Industrial Firms’ Market Power and Credit Market Oligopsony in Developing

by

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ABSTRACT

The theoretical model presented here describes the interactions between a concentrated industrial sector and a perfectly competitive and "bank-oriented" financial system, typical of developing countries. It is shown that an exogenous modification in the degree of concentration in the industrial sector (possibly caused by mergers) does not only affect the equilibrium level of investments and interest rates, but also the transmission mechanism of the monetary policy with composite effects that vary with the level of output and depend on the price demand elasticity and on the elasticity of the credit supply with respect to the lending rate.

Keywords: money supply; credit; money multipliers; Industrial organization and macroeconomics; macroeconomic industrial structure.

JEL Classification: E51, L16.

1. Introduction

Financial markets globalisation has, in many cases, created incentives for mergers and acquisitions, since on the one hand, multinational corporations may implement acquisition policies of local companies in order to enter local markets and, on the other hand, local companies might have incentives to merge in order to face the entry multinational corporations. In many emerging sectors (such as telecommunications, mobile telephones), due to regulations and licenses, barriers to entry may still exist, so that financial markets openness might not always be associated to international free trade, and powerful industrial firms might not be (at least after mergers and acquisitions) price taker. A first purpose of this paper is to consider the macroeconomic impact of mergers and acquisitions not only for what concerns the goods market, but also the financial sector of a developing economy. Many developing economies are characterized by "bank-oriented" financial systems, where the amount of financial flows negotiated in the "spot" financial markets are almost negligible compared to bank intermediation. This means that the situation determined in some industries after the mergers and acquisitions is characterized by strong firms enjoying oligopolistic power on the goods market and oligopsonistic power on the credit market. Strategic interaction between lenders and industrial firms has been studied within an industrial economics perspective (Brander and Lewis 1985, Poitervin, 1989a, 1989b, 1990), but hardly any contribution exists on the macroeconomic implications of industrial firms market power on the credit market. Section 2 introduces a "mesoeconomic" model with industrial firms oligopolistic in the goods market and oligopsonistic in the credit market. Section 3 contains some comparative statics and the main results. Section 4 contains some concluding remarks.

2. The model

The model introduced here is a short run "mesoeconomic" model: it deals with the first impact of monetary policy, and it is constituted by macroeconomic equations and "microeconomic" conditions describing in detail a single industry of relevant size, composed by large firms enjoying market power both on the goods market and in the bank credit market. The banks are assumed to be more numerous than the firms and compete among them under a perfectly competitive regime. The attention is focused on the investments (which are assumed to last for one period only) of the industrial firms. For this reason, it is assumed that the wages and the level of employment are fixed: this could be interpreted as a ceteris

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1 Mazzioli (1998, ch.4) is probably one of the very few contributions in this regard, but it shows weaker and less stringent results than the ones presented here, and the assumptions of the model (in particular for what concerns the financial sector of the economy) may not be referred to developing countries.
paribus assumption, or, alternatively, as an assumption only valid in the short run, since we are dealing with a short run model. The industrial firms' (hereafter “firms”) investments determine the output produced by the industry. In addition, we assume that the model describes an economy with a “banking-oriented” financial system, as usual in developing countries. One of the main “ingredients” of the model must be a convenient framework allowing, on the one hand to formalise the macroeconomic effects of a change in firms’ market power in the industrial sector, and, on the other hand, to include as extreme cases both perfect competition and monopoly. The simplest and more direct way to do the trick is to assume - that there are \( n \) identical firms (each of them owning some of the given \( N \) production units, or factories) behaving as oligopolists à la Cournot on the bank credit market. The individual factory’s investment \( k \) (lasting for one period only ) may be financed either with bank credit (at the interest rate \( r_L \) ) or by issuing bonds (at the interest rate \( r_B \) ). As mentioned before, it is assumed that the overall number \( N \) of production units is fixed. Each of the \( n \) firms therefore raises external finance in order to provide with capital \( k \) each of its \( N/n \) production units, and each production unit is a generic Cobb-Douglas. In this way - by keeping the number of production units in the economy constant - a change in the degree of concentration can be conceptually isolated form any other “entry and exit” effect that might affect the scale of the economy. In order to let \( N/n \) vary with continuity, we allow the possibility for the firms to own a portion of a production unit. In addition, as mentioned before, it is assumed that the firms are oligopolistic in the goods market and produce a final consumption good at the price \( p \). The money base is assumed to be only constituted by the reserves held by the banks at the central bank: this implies that there is no currency and all payments are made with banks deposit. Each of the factories, or production units, owned by the firms may be represented by a Cobb-Douglas in the following way:

\[
y_i = Nk_i \phi \theta
\]

where the subscript “\( i \)”, which identifies the \( i \)-th production unit (or factory), will be ignored in the rest of the paper. For what concerns labour, captured by \( \Phi \), for simplicity we introduce here a “ceteris paribus” assumption.\(^1\) Introducing labour would not have qualitatively changed the result of the paper, but would have considerably complicated its algebra. The optimisation problem of the representative firm may be described as follows:

\[
\begin{align*}
\max \pi & = (N/n) \left[y - w^*L + (1 + r_L)k\right] \\
\text{s.t.} & \quad (N/n)k + K' = S(r_L, r_B, BM)
\end{align*}
\]

where \( \pi \) is the firm’s profits, \( y \) the output produced by each production unit, \( w^* \) the wages and \( L^* \) the labour employed (both fixed in the short run), \( k \) the investment for each production unit, \( S(\cdot) \) is the bank credit supply function\(^2\) (assumed to be a constant elasticity function with respect to \( r_L \) and \( r_B \)), \( BM \) the money base (which - having assumed in our case that there is no currency - is equal to the private banks’ outstanding reserves, figuring - in the central bank balance constraint - as a counterpart for the bonds held by the central bank), \( K' \) the investments made by all the other production units owned by all the other firms; equation (4) is the inverse demand function (assumed for simplicity to be a constant elasticity function with respect to the nominal output \( pY \) and the interest rate \( r_B \)) for the final consumption good produced by the industry under consideration, where \( \psi \) and \( \beta \) are generic positive parameters. In particular \( L \) is a function (assumed to be homogeneous in \( pY \) ) that “capture” the causal link existing between the determination of the industry output \( pY \) and that part of the households’ disposable income spent on the industry final consumption good. This means that the higher the macroeconomic relevance of the industry under consideration, the higher the contribution of the industry output in determining the overall disposable income of the economy, the higher will be the value \( \partial L(\cdot)/\partial Y \). In other words, the industry output affect its demand in two opposite senses: on the one hand (through the term \( Y^\beta \)) it reflects the usual negative relation between the price and the demanded quantity of the good, on the other hand (through the term \( L(\cdot)pY \)) it positively affects the demand for the good through the households' disposable income. Since this is a short run partial equilibrium model, and since the industry output only affects the households disposable income to the extent that our industry is relevant on a macroeconomic point of view, we assume that in (4) the main impact of \( Y \) on \( p \) be negative. Constraint (3) represents the macroeconomic equality between credit supply and firms’ investments. Having assumed that the firms behave as Cournot oligopolists on the goods market and Cournot oligopolists on the credit market, and having assumed that the S.O.C. are satisfied, the F.O.C. are the following:

\[
\begin{align*}
p(d\psi/dk)[1 + 1/n \epsilon_{SP}] & = 1 + r_L \left[1 + (1/n \epsilon_{SP}) \right]
\end{align*}
\]

where \( \epsilon_{SP} \) is the demand price elasticity of the final good, \( \epsilon_{SP} \) is the bank credit supply elasticity with respect to \( r_L \). This means that we have two potential sources of rigidity in the model, one in the

\[p = L(pY)^\psi Y^\beta, \quad \text{where } \psi, \beta > 0 \text{ and } Y = Nk = Nk^\alpha \theta^1(1 - \alpha).
\]

\(^1\) Equation (3) may be interpreted as a special case of the function \( S(r_L, r_B, BM, E^*(\Delta BM)) \), with \( E^*(\Delta BM) = E[\Delta BM]/P[E(\Delta BM)] = 0 \) and \( E^*(\Delta BM) \neq 0 \) (i.e. unanticipated monetary policy). \( E^*(\Delta BM) \) is the private sector expectation concerning the monetary policy intervention (defined as change in the money base) \( F[\Delta BM] \) is the probability distribution function of the expectations with respect to \( E(\Delta BM) \). It is a positive parameter describing the elasticity of the expectations with respect to their monetary intervention \( E^*(\Delta BM) \). Therefore, equation (3) reflects a situation of unanticipated monetary policy.
goods market and one in the credit market.

In general, there is no reason to assume that only one of these sources of rigidity should be taken into account since there is no reason to assume that the firms only use their market power in real transactions, and not in the credit market. We can re-write equation (5) as an implicit function:

\[ f_s(p, k, r_n, n) = 0. \]  

(6)

The rest of the model is composed by the following equations

- Equilibrium on the market for bank credit to the firms:

\[ + - + \]

\[ N_k - S(r_n, r_B, BM) = f_s(r_n, r_B, k, BM) = 0. \]  

(7)

- Equilibrium on the bonds market:

\[ + - + \]

\[ B^H(r_n, r_B) + L^H(r_B, r_L) + BM - BT = f_s(r_n, r_B, BM) = 0. \]  

(8)

We assume, for simplicity, that the interest rate on deposits is null and the households are also the owners of the banking system. \( B^H \) and \( B^M \) represent the demand for bonds by the banks and households respectively, \( BT \) the (given) amount of public debt, \( L^H \) is an excess demand function of households' liabilities with banks. Given the nature of our short-run "first-impact" model, we assume that there is no feedback between the output produced by the industry and the demand for bonds by the households \( B^H(-) \), which amounts to saying that the feedback does exist, but simply does not take place in the short run.

\( L^H \) is defined according to the following assumptions: since we admit that banks lend money to the households, we assume that the sector of bank credit to the households be perfectly competitive and that its interest rate be defined as \( r_n = r_B + h \), where \( h \) is a constant. This assumption consists of aggregating the bonds market and the market for bank credit to the households (both of them perfectly competitive)

\[ \text{and considerably simplifies the algebra of the model, while not qualitatively changing the conclusions of the model. Let us introduce now the equilibrium condition between money demand and supply (9) and the equilibrium condition on the market for the final consumption good (10).} \]

\[ D^H(r_B) - BM/(q(r_n, r_B)) = 0 = f_s(r_L, r_B, BM). \]  

(9)

\[ D^H(-) \text{ is the households' demand for deposits, } q(-) \text{ the total reserves (i.e. the sum of reserve requirements and free reserves) of the banking system, } C = (L/I)^{\alpha} \text{ is obtained by a simple algebraic manipulation of (4).} \]

\[ Y = N_a^{a_k} Y^a \text{ is the output produced by all the existing production units (being fixed in the short run the quantity of labour)). Since the equilibrium conditions on the money and bond markets are linearly dependent, we only consider equation (8).} \]

### 3.2 Comparative statics and main results

\[ * \sim * \]

Let us assume, as usual in financial sector models, that in the excess demand functions for financial assets the partial derivatives with respect to the own interest rates are larger (in absolute value) than the derivatives with respect to alternative interest rates. We get the following system, where \( F \) is the matrix at the left-hand side of the equality:

\[
\begin{bmatrix}
\frac{\partial f_1}{\partial p} & \frac{\partial f_1}{\partial l} & \frac{\partial f_1}{\partial f_1} & 0 \\
\frac{\partial f_2}{\partial k} & \frac{\partial f_2}{\partial f_2} & \frac{\partial f_2}{\partial f_2} & 0 \\
0 & \frac{\partial f_3}{\partial r_n} & \frac{\partial f_3}{\partial r_B} & \frac{\partial f_3}{\partial r_B} \\
0 & \frac{\partial f_4}{\partial r_n} & \frac{\partial f_4}{\partial r_B} & \frac{\partial f_4}{\partial r_B} \\
\frac{\partial f_5}{\partial p} & \frac{\partial f_5}{\partial k} & 0 & 0 \\
\frac{\partial f_6}{\partial p} & \frac{\partial f_6}{\partial k} & 0 & 0 \\
\end{bmatrix} \begin{bmatrix}
dp \\
dk \\
dr_n \\
dr_B \\
dp \\
dk \\
\end{bmatrix} = \begin{bmatrix}
0 & \frac{\partial f_1}{\partial n} \\
0 & \frac{\partial f_2}{\partial n} \\
-1 & \frac{\partial f_3}{\partial BM} \\
0 & \frac{\partial f_4}{\partial BM} \\
0 & \frac{\partial f_5}{\partial n} \\
0 & \frac{\partial f_6}{\partial n} \\
\end{bmatrix} \begin{bmatrix}
\text{dB}M \\
\text{d}n \end{bmatrix} \]

(11)

On the basis of the above-mentioned assumptions we get the following sign pattern:

\[
\begin{bmatrix}
+ & - & - \\
0 & + & + \\
0 & 0 & + \\
- & - & 0 \\
\end{bmatrix}
\]

Since this is a short-run model, for what concerns monetary policy, we get, as expected:

\[
dk/\text{dB}M>0; \ dr_L/\text{dB}M<0; \ dr_B/\text{dB}M<0.
\]

For what concerns the effects of an exogenous change in market structure, we get:
i.e. an increase in the degree of competition (reduction in the degree of concentration) in the industrial sector increases, ceteris paribus, the demand for capital and, as a consequence, the equilibrium level of investments and interest rates. Let us focus our attention on the monetary policy multiplier

$$dk/dBM = \left[\left(1/\det(F)\right) \cdot D_1 \right]$$

where

$$D_1 = \left\{ \left[\left(1/\det(F)\right) \cdot \left(\partial f/\partial r_B\right) \cdot \left(\partial f/\partial p_r\right)\right] - \left[\left(1/\det(F)\right) \cdot \left(\partial f/\partial p_n\right)\right] \right\}$$

(15)

The question is now to analyse whether and how exogenous changes in the market structure affect the transmission mechanism of monetary policy. To do so, we can simply take the derivative of (15) with respect to $n$, which yields the following:

$$d(dk/dBM)/dn = \left[\left(1/\det(F)\right) \cdot \left(\partial D_1/\partial n\right) \cdot dk/dBM\right] = QD + QA.$$  

(16)

where $QD = \left[\left(1/\det(F)\right) \cdot (D_1/\partial n)\right]$ and $QA = \left[\left(1/\det(F)\right) \cdot \left(d\det(F)/dn\right) \cdot dk/dBM\right]$.

$QD$ may be interpreted as the impact that an exogenous change in the market structure induces on the money multiplier and is always negative. $QA$ may be interpreted as the effect determined by an exogenous modification in $n$, “for a given value of the multiplier $dk/dBM$, and its sign is ambiguous. However, its negative terms will be larger in absolute value the larger $IEsLI$ compared to $IEopl$, the highest marginal productivity of physical capital, the more concave the production function of the firms. This also means that the effects on monetary policy might vary with the level of output, as the curvature and marginal productivity of capital change.

4. Concluding remarks

The theoretical model introduced here shows how an exogenous increase in the degree of competition (reduction in the degree of concentration) in the industrial sector affects:

a) positively the equilibrium level of investments in the “concentrated” industrial sector;

b) the transmission mechanism of the monetary policy with composite effects which depend on the price elasticity of the demand for the good produced in the industry under consideration and on the credit supply elasticity with respect to the lending rate. These effects will also be smaller (and more likely to be negative) the larger the marginal productivity of capital and the curvature of the representative firm’s production function.

Mergers and acquisitions are then to be considered among the factors conditioning the transmission mechanism of monetary policy in bank-oriented financial systems.

REFERENCES


will be affected by
\[ \frac{\partial f_1}{\partial B M} = 0 \]

\[ \frac{\partial f_1}{\partial n} \left( \frac{1}{n^{\alpha v}} \right) \frac{\partial v}{\partial k} \frac{\partial (\alpha v)}{\partial p} (-1) \frac{1}{n^2 \varepsilon_{wv}} + (-1) \frac{r}{n^2 \varepsilon_{2L}} < 0 \]

since \( \varepsilon_{wv} < 0 \) and firms' marginal revenue is non-negative.

\[ \frac{\partial f_1}{\partial p} = \frac{\partial v}{\partial k} \left( \frac{1}{1 + \frac{1}{n \varepsilon_{wv}}} \right) > 0 \]

\[ \frac{\partial f_1}{\partial k} = \frac{\partial v}{\partial k} \left( \frac{1}{1 + \frac{1}{n \varepsilon_{wv}}} \right) < 0 \] since \( y(\cdot) \) is a Cobb-Douglas

\[ \frac{\partial f_1}{\partial r_L} = - \left( \frac{1}{1 + \frac{1}{n \varepsilon_{wv}}} \right) < 0 \]

\[ \frac{\partial f_1}{\partial B} = 0 \]

\[ \frac{\partial f_2}{\partial B M} = \frac{\partial S(\cdot)}{\partial B M} > 0 \]

\[ \frac{\partial f_2}{\partial n} = 0 \]

\[ \frac{\partial f_2}{\partial \varepsilon} = 0 \]

\[ \frac{\partial f_2}{\partial \varepsilon} = 0 \]
\[ \frac{\partial f_1}{\partial k} = N > 0 \]

\[ \frac{\partial f_2}{\partial r_L} = -\partial S(\cdot, \partial r_L) < 0 \]

\[ \frac{\partial f_2}{\partial r_B} = -\partial S(\cdot, \partial r_B) > 0 \]

\[ \frac{\partial f_3}{\partial BM} = -1 < 0 \]

\[ \frac{\partial f_3}{\partial n} = 0 \]

\[ \frac{\partial f_3}{\partial n} = 0 \]

On the basis of the assumptions on \( H' (\cdot) \) contained in the text of the paper.

\[ \frac{\partial f_4}{\partial k} = 0 \]

\[ \frac{\partial f_4}{\partial r_L} = \partial b^H / \partial r_L + \partial L^b H / \partial r_L < 0 \]

This happens because we have

\[ \frac{\partial f_4}{\partial r_L} = \alpha N A k^{x-1} \Phi < 0 \]

Having assumed that the derivatives with respect to the own rates are larger in absolute value than the cross-derivatives and taking into account the explanations contained in footnote 14 of the text of the paper, we get:

\[ L^b H^1 (r_L, r_B, r_H (r_B)) - L^b H^1 (r_H (r_B), r_B) > 0 \]

Which, by the way, also means that

\[ \partial L^b H / \partial r_H > 0 \]

\[ \frac{\partial f_4}{\partial r_B} = \partial b^H / \partial r_B + \partial b^H / \partial r_B + \partial L^b H / \partial r_B > 0 \]

\[ \frac{\partial f_4}{\partial BM} = 0 \]

\[ \frac{\partial f_4}{\partial n} = 0 \]

\[ \frac{\partial f_4}{\partial p} = \frac{1}{\beta} C \cdot \beta^{(1/\beta + 1)} < 0 \]
\[
\frac{\partial f_t}{\partial r_B} = 0
\]

\[
\frac{\partial^2 f_t}{\partial r_B \partial \pi} = 0
\]

\[
\frac{\partial^2 f_t}{\partial r_J \partial \pi} = \frac{1}{\pi^2 \varepsilon_{\pi}} > 0
\]

\[
\frac{\partial^2 f_t}{\partial \pi \partial \varphi} = p \frac{\partial y}{\partial \pi^2} (-1) \frac{1}{\pi^2 \varepsilon_{\varphi}} < 0
\]

\[
\frac{\partial^2 f_t}{\partial \varphi \partial \alpha} = \frac{\partial y}{\partial \varphi} (-1) \frac{1}{\pi^2 \varepsilon_{\varphi}} > 0
\]

\[\det(F) = (+1) \left( \frac{\partial y}{\partial J} \frac{\partial r_J}{\partial \alpha} \right) \left( \frac{\partial^2 f_t}{\partial r_B \partial \pi} \right) \left( \frac{\partial^2 f_t}{\partial r_B \partial \varphi} \right) \left( \frac{\partial^2 f_t}{\partial r_J \partial \pi} \right) \left( \frac{\partial^2 f_t}{\partial r_J \partial \varphi} \right) \left( \frac{\partial^2 f_t}{\partial r_B \partial r_J} \right)
\]

\[\det(F) = (+1) \left( \frac{\partial y}{\partial J} \frac{\partial r_J}{\partial \alpha} \right) \left( \frac{\partial^2 f_t}{\partial r_B \partial \pi} \right) \left( \frac{\partial^2 f_t}{\partial r_B \partial \varphi} \right) \left( \frac{\partial^2 f_t}{\partial r_J \partial \pi} \right) \left( \frac{\partial^2 f_t}{\partial r_J \partial \varphi} \right) \left( \frac{\partial^2 f_t}{\partial r_B \partial r_J} \right)
\]

\[\det(F) = (+1) \left( \frac{\partial y}{\partial J} \frac{\partial r_J}{\partial \alpha} \right) \left( \frac{\partial^2 f_t}{\partial r_B \partial \pi} \right) \left( \frac{\partial^2 f_t}{\partial r_B \partial \varphi} \right) \left( \frac{\partial^2 f_t}{\partial r_J \partial \pi} \right) \left( \frac{\partial^2 f_t}{\partial r_J \partial \varphi} \right) \left( \frac{\partial^2 f_t}{\partial r_B \partial r_J} \right)
\]

The sign of \(d\det(F)/d\pi\) is uncertain.

The term
\[(-1) \left( \frac{\partial y}{\partial J} \frac{\partial r_J}{\partial \alpha} \right) \left( \frac{\partial^2 f_t}{\partial r_B \partial \pi} \right) \left( \frac{\partial^2 f_t}{\partial r_B \partial \varphi} \right) \left( \frac{\partial^2 f_t}{\partial r_J \partial \pi} \right) \left( \frac{\partial^2 f_t}{\partial r_J \partial \varphi} \right) \left( \frac{\partial^2 f_t}{\partial r_B \partial r_J} \right)
\]

is negative, i.e. it contributes to make positive \(d\delta_k/dBM/d\pi\); the algebraic sum of the terms containing \(\frac{\partial^2 f_t}{\partial r_B \partial r_J} \), i.e.
\[(+1) \left( \frac{\partial y}{\partial J} \frac{\partial r_J}{\partial \alpha} \right) \left( \frac{\partial^2 f_t}{\partial r_B \partial \pi} \right) \left( \frac{\partial^2 f_t}{\partial r_B \partial \varphi} \right) \left( \frac{\partial^2 f_t}{\partial r_J \partial \pi} \right) \left( \frac{\partial^2 f_t}{\partial r_J \partial \varphi} \right) \left( \frac{\partial^2 f_t}{\partial r_B \partial r_J} \right)
\]

is positive because
\[\left| \frac{\partial^2 f_t}{\partial r_B \partial r_J} \right| < \left| \frac{\partial y}{\partial J} \frac{\partial r_J}{\partial \alpha} \right| \text{ and } \left| \frac{\partial^2 f_t}{\partial r_B \partial r_J} \right| < \left| \frac{\partial^2 f_t}{\partial r_B \partial r_J} \right|
\]

since it has been assumed that the derivatives with respect to the own interest rate are larger in absolute value than the cross-derivatives.

This means that the term \(\Delta \) in equation (16) is smaller and more likely to be negative the larger \(\varepsilon_{\pi}\) with respect to \(\varepsilon_{\varphi}\) and the larger the marginal productivity of capital and the degree of concavity of the production function.

\[d\delta_k/dBM = \left( \frac{\partial^2 f_t}{\partial r_B \partial \pi} \right) \left( \frac{\partial^2 f_t}{\partial r_B \partial \varphi} \right) \left( \frac{\partial^2 f_t}{\partial r_J \partial \pi} \right) \left( \frac{\partial^2 f_t}{\partial r_J \partial \varphi} \right) \left( \frac{\partial^2 f_t}{\partial r_B \partial r_J} \right)
\]

\[d\delta_r/dBM = \left( \frac{\partial^2 f_t}{\partial r_B \partial \pi} \right) \left( \frac{\partial^2 f_t}{\partial r_B \partial \varphi} \right) \left( \frac{\partial^2 f_t}{\partial r_J \partial \pi} \right) \left( \frac{\partial^2 f_t}{\partial r_J \partial \varphi} \right) \left( \frac{\partial^2 f_t}{\partial r_B \partial r_J} \right)
\]

\[d\delta_B/dBM = \left( \frac{\partial^2 f_t}{\partial r_B \partial \pi} \right) \left( \frac{\partial^2 f_t}{\partial r_B \partial \varphi} \right) \left( \frac{\partial^2 f_t}{\partial r_J \partial \pi} \right) \left( \frac{\partial^2 f_t}{\partial r_J \partial \varphi} \right) \left( \frac{\partial^2 f_t}{\partial r_B \partial r_J} \right)
\]

because we have
\[\left| \frac{\partial^2 f_t}{\partial r_B \partial r_J} \right| < \left| \frac{\partial y}{\partial J} \frac{\partial r_J}{\partial \alpha} \right| \text{ and } \left| \frac{\partial^2 f_t}{\partial r_B \partial r_J} \right| < \left| \frac{\partial^2 f_t}{\partial r_B \partial r_J} \right|
\]

since it has been assumed that the derivatives with respect to the own interest rate are larger in absolute value than the cross-derivatives.


