
by

Carlo Alberto Magni

Settembre 2005

Università degli Studi di Modena e Reggio Emilia
Dipartimento di Economia Politica
Via Berengario, 51
41100 Modena (Italia)
e-mail magni.carloalberto@unimore.it
The use of NPV and CAPM for capital budgeting is not a good idea. A Reply to De Reyck (2005)

Carlo Alberto Magni
Università di Modena e Reggio Emilia, Dipartimento di Economia Politica
viale Berengario 51, 41100 Modena, Italy
tel. 0039-59-2056777, fax 0039-59-2056937, Email:magni@unimore.it

Abstract
In Magni [Eur. J. Operat. Res. 137 (2002) 206] I present some inconsistencies implicit in the net-present-value criterion, as currently used in finance. In particular, I present two theses: (A) The use of NPV methodology is self-contradictory; (B) the equivalent-risk principle is inapplicable. This paper develops and strengthens the first thesis showing that the the use of CAPM for capital budgeting is at odds with arbitrage theory, and that the NPV rule is meaningless even in the simplest case, because net present value is any number one wants it to be. Cognitively, this amounts to say that the NPV procedure leaves decision makers subject to a framing bias; financially, this amounts to say that additivity does not hold. De Reyck's [Eur. J. Operat. Res. 161 (2005) 499] objection to my second thesis is shown to be invalid (not only because he does not cope at all with my second thesis, but also) because he mistakes a project's expected rate of return for a project's cost of capital and confuses the notion of risk-equivalence implicit in the CAPM with the notion of risk-equivalence implicit in arbitrage theory.

Keywords. Finance, Investment Analysis, Net Present Value, Capital Asset Pricing Model, arbitrage.

Introduction
The Net Present Value (NPV) rule states that project valuation should be accomplished abiding by what I like to name the equivalent-risk (ER) tenet: Decision makers involved in capital budgeting decisions should only compare assets having equivalent risk. Therefore, a project should be valued by discounting its cash flows with a risk-adjusted rate of return reflecting the expected rate of return of an asset which is equivalent in risk to the project under valuation. In Magni (2002) I present two theses, henceforth labelled A and B respectively: (A) "inconsistencies and antinomies arise when applying the above mentioned rule" (Magni, 2002, p. 206), (B) "it is actually impossible to compare alternatives equivalent in risk and any decision maker cannot prevent herself to violate the above tenet" (ibidem). De Reyck (2005) holds that my arguments supporting thesis B are flawed. This paper shows that De Reyck's argument is biased and adds some irons in the fire claiming that arbitrage theory and Capital Asset Pricing Model (CAPM) are incompatible when used for capital budgeting problems. Also, the NPV rule is meaningless, because the net present
The value of a project is any real number. These findings are unexpected results and they are not yet recognized in the literature.

The paper is structured as follows. Section 1 reminds the reader about the concept of rate of return and the meaning of “equivalent in risk” in the CAPM. Although this is basic finance, all arguments in this paper stem from logical deduction of such fundamental concepts. Section 2 shows that the use of CAPM for capital budgeting is incompatible with arbitrage theory, since the notions of risk implicit in the two models differ. Section 3 revisits De Reyck’s (2004) Proposition disclosing the biases that impair De Reyck’s argument and offers some Propositions aimed at shedding lights on the issue at hand. Section 4 shows that the notion of net present value is ambiguous and self-contradictory, since the NPV of a project is not unique. Furthermore, it is meaningless, since it is any real number one wants it to be. Section 5 briefly highlights a methodological error in De Reyck (2005). Some remarks conclude the paper.

I will be hereafter concerned with one-period projects $x$ and $y$, which pay off, at time 1, the random cash flows $X$ and $Y = X + K$ respectively, and whose costs are $I_x$ and $I_y$ respectively. I assume the existence on the market of a security $\delta$ having the same beta as project $x$, and of a security $\pi$ which exactly replicates project $x$’s cash flows in every state of nature. I will make use, among others, of the following notations:

$$\Delta = \text{value of security } \delta \text{ at time 1 (or, equivalently, cash flow released by } \delta \text{ if it is sold in the market at time 1).}$$

$$\Pi = \text{value of security } \pi \text{ at time 1 (or, equivalently, cash flow released by } \pi \text{ if it is sold in the market at time 1).}$$

$$r_f = \text{risk-free rate.}$$

$$r_m = \text{market rate of return}$$

$$\sigma_m^2 = \text{variance of market rate of return}$$

$$r_l = \text{asset } l\text{'s rate of return}$$

$$V_l = \text{value of asset } l$$

$$I_l = \text{cost of asset } l$$

$$\beta_l = \frac{\text{cov}(r_l, r_m)}{\sigma_m} = \text{systematic risk of asset } l \quad l \in \{x, y, \delta, \pi\}$$

where cov stands for covariance. Furthermore, the symbols $i_x$ and $i_y$ represent the costs of capital of project $x$ and $y$ respectively, and a bar over any symbol means that expectation is taken. As a last assumption, securities $\delta$ and $\pi$ lie on the Security Market Line (i.e. they are in equilibrium).

1 NPV and CAPM for capital budgeting

It is standard in finance to solve capital budgeting problem by using the net-present-value rule alongside the Capital Asset Pricing Model (CAPM) (see Rubinstein, 1973; Copeland and Weston, 1988; Brealey and Myers, 2000, or any other textbook): A project $x$ should be undertaken if and
only if its Net Present Value (NPV), calculated by discounting cash flows at the cost of capital, is positive. The cost of capital is also labelled risk-adjusted rate of return and is an opportunity cost, representing the expected rate of return of a security lying on the Security Market Line (SML) and equivalent in risk to the project. Risk-equivalence has a precise meaning, which is worth repeating even if it is commonplace in the literature:

**Definition 1.** Project $x$ has the same risk as asset $\delta$ if $\beta_x = \beta_\delta$.

The random rate of return of one-period project $x$ is found by computing the ratio of its payoff $X$ to the cost $I_x$ and then subtracting one:

$$r_x = \frac{X}{I_x} - 1. \quad (1)$$

Likewise, the random rate of return of $\delta$ is

$$r_\delta = \frac{\Delta}{I_\delta} - 1 = \frac{\Delta}{V_\delta} - 1 \quad (2)$$

where the second equality holds since $\delta$ is in equilibrium (lies on the SML). Let us now adopt Definition 1 as our definition of risk. Using the fact that $\beta_x = \beta_\delta$ the CAPM gives us the following relation:

$$i_x = r_\delta = r_f + \beta_\delta(r_m - r_f). \quad (3)$$

Eq. (3) is routine in finance and the first equality explicitly underlines that a project’s cost of capital coincides with the expected rate of return of an equivalent-risk asset lying on the SML. Therefore, the value of project $x$ is

$$V_x = \frac{X}{1 + i_x} = \frac{X}{1 + r_\delta}. \quad (4)$$

From eq. (4), we get to

$$i_x = \frac{X}{V_x} - 1 \quad (5)$$

which highlights the dependence of a project’s cost of capital on value. I will often stress that rate of return depends on cost and that cost of capital depends on value, as eqs. (1) and (5) manifest.

The NPV rule tells us that $x$ should be undertaken if and only

$$-I_x + V_x = -I_x + \frac{X}{1 + i_x} > 0 \quad (6)$$

or, in CAPM form,

$$\bar{r}_x > i_x = r_f + \beta_\delta(r_m - r_f) \quad (7)$$

since $\bar{X} = I_x(1 + \bar{r}_x)$. In words, eq. (7) may be restated as

\[x’s \ expected \ rate \ of \ return > x’s \ cost \ of \ capital\]

or, equivalently,

\[x’s \ expected \ rate \ of \ return > \delta’s \ expected \ rate \ of \ return.\]

The NPV rule boils then down to a comparison that is the direct consequence of an equivalent-risk (ER) tenet, according to which a decision maker is allowed to compare only alternatives that are equivalent in risk.
2 CAPM and arbitrage

The notion of risk-equivalence implied by Definition 1 strictly depends on the notion of beta. But beta is not the only measure of risk finance relies on. Arbitrage theory provides us with the following notion of risk-equivalence:

Definition 2. Project $x$ has the same risk as asset $\pi$ if $x$'s payoffs are replicable by $\pi$ in every state of nature.

The following Proposition shows that Definitions 1 and 2 are incompatible.

Proposition 1. Let $x$ be a one-period project paying off the random sum $X$ and let $\pi$ be a twin security, such that $X = a\Pi$ for some nonzero $a$. Then $\beta_x \neq \beta_\pi$, (as long as $x$ is not in equilibrium, i.e. $V_x \neq I_x$).

Proof. The assumption $X = a\Pi$ implies $V_x = aV_\pi$. Hence,

$$\beta_x = \frac{\text{cov}(X - 1, r_m)}{I_x \sigma_m^2} = \frac{\text{cov}(X, r_m)}{I_x \sigma_m^2}$$

$$\neq \frac{\text{cov}(X, r_m)}{V_x \sigma_m^2} = \frac{\text{cov}(a\Pi, r_m)}{aV_{\pi} \sigma_m^2} = \beta_\pi$$

Remark 1. By modus tollens, it is also obvious that, ceteris paribus, if $x$ and $\pi$ had the same beta, then $\pi$ would not be a twin security of $x$. As a result, security $\delta$, having the same beta as $x$, does not replicate $x$.

Proposition 1 shows that the definitions of risk implied by CAPM and arbitrage theory conflict each other. This implies that the very concept of value is ambiguous, as the following Proposition shows:

Proposition 2. The notion of value in the CAPM is different from the notion of value in arbitrage theory, i.e.

$$V_x^C \neq V_x^A$$

where $V_x^C$ is the value of project $x$ obtained by application of the CAPM and $V_x^A$ is the value of $x$ obtained by application of arbitrage theory.

Proof. As $\delta$ and $\pi$ lie on the SML, we have $r_\delta = r_f + \beta_\delta (r_m - r_f)$ and $r_\pi = r_f + \beta_\pi (r_m - r_f)$. We also have $\beta_x = \beta_\delta$ (by definition of asset $\delta$) and $\beta_x \neq \beta_\pi$ (by Proposition 1). The latter implies $\beta_\delta \neq \beta_\pi$, which in turn implies $\bar{r}_\delta \neq \bar{r}_\pi$, which leads to

$$V_x^C = \frac{X}{1 + i_x} = \frac{X}{1 + \bar{r}_\delta} \neq \frac{X}{1 + \bar{r}_\pi} = \frac{a\Pi}{1 + \bar{r}_\pi} = V_x^A.$$ 

\footnote{Ambiguity lies in other economic concepts as well. See Magni (2003, 2004, forthcoming) for ambiguity in the notion of excess profit.}
We have two different notions of risk-equivalence, expressed in Definition 1 (linked to the CAPM) and Definition 2 (derived from arbitrage theory), as well as two different notions of value. Contrary to what is commonly thought, the two notions are incompatible. In particular, risk in the CAPM depends on beta, which depends on cost, risk in arbitrage theory depends on cash flows, which do not depend on cost.

3 Linking values of two projects

All De Reyck's (2005) results, intended to defend NPV, rest on a Proposition which I here restate, adjusting symbols for coherence:

De Reyck's Proposition 1. Let project x be a one-period project resulting in cash flows $X = (X_1, X_2)$ with probabilities $p$ and $1 - p$, respectively, and let $i_x$ be project x's cost of capital, obtained through the market valuation of a security or project with exactly the same payoff pattern. Project y with cash flows $Y = (Y_1, Y_2)$ with probabilities $p$ and $1 - p$ can then be valued as follows:

$$V_y = \frac{V - \varrho(Y, X)(i_x - r_f)V_x \sigma(Y)}{1 + r_f}$$  \hspace{1cm} (9)

with $\varrho(Y, X) \in \{-1, 1\}$.

As throughout this paper our assumption is $Y = X + K$, De Reyck's Proposition 1 becomes

De Reyck's Proposition 2. Let project x be a one-period project resulting in cash flows $X = (X_1, X_2)$ with probabilities $p$ and $1 - p$, respectively, and let $i_x$ be project x's cost of capital, obtained through the market valuation of a security or project with exactly the same payoff pattern. Project y with cash flows $Y = (X_1 + K, X_2 + K)$ with probabilities $p$ and $1 - p$ can then be valued as follows:

$$V_y = \frac{V - (i_x - r_f)V_x}{1 + r_f}.$$  \hspace{1cm} (10)

I now show that De Reyck's Proposition is flawed (to this end, it suffices to disprove De Reyck's Proposition 2). As a preliminary result, the following Proposition shows that two random cash flows differing by a constant may have the same beta.

Proposition 3. Let projects x and y be one-period projects resulting in the random cash flows $X$ and $Y = X + K$ respectively. If $i_x = i_y$, then $\beta_x = \beta_y$. 

\footnote{If $x$ were in equilibrium, then we would have $V^C_x = V^A_x$. In other words, arbitrage theory and CAPM do value assets in equilibrium in the same way.}
Proof.

\[
\beta_y = \frac{\text{cov}(r_y, r_m)}{\sigma_m^2} \\
= \frac{\text{cov}(Y, r_m)}{I_y \sigma_m^2} \\
= \frac{\text{cov}(X + K, r_m)}{I_y \sigma_m^2} \\
= \frac{\text{cov}(X, r_m)}{I_x \sigma_m^2} \\
= \frac{\text{cov}(r_x, r_m)}{\sigma_m^2} = \beta_x
\] (11)

An obvious consequence of the above Proposition is

Corollary 1. If the assumptions of the above Proposition hold, then \( i_x = \bar{r}_d = i_y \).

Proposition 4. Let project \( x \) be a one-period project which pays off \( X \) and let \( i_x \) be project \( x \)'s cost of capital, calculated assuming a notion of risk as defined in Definition 1. If project \( y \) pays off the sum \( Y = X + K \), then it may be valued as follows:

\[
V_y = \frac{\bar{Y} - \frac{1 + \bar{r}_x (i_x - r_f) I_x}{1 + r_f}}{1 + r_f}
\] (12)

or, equivalently,

\[
V_y = \frac{\bar{Y} - \frac{1 + \bar{r}_y (i_y - r_f) I_y}{1 + r_f}}{1 + r_f}
\] (13)

Proof. We have

\[
V_y = \frac{\bar{Y}}{1 + i_y} = \frac{\bar{Y}}{1 + r_f + \beta_y (\bar{r}_m - r_f)}
\]
whence

\[
V_y = \frac{\bar{Y} - V_y \beta_y (\bar{r}_m - r_f)}{(1 + r_f)} \\
= \frac{\bar{Y} - V_y \frac{\text{cov}(Y, r_m)}{\text{Cov}(r_m)} (\bar{r}_m - r_f)}{(1 + r_f)} \\
= \frac{\bar{Y} - \frac{1 + \bar{r}_y}{1 + t_y} \frac{\text{cov}(Y, r_m)}{\text{Cov}(r_m)} (\bar{r}_m - r_f)}{(1 + r_f)} \\
= \frac{\bar{Y} - \frac{1 + \bar{r}_x}{1 + t_x} \frac{\text{cov}(X+K, r_m)}{\text{Cov}(r_m)} (\bar{r}_m - r_f)}{(1 + r_f)} \\
= \frac{\bar{Y} - \frac{1 + \bar{r}_x}{1 + t_x} \beta_x I_x (\bar{r}_m - r_f)}{(1 + r_f)} \\
= \frac{\bar{Y} - \frac{1 + \bar{r}_x}{1 + t_x} (t_x - r_f) I_x}{(1 + r_f)}.
\]

(14)

To get to (13), consider the fourth step of (14) and divide and multiply by $I_y$ the ratio $\frac{\text{cov}(Y, r_m)}{\sigma_m^2}$.

Remark 2. Note that the above result holds whatever the value of $I_x$ and $I_y$.

Comparing eqs. (12)-(13) with eq. (10) it is clear that De Reyck’s Proposition does not hold. The reason is that he does not correctly apply the standard definition of a project’s rate of return. Let us see things in details. At first, De Reyck correctly affirms that “finance theory and the Capital Asset Pricing Model ... define risk as a function of returns, not cash flows” (p. 501, italics added) and that “Risk, as defined in the Capital Asset Pricing, is defined in terms of return generated by the project” (p. 504, italics added). In actual facts, he does not comply with his very words: In his Proposition’s proof he always replaces rate of return generated by the project with rate of return generated by the equivalent-risk alternative (i.e. cost of capital) or vice versa. For project $y$, he defines $r_y=(r_{y1}, r_{y2})$ as “the returns of the project in each project state” (p. 501) and, for project $x$, writes that $r_x=(r_{x1}, r_{x2})$ “denote the project returns in the different scenarios” (p. 502), but his equations are not consistent with these very statements. For example, he uses the relation $V_y = \frac{\bar{Y}}{1 + \bar{r}_y}$ which is incorrect, given that a project’s value depends on cost of capital, not on the project’s expected rate of return. Also, his eq. (2.7) is:

$$\bar{r}_y = r_f + \beta_y (\bar{r}_m - r_f).$$
But the left-hand side of the equality is, by his very definition, the expected return generated by project \( y \), whereas the right-hand member is project \( y \)'s cost of capital. The two are, in general, different. To avoid pedantry, Table 1 collects incorrect expressions in De Reyck's proof and replaces them with the corresponding correct expressions.

<table>
<thead>
<tr>
<th>De Reyck</th>
<th>Correct</th>
<th>Error location</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_y = \frac{\bar{V}}{1+r_y} )</td>
<td>( V_y = \frac{\bar{V}}{1+r_y} )</td>
<td>p. 501, eq. (2.6)</td>
</tr>
<tr>
<td>( \bar{r}_y = r_f + \beta_y(\bar{r}_m - r_f) )</td>
<td>( i_y = r_f + \beta_y(\bar{r}_m - r_f) )</td>
<td>p. 501, eq. (2.7)</td>
</tr>
<tr>
<td>( r_y = \frac{\bar{V}}{V_y} - 1 )</td>
<td>( r_y = \frac{\bar{V}}{V_y} - 1 )</td>
<td>p. 501, column right, line 10</td>
</tr>
<tr>
<td>( \text{cov}(r_y, r_m) = \frac{\text{cov}(Y_r, r_m)}{V_y} )</td>
<td>( \text{cov}(r_y, r_m) = \frac{\text{cov}(Y_r, r_m)}{V_y} )</td>
<td>p. 501, column right, line 11</td>
</tr>
<tr>
<td>( V_y = \frac{\bar{V}}{1+(r_f + \frac{\text{cov}(Y_r, r_m)}{V_y}(\bar{r}_m - r_f))} )</td>
<td>( V_y = \frac{\bar{V}}{1+(r_f + \frac{\text{cov}(Y_r, r_m)}{V_y}(\bar{r}_m - r_f))} )</td>
<td>p. 501, column right, line 12</td>
</tr>
<tr>
<td>( i_x = \bar{r}_x = r_f + \beta_x(\bar{r}_m - r_f) )</td>
<td>( i_x = \bar{r}_x = r_f + \beta_x(\bar{r}_m - r_f) )</td>
<td>p. 501, eq. (2.9)</td>
</tr>
<tr>
<td>( \beta_x = \frac{\text{cov}(r_x, r_m)}{V_x \sigma_m^2} )</td>
<td>( \beta_x = \frac{\text{cov}(r_x, r_m)}{V_x \sigma_m^2} )</td>
<td>p. 501, eq. (2.10)</td>
</tr>
</tbody>
</table>

Keeping on neglecting that the rate of return of a project depends on cost, De Reyck destroys the validity of his proof. He evidently confuses expected rate of return of project \( x \) with cost of capital of project \( x \). The former is the expected rate of return actually produced by project \( x \), the latter is the rate of return that the equivalent-risk asset \( \delta \) lying on the SML is expected to release, or equivalently, the expected rate of return that project \( x \) would generate if it were in equilibrium. In a nutshell:

project \( x \)'s rate of return depends on cost (see this paper's eq. (1)), project \( x \)'s cost of capital depends on value (see this paper's eq. (5)). Equivalently, beta depends on cost, not on value.4

3In his eq. (2.10) De Reyck introduces the symbol \( P(\bar{e}) \) which he does not define. The same symbol appears in eq. (2.11). However, it is evident that it is a typo and that that such a symbol stands for the value of \( x \).

4That betas and actual rate of returns depend on cost, not on values, should be obvious. However, Rubinstein (1973) underlines this fact by using the evocative symbol “\( \text{COST}_x \)” in his classical paper. His rule at p. 171 is identical to eq. (7) in this paper and his definition of project's rate of return at p. 172 is just the same as eq. (1) in this paper. Copeland and Weston (1988, pp. 414–418) show an example of capital-budgeting problem, giving a correct expression of the rate of return as a function of cost (ibidem, p. 416), and providing a correct expression of
Remark 3. The reader may think that eq. (10) may be found back by picking $I_x = V_x$ and $I_y = i_y$ in eq. (12). This boils down to assume that both projects $x$ and $y$ lie on the SML, i.e. they are in equilibrium with all the other securities of the market. However, this assumption is not legitimate unless one also assume $I_x \neq I_y$, as the following Proposition holds:

**Proposition 5.** Let $x$ and $y$ be projects that pay off the sum $X$ and $X + K$ respectively, and let $I_x = I_y = 1$. Then, at least one of them is not in equilibrium (i.e. at least one of them does not lie on the SML).

Proof. By Corollary 1, we have $i_x = i_y$. Then $V_x = \frac{X - (i_x - r_f)V_x}{1 + r_f}$ and $V_y = \frac{X + K + (i_x - r_f)V_x}{1 + r_f}$. If both were in equilibrium (lying both on the SML) we would have $V_x = I = V_y$, so that $X = X + K$, which is absurd (as long as $K \neq 0$).

Remark 4. As an alternative proof, just take directly (10) and assume $I_x = I_y = V_x = V_y$. Then,

$$V_x = V_y = \frac{Y - (i_x - r_f)V_x}{1 + r_f} = \frac{X + K - (i_x - r_f)V_x}{1 + r_f} = \frac{X - (i_x - r_f)V_x}{1 + r_f} + \frac{K}{1 + r_f} = \frac{X}{1 + i_x} + \frac{K}{1 + r_f} = V_x + \frac{K}{1 + r_f},$$

which is absurd.

We may then retrieve eq. (10) as a particular case of eq. (12) with the following Proposition (whose name will be motivated in the subsequent Remark):

**Useless Proposition.** Let one-period projects $x$ and $y$ pay off $X$ and $Y = Z + K$ respectively, and let $i_x$ be project $x$'s cost of capital, calculated assuming a notion of risk as defined in Definition 1. If both projects lie on the SML and if $I_x \neq I_y$, then $y$ may be valued as follows:

$$V_y = \frac{Y - (i_x - r_f)V_x}{1 + r_f}$$

the beta as a function of cost (ibidem, eq. (12.30) and eq. (12.31)). Their calculations in Table 12.2 and 12.3 are consistent with this correct approach and the problem is correctly solved. However, it is worth noting that in the first equality of p. 417 the authors denote cost of capital by using the same symbol "r" they have previously used for the actual rate of return. This equality is not used to solve the problem (it just serves the function to introduce the cost of capital), so no problem arises. However, one wonders whether Copeland and Weston's is just a typo or, rather, a Freudian lapsus.
Remark 5. The above Proposition assumes that $x$ and $y$ are in equilibrium. Barring the fact that this is an extremely rare case, if both projects lie on the SML, what's the point in determining the value of $y$? One does not need any particular Proposition to value it, since we simply have $V_y = I_y$, i.e. cost coincides with value and project $y$ (as well as project $x$) has a zero NPV so that the capital budgeting problem is trivial. In other terms, my above Proposition (which aims at retrieving De Reyck's eq. (10) taking the correct assumptions) is totally useless!

Remark 6. The reader may note that while my Proposition 4 uses Definition 1 to measure risk, De Reyck adopts the notion of risk implicit in arbitrage theory (he writes of a security with "exactly the same payoff pattern"). Here comes a further error: If the notion of risk employed is that derived from Definition 2, then one should not use the CAPM relation $i_x = r_f + \beta_x(\bar{r}_m - r_f)$, which is equivalent to chooses $V_x^C$ as the value of project $x$. One should instead use $V_x^A$ or, which is the same, pick $i_x = \bar{r}_x$, where $\bar{r}_x \neq \bar{r}_y$ (see Proposition 2). So, De Reyck commits two main errors: In first place he uses the CAPM in an incorrect way (mistaking a project's expected rate of return for its cost of capital); in second place his assumptions presuppose a notion of risk that dismisses the CAPM. As far as I can tell, the latter mistake shows that De Reyck is not aware that the CAPM notion of risk and the arbitrage notion of risk collide.

In the light of what we have seen, my thesis A remains untouched by De Reyck's paper and is now reinforced by the following one: The use of CAPM for capital budgeting evaluations is incompatible with arbitrage theory.

4 Meaninglessness

In the previous section we have seen how to correctly value project $y$ and have pointed out the flaws in De Reyck's arguments. This section takes a radically different route and, quite unexpectedly, shows that all this debating on valuing projects is an idle issue, since the very notion of net present value is meaningless.

Lemma 1. Let $x$ and $y$ be two projects paying off the random sums $X$ and $Y = X + K$ $K \in \mathbb{R}$ after one period. Let $I = I_x = I_y$ be the initial outlay for both $x$ and $y$ and let $i_x$ be the cost of capital for project $x$. Then the Net Present Value of project $y$ is simultaneously given by

$$NPV_y = -I + V_x + \frac{K}{1 + i_x}$$

and

$$NPV^y = -I + \frac{V}{1 + i_y} + \frac{K}{1 + r_f},$$

with $NPV_y \neq NPV^y$ (as long as $r_f \neq i_x$ and $K \neq 0$).

Proof. The NPV of $y$ is

$$NPV_y = -I + \frac{V}{1 + i_y}. \quad (15)$$

But from eq. (11) we have $\beta_y = \beta_x$ so that $i_y = i_x$ (Corollary 1). Hence,

$$NPV_y = -I + \frac{V}{1 + i_x} = -I + \frac{X + K}{1 + i_x} = -I + V_x + \frac{K}{1 + i_x}. \quad (16)$$
At the same time, $Y$ may be seen as a portfolio of project $x$ and an asset yielding the risk-free sum $K$. Then the NPV of $y$ must also be

$$\text{NPV}_Y = -I + \frac{X}{1+i_x} + \frac{K}{1+r_f} = -I + V_x + \frac{K}{1+r_f}$$

(17)

Lemma 1 shows that the NPV rule based on Definition 1 is self-contradictory (this is just my thesis A). But, as the following Proposition shows, the inconsistency is even stronger: Project $y$ may be arbitrarily determined by the decision maker himself!

**Proposition 6.** The Net Present Value of project $y$, whose payoff is $Y=X+K$, is any real number.

**Proof.** In general, we may write $Y = X+\alpha+(K-\alpha)$ where $\alpha$ is any real number. We may discount $X+\alpha$ at the rate $i_x=i_y$ (Corollary 1); the certain sum $(K-\alpha)$ is obviously discounted at the risk-free rate. This means that the NPV of $y$ is a function of $f(\alpha)$ such that

$$f(\alpha) = -I + \frac{X+\alpha}{1+i_x} + \frac{K-\alpha}{1+r_f}$$

(18)

Eq. (18) describes a monotonic function whose image is the set of all real numbers (as long as $i_x \neq r_f$).

It is worth noting that $f(K) = \text{NPV}_y$ and $f(0) = \text{NPV}_y$, so that Lemma 1 shows two NPVs among many infinite possible ones. As a result, the use of the NPV+CAPM methodology is invalid even in the simplest case, as the following Corollary shows:

**Corollary 2.** Let $x$ be any one-period project. The NPV of the project is arbitrarily chosen by the decision maker himself.

**Proof.** Choose an arbitrary real number, say $G$, and pick $K=0$ and $\alpha = f^{-1}(G)$ in (18).

**Remark 7.** The above result depends on the fact that the decision maker may always shape cash flows in the way he prefers. Any project $x$ with cost $I_x$ and final cash flow $X$ may be reinterpreted by the decision maker as a portfolio consisting of a project whose cost is $I_x$ and whose payoff is the random sum $X-\alpha$, plus an asset yielding the certain sum $\alpha$. The project paying $X-\alpha$ has the same beta as project $x$ (as the costs are equal), so the costs of capital coincide. The risk-free sum must be discounted at the risk-free rate for obvious reasons. Hence, the indeterminacy of the NPV, which changes as $\alpha$ changes, i.e. as the framing of the decision process is changed by the decision maker (see also Magni, 2002, section 4). This boils down to say that the NPV+CAPM methodology leaves decision makers subject to a framing bias (see Tversky and Kahneman, 1981; Kahneman, Slovic and Tversky, 1982; Kahneman and Tversky, 1984).

**Remark 8.** This instance of framing effect may be called the “additivity bias”. Finance heavily relies on the concept of value additivity or NPV additivity. Additivity is preserved if arbitrage theory is applied in valuing assets, but as eqs. (16) and (17) show, the CAPM does not comply with this principle:

$$V_{x+K} \neq V_x + V_K$$
or, in terms of eq. (18),
\[ V_{x-a} + V_a \neq V_{x-a+a} = V_x \]
which signals the absurdity of the evaluation, given the arbitrary value of \( \alpha \).

Let us now focus on the De Reyck's (2005) "clarifying example" (pp. 503–504) which, actually, clarifies the types of error he commits. At pp. 503–504 of his paper he considers two one-period projects paying off the sums \( \bar{x} \) and \( \bar{z} \) such that \( \bar{x} = \bar{z} + 20 \). In particular, \( \bar{x} \) pays off 20 or 40 with equal probabilities, \( \bar{z} \) releases 0 ad 20 with equal probabilities. I will henceforth denote with \( x \) and \( z \) these two projects. De Reyck assumes a cost of capital equal to \( i_x = 10\% \) so that he correctly writes that the value of \( x \) is \( V_x = \frac{30}{1.1} = 27.27 \).\(^5\) A main error is made when he writes that the returns generated by \( x \) are \( \frac{20}{V_x} - 1 = -26.66\% \) and \( \frac{40}{V_x} - 1 = 46.66\% \). We know that the (rates of) return generated by the project depend on cost, not on value. If De Reyck does not give us the cost of the project, how can he calculate the returns generated by the project? To De Reyck, the returns are \(-26.66\% \) and \(46.66\% \) regardless of the cost the investor pays to undertake the project!\(^6\)

Now, let us repair this major flaw assuming that the cost for \( x \) is \( I_x = 16 \). This means that the returns generated by \( x \) are \( \frac{20}{16} - 1 = 25\% \) and \( \frac{40}{16} - 1 = 150\% \). The NPV of \( x \) is \(-16 + 27.27 = 11.27 \). As for \( z \) De Reyck applies his Proposition to value it and then calculates its (rates of) return in terms of the value obtained (so committing two errors in one shot).\(^7\) But, again, returns depend on cost, not on value. Let us then assume that the cost of \( z \) is \( I_z = 16 \). Then, the returns of this project are simply \( \frac{30}{16} - 1 = 0\% \) and \( \frac{40}{16} - 1 = 25\% \). As for the value of \( z \), obviously, eq. (10) may not be applied, so the value \( 8.225 \) provided by De Reyck is wrong (the variances of the returns he shows us are also wrong, as they are functions of cost, not of value). To sum up, De Reyck thinks that a rate of return of a project is independent of cost and dependent of value, whereas it is the other way round: A project's rate of return is independent of value and dependent on cost.\(^8\)

Finally, I point out a further related fault in his example: He writes that "despite the fact that the variance of \( \bar{x} \) and \( \bar{z} \) are the same, their risk is different" (ibidem, p. 504). This statement is unwarranted because he does not specify the costs of the projects. And if costs are not known, neither are betas. As I have assumed \( I_x = I_z = 16 \), the two projects are such that the variances of the cash flows are the same, the variances of the rates of return are the same, and the betas are the same as well, so that the risk (in the sense of Definition 1) is the same:

\[
\beta_x = \frac{\text{cov}(r_x, r_m)}{\sigma_m^2} = \frac{\text{cov}(\bar{x}, r_m)}{\sigma_m^2} = \frac{\text{cov}(\bar{z} + 20, r_m)}{\sigma_m^2} = \frac{\text{cov}(\bar{z}, r_m)}{\sigma_m^2} = \frac{\text{cov}(r_x, r_m)}{\sigma_m^2} = \beta_z.
\]

\(^5\)It is worth noting that the (correct) valuation of project \( x \) in this example contradicts the (incorrect) valuations made in the proof of his Proposition (e.g. see Table 1 in this paper, line 1).

\(^6\)Even the wording De Reyck adopts in writing interestingly betrays his thinking. He defines \( \bar{x} \) as a payoff and then writes of "returns generated by \( \bar{x} \)" (p. 504, column left, line 3), instead of "returns generated by the project". Return of a payoff is nonsense, return always refers to an asset (projects, securities etc.) and one-period assets are univocally defined by two variables: Cost and payoff. Thus, De Reyck disregards cost even in his wording. But, true as it is, to speak of a payoff's return is to utter no word.

\(^7\)First error: Application of an incorrect Proposition. Second error: Neglect of cost in computing rate of return.

\(^8\)This is just the reason rates of return are called internal: They only depend on cash flows paid to and received from the project, that is on cash flows which are intrinsic to the project (the external financial milieu is not relevant, and value just depends on external variables).
As for the values of the two projects, it is not worth the trouble of valuing them. Corollary 2 allows you to choose whatever value you like.9

The final consequences are that the NPV is a contradictory and ambiguous notion and the CAPM is unreliable for capital budgeting purposes.

5 A methodological error

De Reyck’s paper is flawed from a methodological point of view as well. In his paper’s Abstract, after having correctly stated that I claim that “the so-called equivalent-risk tenet of finance ... is impossible to implement” he writes, at p. 499:

we show that the main thesis of the paper is incorrect.

In the Introduction (p. 599), he also writes:

we refute Magni’s claim that it is impossible to implement the equivalent-risk tenet of finance.

In the Conclusions (p. 504), he once again writes:

the claim made by Magni (2002) that the equivalent-risk tenet of finance is impossible to implement, is incorrect.

All these statements makes the reader think that he is concerned with my thesis B, according to which the ER tenet is inapplicable. In actual facts, De Reyck’s paper does not focus on thesis B. He focuses only on my paper’s sections 2, 3 and 4, where thesis A is coped with and where thesis B is not mentioned at all. The only part where thesis B is stated and shown to hold is the second part of my paper (Magni, 2002, p. 213 and following), whose arguments are independent of the first part’s results. In this way De Reyck misguides the reader and commits a significant methodological error: He faces a paper where two theses are presented (A and B), claims that he will disprove B but, in contrast, concentrates his efforts on thesis A presenting a (flawed) Proposition, and finally asserts that his conclusions invalidate thesis B.

We can then conclude by claiming that my thesis A remains unchallenged and is now reinforced by the results shown in this paper. As for my thesis B, it is evidently untouched as well, since De Reyck’s objections do not refer to it (see Magni, 2005, for a thorough treatment of thesis B).

6 Concluding Remarks

The use of CAPM for capital budgeting is standard in finance: A project is to be undertaken if its NPV discounted at the risk-adjusted rate of return is positive. The risk-adjusted rate of return (cost of capital) is the expected rate of return of an asset lying on the SML equivalent in risk to the project at hand. Equivalent in risk means that the project and the asset have the same beta.  

9This also means that although \( z \) pays 20 more than \( x \) in every state of nature and the cost is the same for both, we may arbitrarily decide that \( x \) is preferable than \( z \)!
In my 2002 paper I introduce two theses: (A) The NPV is self-contradictory, (B) it is impossible to apply the equivalent-risk tenet. This paper only deals with thesis A and reinforces it. In particular, it shows that

- the notions of risk and value implicit in the CAPM differs from that implicit in arbitrage theory
- the use of CAPM for capital budgeting is incompatible with the use of no-arbitrage arguments
- decision makers may frame a sum the way they prefer, which gives rise to different valuations for the same project. In other terms, they are trapped in a framing bias
- the additivity principle is not fulfilled
- the very notion of net present value is senseless, since the NPV of a project is anything one wants it to be.

While all these results have been directly proved, I have also directly disproved De Reyck’s Proposition, on which all his objections are grounded. Its attack to my position is fully biased for three evident reasons:

- in the assumptions of his Proposition he uses the notion of risk implicit in arbitrage theory, but in the proof he uses the notion of risk derived from the CAPM
- he mistakes expected rate of return for cost of capital
- he commits a methodological error: He claims his paper invalidates my thesis B but he completely disregards it and deals with my thesis A.

Indeed, we all may claim that to use NPV and CAPM for capital budgeting is not a good idea!

References


5. Paolo Bosi e Paolo Silvestri [1986] "La distribuzione per aree disciplinari dei fondi destinati ai Dipartimenti, Istituti e Centri dell'Università di Modena: una proposta di riforma", pp. 25
24. Fernando Vianello [1987] "Effective Demand and the Rate of Profit. Some Thoughts on Marx, Kalecki and Sraffa", pp. 41
27. Giovanni Procacci [1988] "The State and Social Control in Italy During the First World War", pp. 18
29. Maria Cristina Marcuzzo (a cura di) [1988] "Richard F. Kahn A disciple of Keynes", pp. 118
43. Giovanna Procacci [1989] "State coercion and worker solidarity in Italy (1915-1918): the moral and political content of social unrest", pp. 41
44. Carlo Alberto Magni [1989] "Reputazione e credibilità di una minaccia in un gioco bargaining", pp. 56
47. Paolo Bosi, Roberto Golinnelli, Anna Stagni [1989] "Le origini del debito pubblico e il costo della stabilizzazione", pp. 26
48. Roberto Golinnelli [1989] "Note sulla struttura e sull'impiego dei modelli macroeconomici", pp. 21
52. Paolo Silvestri [1989] "Il bilancio dello stato", pp. 34
54. Michele Lalla [1990] "The Aggregate Escape Rate Analysed through the Queuing Model", pp. 23
55. Paolo Silvestri [1990] "Sull'autonomia finanziaria dell'università", pp. 11
63. Andrea Ginzburg [1990] "Debito pubblico, teorie monetarie e tradizione civica nell'Inghilterra del Settecento", pp. 30
64. Mario Forni [1990] "Incertezza, informazione e mercati assicurativi: una rassegna", pp. 37
67. Paolo Bertolini [1990] "La situazione agro-alimentare nei paesi ad economia avanzata", pp. 20
74. Enrico Giovannetti [1990] "Illusioni ottiche negli andamenti delle grandezze distributive: la scala mobile e l’appattamento delle retribuzioni in una ricerca", pp. 120
77. Antonietta Bassetto e Costanza Torricelli [1990] "Una riqualificazione dell’approccio bargaining alla selezione di portafoglio", pp. 4
78. Antonietta Bassetto e Costanza Torricelli [1990] "Il portafoglio ottimo come soluzione di un gioco bargaining", pp. 15
80. Francesca Bergamini [1991] "Alcune considerazioni sulle soluzioni di un gioco bargaining", pp. 21
89. Maria Cristina Marcuzzo [1992] "La relazione salari-occupazione tra rigidità reale e rigidità nominali", pp. 30
90. Mario Biagioli [1992] "Employee financial participation in enterprise results in Italy", pp. 50
99. Marcello D’Amato e Barbara Pietrosetti [1994] "The relationship(s) among Wages, Prices, Unemployment and Productivity in Italy!", pp. 30
101. Barbara Pietrosetti [1994] "Using a VECM to characterise the relative importance of permanent and transitory components", pp. 28
102. Gian Paolo Castelli e Gabriele Pastrello [1994] "Polsh recovery: from the slump to an old dilemma", pp. 20
105. Giuseppe Marotta [1994] "Credit view and trade credit: evidence from Italy", pp. 20
118. Mario Forni e Marco Lippi [1995] “Permanent income, heterogeneity and the error correction mechanism.” pp. 21
140. Mauro Dell’Amico, R.J.M. Vanseven [1996] “Flow and open shop scheduling on two machines with transportation times and machine-independent processing times in NP-hard, pp. 10
145. Paolo Bertolini [1996] “La modernizzazione dell’agricoltura italiana e le cas e dell’Emilie Romagne” pp. 20
146. Enrico Giovannetti [1996] “Organizzazione industriale e sviluppo locali: le cas e dell’agroindustria in Emilie Romagne” pp. 18
147. Maria Elena Bonetti e Roberto Golinielli [1996] “Le determinanti del leverage delle imprese: una applicazione empirica ai settori industriali dell’economia italiana” pp. 31
161. David Lane [1996] “Is what is good for each best for all? Learning from others in the information contagion model” pp. 18
167. Marcello D’Amato e Barbara Pistoretti [1996] “So many Italian: Statistical Evidence on Regional Cohesion” pp. 31
173. Mauro Dell’Amico [1997] “A Linear Time Algorithm for Scheduling Outforests with Communication Delays or Two Processor-pp. 18
175. Paolo Bosi e Massimo Matteuzzi [1997] “Nuovi strumenti per l’assistenza sociale” pp. 31
176. Mauro Dell’Amico, Francesco Maffioli e Marco Trubian [1997] “new bounds for optimum traffic assignment in satellite communication” pp. 21
186. Mauro Dell’Amico e Francesco Maffioli [1997] “Combining Linear and Non-Linear Objectives in Spanning Tree Problem” pp. 21
203. Stefano Bordoni [1997] “Supporti Informatici per la Ricerca delle soluzioni di Problemi Decisionali” pp. 30
212. Alberto Roverato [1997] “Asymptotic prior to posterior analysis for graphical gaussian models” pp. 8

Andrea Ginzburg e Antonio Ribba [2001] “Vizi e virtù del monetarismo democratico: un promemoria per il futuro” pp. 31


Carlo Mazzaferrro e Stefano Toso [2001] “La spesa per previdenza ed assistenza: riforme in corso e nuovi scenari” pp. 16

Silvia Giannini e Maria Cecilia Guerra [2001] “Requiem per la riforma Visco” pp. 25

Andrea Francalanci e Stefano Toso [2001] “Spera sociale e meccanismi di mercato: i buoni servizi (vouchers)” pp. 25

Maria Elena Boncompini, Silvia Giannini, Maria Cecilia Guerra e Angela Tiraferrì [2001] “Incentivi agli investimenti e tassazione del reddito di impresa: una valutazione delle recenti innovazioni normative” pp. 33

Marina Mura [2001] “Growth, Trade and Unemployment” pp. 34

Tindara Addabbo F. Olivier [2001] “Offerta di lavoro e servizi all’uomo in Italia” pp. 23


Luigi Brighi e Marcello D’Amato [2001] “Two-Dimensional Screening: A Case of Monopoly Regulation” pp. 20


Enrico Giovannetti [2001] “La divisione del lavoro è limitata dalla divisione del lavoro” pp. 26

Paola Bertolini, Michele Bruni e Enrico Giovannetti [2001] “Struttura produttiva e mercato del lavoro nell’agroindustria: evoluzione tecnologica e bisogni formativi” pp. 174


Paola Bertolini e Montanari Marco [2001] “Valutazione dell’allargamento dell’unione europea ad Est attraverso un modello gravitazionale” pp. 20


Carlo Mazzaferrro [2001] “Uno schema per la valutazione del trattamento fiscale del risparmio pensionistico” pp. 16


Anna Maria Sala [2001] “Marchio di qualità e servizi turistici” pp. 33

Michele Lalla [2001] “Struttura e cambiamento nelle relazioni tra le imprese metalmeccaniche nella provincia di Modena. Il Distributions degli addetti e posizioni per le stime dei parametri” pp. 24


Elena Pirani e Margherita Russo [2001] “Struttura e cambiamento nelle relazioni tra le imprese metalmeccaniche nella provincia di Modena III. Aspetti metodologici dell’indagine empirica: fase di rilavazione, controlli e statistiche preliminari” pp. 52


Margherita Russo e Elena Pirani [2001] “Struttura e dinamica dei cambiamenti nelle relazioni tra le imprese metalmeccaniche in provincia di Modena. IV Primi risultati dell’indagine empirica” pp. 88

Giovanni Salinas [2002] “La certificazione come strumento per la politica industriale. L’esperienza dell’Emilia Romagna” pp. 28


Paolo Bosi e M. Cecilia Guerra [2002] “The Role of Tax Incentives in Voluntary Pension Schemes in Italy: what can other Countries learn from this?” pp. 23


Bertella Farnetti [2002] “George Keenan e la divisione dell’Europa dopo la seconda guerra mondiale” pp. 111


Antonella Picchio [2002] “Pieno, carote, panne e rose: salario netto e di sostituzione nelle carte dell’archivio di Stato” pp. 45


Massimo Gatti e Costanza Torricelli [2002] “Quanto reale è il potere delle opzioni reali? Le imprese Tnt e il caso Tiscali” pp. 30


Giovanni Mottura [2002] “NON SOLO BRACCIA: Condizioni di lavoro e percorsi di inserimento sociale degli immigrati in un’area ad economia diffusa” pp. 165

Alberto Rinaldi [2002] “The Emilian Model Revisited: Twenty Years After” pp. 28

Anna Maria Sala [2002] “Marchio di qualità e servizi turistici. L’offerta alberghiera” pp. 69

Carlo Alberto Magni [2002] “Antinomie e illusioni cognitive nel criterio del valore attuale” pp. 28


Nicola Walter Palmeiri [2002] “Diritto della comunicazione e dell’informazione” pp. 280

Paolo Bertella Farnetti [2002] “Condizioni del Paese e la Lotta per gli Stati Uniti d’Europa” pp. 83

476 Anna Maria Sala [2004] “Sistema urbano e dinamiche inediativi Una verifica empirica” pp. 83
477 Paolo Bosi [2005] “Paradigmi economici e riforma dei welfare nelle politiche europee” pp. 16
478 Claudio Marra [2005] “Percorsi, aspettative e valutazioni nell’esperienza lavorativa degli immigrati stranieri in Emilia Romagna: i casi di Modena e Reggio Emilia” pp 39
479 Paolo Bosi, M.Cecilia Guerra, Paolo Silvestri [2005] “Il finanziamento dei servizi per la non autosufficienza nel quadro della riforma del Titolo V” pp. 40
482 Giada Luca Di Lorenzo e Giuseppe Mareta [2005] “Una politica monetaria meno efficace con l’UME? Evidenza dal passthrough nei tassi d’interesse attivi” pp. 28
485 Margherita Russo e Federica Ronsi [2005] “Stimolare l’innovazione con strumenti innovativi: reti di partenariato e sviluppo locale nei programmi comunitari” pp. 32
486 Luigi Brighi e Reinhard John [2005] “A Hypothesis Guaranteeing the Weak Weak Axiom” pp. 11
487 Massimo Baldini e Luca Beltrametti [2005] “Modelli di finanziamento di un fondo pubblico per la non autosufficienza” pp. 26
488 Michele Lalla e Sandra Paterlini [2005] “Duration Models and Differential Evolution in the Analysis of Large Data Sets” pp. 29
491 Vittorio Morriga, Silvia Mussioli, Costanza Torricelli [2005] “The no Arbitrage Condition in Option Implied Trees: Evidence From The Italian Index Options Market” pp. n. 29
492 Gisella Facchinetti e Pacchiarotti Nicoletta [2005] “Economic principle on fuzzy profit by weighted average value” pp. 10
493 Massimo Baldini, Paolo Bosi, Maria Cecilia Guerra e Paolo Silvestri [2005] “L’impatto distributivo dei tributi locali: un’applicazione sul Comune di Modena” pp. 25
494 Andrea Giuntini [2005] “Boom e infrastrutture a Modena. La Camera di Commercio e l’autostrada del Brennero” pp. 30