Sources} \\
C_s & D_s(\delta) \\
w_s(x) & E_s
\end{array}
\]

for (i),

\[
\begin{array}{l|l}
\text{Uses} & \text{Sources} \\
C^* & E^*
\end{array}
\]

(4.3)  
(4.4)
for (ii). We have \( E_s + D_s(\delta) = C_s + w_s(x) \) and \( E^* = C^* \) for all \( s \). Using (4.1) and (4.2) the two alternative dynamic systems are then expressed by the following recurrence equations:

\[
E_s = E_{s-1} + (i w_{s-1}(x) - \delta D_{s-1}(\delta) + i C_{s-1}) \\
E^* = E^{*-1} + i C^{*-1}
\]

(4.5)

for cases (i) and (ii) respectively. The second addends of both equalities in (4.5) is the income in period \( s \).\(^5\) This means that for period \( s \) we have two alternative incomes, depending on the choice made. The residual income is the incremental profit of (i) over (ii), that is the difference between the two alternative profits drawn from the alternative dynamic systems. In other words, \( E_s - E_{s-1} \) is the periodic gain associated with case (i), \( E^* - E^{*-1} \) is the gain associated with case (ii); then the residual income is the differential gain of (i) compared with (ii). I define \textit{periodic Systemic Value Added} (SVA\(_s\)) such a differential gain:

\[
\text{SVA}_s = [(E_s - E_{s-1}) - (E^* - E^{*-1})].
\]

(4.6)

Summing for \( s \) we obtain what I shall name the \textit{overall Systemic Value Added} (SVA), which is just the NFV of the project seen with our 'systemic eyes':

\[
\text{SVA} = \sum_{s=1}^{n} [(E_s - E_{s-1}) - (E^* - E^{*-1})] = E_n - E^0 = \text{NFV}.
\]

(4.7)

So, the decision maker will accept the project if and only if

\[
\text{SVA} = \text{NFV} = \text{NPV}(1 + i)^n > 0.
\]

PS's model and the SVA model give rise to different partition. From (4.5) we have

\[
\text{SVA}_s = x w_{s-1}(x) - \delta D_{s-1}(\delta) - i (C_{s-1} - C^*)
\]

(4.8)

and

\[
\text{EVA}_s \neq x w_{s-1} - i (C_{s-1} - C^*)
\]

as well as

\[
G_s \neq x w_{s-1} - i (C_{s-1} - C^*).
\]

The SVA model is grounded on a \textit{systemic way of reasoning}: the net worth is a system structured in accounts whose value evolves diachronically following different laws. The algebraic sum of the accounts constitutes the value of the whole net worth. This enables

\(^{5}\)I will also use the terms profit, gain, return, interest as synonyms of income.
us to avoid compounding, whereas PS's model rests on the concept of Net Final (Present) Values and on capitalization processes. In a sense, by using a systemic perspective we are able to sum cash regardless of its maturity. This result, far from being illicit, suggests that we can create a cognitive outlook where there is no need of capitalization factors: time dimension is grasped by means of the system's diachronic evolution.

5. Zeligs' EVA and SVA

In the light of what we have seen, the Zeligs are rational agents following a systemic outlook for measuring excess-profit. To show it, consider She-Zelig's EVA in (2.1.4), where \((w_{s-1} - D_{s-1})\) is replaced by \((w_{s-1}^S - D_{s-1}^S)\). Just note that

\[
    w_{s-1}^S = - \sum_{k=0}^{n-1} a_k (1 + i)^{s-1-k},
\]

\[
    D_{s-1}^S = \sum_{k=0}^{n-1} f_k (1 + i)^{s-1-k}
\]

whence, using (4.1) and (4.2),

\[
    C^S - C_s = w_{s-1}^S - D_{s-1}^S.
\]

Therefore, we have, from (4.8),

\[
    \text{SVA}_s = \text{EVA}_s^S.
\]

As for He-Zelig, he is an arrant rascal, for he just disguises the SVA as an Economic Value Added. Using (5.1) and (3.3), we have\(^6\)

\[
    x^H_s = i + \sum_{k=0}^{n-1} w_{s-1}^H [\Phi_k(i) - x \Phi_k(x)]
\]

\[
    = i + x w_{s-1}^H (x) - i w_{s-1}^H
\]

\[
    = x w_{s-1}^H (x) w_{s-1}^H
\]

Analogously, using (5.2) and (3.4) we easily get to

\[
    \delta^H_{s-1} = \delta \frac{D_{s-1}^H(\delta)}{D_{s-1}^S}.
\]

\(^6\)Remember that \(w_{s-1}^S = w_{s-1}^H\) and \(D_{s-1}^S = D_{s-1}^H\).
Hence, simple substitutions provide us with

$$EVA^S_e = EVA^S$$

whence

$$SVA_e = EVA^H_e.$$

The Zeligs are then systemically-minded agents: They translate the concept of residual income by adopting a systemic perspective. Such a perspective is not, in my opinion, less significant than the one sussumed by the EVA model, it is just different. But they do not only offer us a new way of treating the concept of excess profit; showing us their seemingly irrational EVA’s they are trying to communicate us some intriguing relations between the SVA model and the EVA model. To investigate the issue thoroughly, I have interviewed the two Zeligs some time ago. I herewith report a summary of what they told me that day.

She-Zelig: “I aim at measuring residual income for project $P$, that is I am searching a formal way of decomposing its NFV. Speaking in differential terms, at time $s$ my net return from the project is $xw_{s-1}(x) - \delta D_{s-1}$. What is the foregone return (the so-called opportunity cost)? Well, if I select opposite case (ii), my account $C$’s value at time $s$ will be reduced by the sum $C^{*s-1} - C_{s-1}$ ($=w_{s-1}^S - D_{s-1}^S$) with respect to the alternative case. Such value is the sum to which I renounce in order to receive the just mentioned return from the project. On this sum I would earn interest at the rate $i$. Hence, the foregone return in period $s$ is $i(C^{*s-1} - C_{s-1})$ ($=i(w_{s-1}^S - D_{s-1}^S)$). What am I then doing? I am just calculating a modified Economic Value Added where the foregone return is not the project’s outstanding capital times the rate $i$ but the latter times the differential value of account $C$.”

He-Zelig: “I aim at measuring residual income for project $P$, that is I am searching a way of decomposing its NFV. The net profit is $xw_{s-1}(x) - \delta D_{s-1}$ and, like my mate, I regard as foregone return the differential gain $i(C^{*s-1} - C_{s-1})$ ($=(i w_{s-1}^H - D_{s-1}^H)$). Since $xw_{s-1} - \delta D_{s-1} = x_{s-1}^H w_{s-1}^H - \delta_{s-1}^H D_{s-1}^H$, the systemic perspective I rely on leads me to an Economic Value Added of a ‘shadow’ project whose net cash flows are $a_{s-1}^H + f_{s-1}^H$. In other words, in period $s$ I can invest capital $w_{s-1}^H$ at the rate $x_{s-1}^H$ financing such an investment by borrowing $D_{s-1}^H$ at the rate $\delta_{s-1}^H$. Obviously, in this case that part of my net worth invested in the project is only $w_{s-1}^H - D_{s-1}^H$. The latter sum can be alternatively be invested in asset $C$, whose rate of return is $i$. What am I doing then? I am just calculating the EVA of a ‘shadow’ project.”

On the basis of these words, we can say that She-Zelig’s EVA is just a modified EVA of the original project and He-Zelig’s EVA is, so to say, the original EVA of a modified (shadow) project.
6. Remarks and Conclusions

Our alleged psychopaths have been shown to be perfectly rational agents. In my opinion, their provocation must be taken seriously. I find their model rather interesting in more than one sense. They offer a new way of looking at the concept of excess profit. As far as I know, the literature provides us with Stewart's and Peccati's models which represent the two sides of the same medal. It is taken for granted that residual income is an unambiguous concept and the way of reasoning of Stewart and Peccati seem to be the natural one for translating such a concept in a formal way. But there is no natural way of mathematically representing it, for what is exactly excess profit? It formally translates, via a mathematical subtraction, the comparison between two courses of action. But which one is the correct pair of alternatives? Stewart's and Peccati's or the SVA's? In my opinion, it is a conventional matter. It depends on which information the evaluator is willing to draw from the concept of excess profit. That is, it depends on the cognitive perspective the evaluator adopts or, in other terms, the cognitive outlook with which he/she frames the evaluation process. The issue at hand is a subtle one and I shall not dwell on it (after all, I am not a cognitive psychologist). I only stress that, in my opinion, the hub lies in the fact that the concept of residual income derives from a counterfactual conditional of the kind:7

'If it were not A, then it would be B'

or

'If it had not been A, then it would have been B.'

The question is: what is A and what is B in our case? A and B are evidently two mutually exclusive situations, but it is not obvious how to interpret A and B. Think of PS's model (assuming, for convenience, $D_s = 0$ for all $s$). PS-minded evaluators say:

“We can invest $w_{s-1}$ either at the rate $x$ or at the rate $i$.”

Now think of the systemic approach. Systemic-minded evaluators say:

“We can either invest $w_{s-1}$ at the rate $x$ or $w^{S}_{s-1}$ at the rate $i$.”

In terms of the counterfactual conditional above mentioned we have:

\[ A = \text{"investing } w_{s-1} \text{ at the rate } x" \quad B = \text{"investing } w_{s-1} \text{ at the rate } i" \] (6.1)

for PS-minded evaluators, and

\[ A = \text{"investing } w_{s-1} \text{ at the rate } x" \quad B = \text{"investing } w^{S}_{s-1} \text{ at the rate } i" \] (6.2)

for systemic-minded evaluators.8 It is my opinion that there is no unique economically significant interpretation so as to decide which one is the correct one, for A and B are

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7 Writing about opportunity cost (which is the basis of the concept of excess profit) Buchanan (1969) uses the terms “might be” (p. 46) and “might have been” (p. vii).

8 (6.2) shows She-Zelig's version of A and B.
intrinsically ambiguous. The excess profit is a residual income but it is unclear what 'residual' mean in terms of alternatives of action. PS's interpretation is, so to say, project-oriented, since it focuses on the outstanding capital of the project assuming for it alternative rates of return, disregarding the investor's wealth; conversely, the systemic interpretation is wealth-oriented, for it describes two different stories of the investor's wealth and recognizes that the foregone return depends on the differential value in account C: In fact, the two alternative net worths at the beginning of period s can be written as

\[ E_n = C_{s-1} + w_{s-1} \]
\[ E^s = C_{s-1} + (C^{s-1} - C_{s-1}). \]

Now, the term \( C_{s-1} \) is shared by both alternatives, so the differential terms are represented by the second addends. In the first case \( w_{s-1} \) yields profit at a rate \( x \); in the second case \( (C^{s-1} - C_{s-1}) \) yields profit at a rate \( i \). Hence, the SVA.

So we have now two valuable models capable of formally grasping the notion of excess profit. Are there other interpretations? It might be, maybe reframing the evaluation process in another different way. However, in my conventionalist view, there is no 'best model', but just different ways to translate the same concept. It is just a matter of convention to choose one or the other, a convention regarding the way we are willing to shape the evaluation process. Actually, there is no way of attaching objectivity to either model.

The project-oriented evaluator focuses on the two alternative rates, the wealth-oriented evaluator focuses not only on the alternative rates but also on the alternative values of the net worth: That is, he/she considers that the situation of his/her wealth (in particular, of account C) will depend on the choice made at time 0. To select the model to be used we have to choose it on the basis of our subjective interpretation of the notion of excess profit.

The fact that only one interpretation has been adopted in the literature is just due, in my opinion, to the fact that no other interpretation has been searched for. Poincaré (1902) writes that the reason why a convention is adopted relies on its simplicity and easiness of applicability. In this way, Poincaré implies that a convention is deliberately (even when implicitly) adopted by scientists. Conversely, it is my convincement that a convention is sometimes adopted unconsciously: This is what happens when it is widely accepted that there is one only (objective) way of taking account of some events or concepts and this is what in my opinion has happened with the concept of residual income in both theoretical economics and finance.9

Future researches can be addressed to investigate more thoroughly these two interpretations: From a financial point of view, some whimsicalities seem to arise in the description of the investor's financial system if PS's approach is adopted (see Magni, 2000). From a

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9I agree with Duhem (1914) who is perfectly aware of this aspect in scientific research: More than one passage of his La théorie physique is devoted to show that in science the same 'practical fact' may be described by more than one 'theorical fact', i.e. more than one symbolic proposition.
theoretical point of view it could be worth re-analysing the concept of residual income focusing on its ambiguities as well as investigating the interrelations between such a concept and the Net Final (Present) Value, which I have here just mentioned while showing the equivalence between Peccati's model and Stewart's. Further, other interpretations of the notion of residual income could be searched for alongside possible relations with the existing ones.

Finally, another aspect is worth taking account for, in my opinion. He-Zelig has shown us an interesting way of measuring residual income. He introduces the concept of ‘shadow project’ to which he applies the EVA procedure obtaining the \( SVA \), of the original project. He-Zelig is not only willing to making fun of the student, he is actually informing us that the SVA model can be seen as an EVA model. In terms of the counterfactual conditional above mentioned he shapes A and B so that they coincide with those in (6.2) but disguise them in such a way that they seem to mirror (6.1), for the capital invested is the same, only the rates differ:

\[
A = "investing \ w_{x-1}^H \ at \ the \ rate \ x^H" \quad B = "investing \ w_{i}^H \ at \ the \ rate \ i". \quad (6.3)
\]

In other words, a ‘wealth-oriented’ residual income can be seen as a ‘project-oriented’ residual income if we shift from the project to its shadow project. We are therefore tempted to ask whether such a connection may be iterated backward, that is: May the original project be seen as the ‘shadow project’ of some other project? The answer is positive and the very EVA, of the original project is the SVA, of that project of which the original one is the shadow. (see Magni, 2000). Therefore, the EVA model can be seen in turn as a systemic model. Therefore, SVA and EVA, though alternative, seem to bear strong relations to each other and these aspects deserve, in my opinion, careful attention. Other possible developments are in the sense of introducing variable rates (in the sense of Teichroew et al., 1965a, 1965b) and multiple accounts, so enriching the structure of the financial system of the decision-maker and the complexity of the evaluation process.

The final hope is that this episode will attract attention not only by economists and financial analysists but also by decision-theory scholars and cognitive psychologists as well as financial mathematicians: The issue at hand may be studied from multiple points of view and will surely benefit from interdisciplinary contributions, which could shed light on many important aspects.

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