The Put-Call Parity in the Index Options Markets

Further results for the Italian Mib30 Options market

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Abstract

The birth and success of index option markets have fostered empirical research on their efficiency. While most of the literature focuses on North American markets, studies on European markets are still limited. The aim of the present paper is to provide further evidence on a European market, the Italian index option market (MibO), by testing the validity of the most famous no-arbitrage relationship in the option markets: the Put-Call parity (PCP). The growth of the market, new facts (such as the transition to the Euro and new market rules) and the availability of a broader and better quality high frequency data set make our work different from the previous study on the same market by Cavallo and Mammola(2000). Our analysis highlights the role of frictions in the tests of the PCP and points at a substantial and increased efficiency of the Italian index option market.

Keywords: index options, market efficiency, put-call parity

JEL Classification: G13, G14

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1. **Introduction**

Efficiency is of uttermost importance for the functioning and the development of financial markets and is commonly tested by means of the no-arbitrage relationships that must hold among financial assets. Among the latter, the Put Call Parity (PCP) particularly lends itself to the empirical investigation of the cross-market (option and stock) efficiency.

The PCP is a no-arbitrage relationship that must hold between the prices of a European call and a European put written on the same underlying and having the same strike and time to expiration. Since the seminal paper by Stoll(1969) the relationship has been extended in many directions and widely tested, especially for US stock option markets[1]. The introduction and success of index options both in the US and in Europe have called for attention of empirical research to these markets. While most of the literature on index options has focused on US markets (e.g. Ackert and Tian(2001), Evnine and Rudd(1985), Kamara and Miller(1995)), since the mid 90s a few contributions have investigated the validity of the PCP in some relatively new European index option markets. As far as we know, only a few recent papers[2], propose efficiency tests on European markets and specifically: Capelle-Blancard and Chaudhury (2001) for the French index (CAC40) option market, Mittnik and Rieken(2000) for the German index (DAX) option German and Cavallo and Mammola (2000) for Italian index (Mib30) option market.

The empirical literature on the efficiency of European option markets is not only still limited in number of contributions, but also relates – except for the work of Capelle-Blancard and Chaudhury (2001) – on a pre-Euro period, which normally represents the infancy of the index option market under investigation.[3]

The aim of the present paper is to provide further results on the efficiency of the Italian Mib30 index option (MibO) market. Many changes with respect to the period analysed by Cavallo and Mammola(2000) make a new investigation of this market interesting. On one hand, the market has experienced the introduction of the Euro and some new market rules that may have affected the market efficiency. On the other, the growth of the market, which was substantial until 2001, allows to empirically test efficiency on a wider range of options as opposed to the previous

[1] Two are essentially the types of extensions proposed in the literature: the first type implies modifications in the equation representing the PCP to account for, e.g., dividends and transaction costs; the second one accounts for the early exercise and transforms the equality representing the PCP into an inequality condition that holds for American-type of options. Among the many papers that contributed in the two above-mentioned directions by also performing empirical tests of the different relationships, see: Merton(1973), Gould and Galai(1974), Bhattacharya(1983), Klemkosky and Resnick(1979, 1980) and Nisbet(1992) on the London Traded Option Market.

[2] Earlier works include Chesney et al.(1995) tests on the Swiss option index market and Puttonen(1993) on the Finnish option index market.

[3] The Dax index option was introduced in August 1991 and the analysis by Mittnik and Rieken(2000) relates to the period February 1992-September 1995. The MIB30 index option was introduced in 1995 and the analysis by Cavallo and Mammola(2000) relates to the period July 1996-February 1997.

study where only one-month at the money options are considered. This, coupled with the period analysed, also allows the authors to bypass the dividend problem. Moreover our study is conducted by means of high frequency data, which allow the high synchronicity fundamental for the tests of the arbitrage strategy.[4] Eventually, by considering a more recent period, one can see whether there has been an increase in efficiency or a decrease.

The paper is organised as follows. In section 2 the methodology and the literature on market efficiency are reviewed. Section 3 discusses the issues involved in the construction of the high frequency data set used in our work. The efficiency tests and the results for the MibO market are presented in section 4. Last section concludes.

2. **Tests of efficiency: literature and methodology**

In this section, we review the methodology used in the empirical literature testing the efficiency of index options markets by focussing on studies for the relatively new European markets. In particular, we will focus on the empirical tests of efficiency which are disjoint from any test of validity of the pricing model, i.e. tests based on given stock and option market prices and aimed at checking the violations of no-arbitrage relationships among these prices. More specifically, we will restrict our attention to the PCP, which is the most famous condition of cross market efficiency between the underling and the option market.

Among the most recent, efficiency tests are proposed by Capelle-Blancard and Chaudhury (2001) for the French index (CAC40) option market, Mittnik and Rieken(2000) for the German index (DAX) option and Cavallo and Mammola (2000) for Italian index (MIB30) option market.

In order to review the methodology used in this latter literature and to throw light on the choices made in the present paper, it is worth recalling the PCP condition for index options.[5] In its basic version, i.e. under the assumptions that the markets are frictionless, the underlying pays no dividends and the lending and borrowing rates are equal, the PCP implies that, in an efficient market, the following equality must hold:

$$c + Xe^{-rT} = p + I_0 \qquad (1)$$

where:

I_0 = current level of the underlying index;

X = strike price;

[4] Options and index data were kindly provided by Borsa Italia S.p.A..

[5] Although, to the end of the present paper, we only review possible tests of the PCP, it should be mentioned that the papers cited also test different types of arbitrage restriction (such as lower boundary conditions and various types of arbitrage strategies).

T = time to expiration of the option;

r = risk-free interest rate;

c = call price*;*

p = put price.

By contrast, if the market is inefficient, one can gain arbitrage profits by means of either a *long (conversion)* or a *short (reversion)* strategy depending on whether the overvalued option is a call or a put respectively. Therefore, efficiency tests based on the PCP essentially imply testing one of the following:

$$long \quad c + Xe^{-rt} - p - I_0 \geq 0 \tag{2}$$

$$short \quad p + I_0 - c - Xe^{-rt} \geq 0 \tag{3}$$

If in theory the issue is quite simple, in practice many are the problems to be tackled when performing a test with real market data. In the following, we will discuss each of these problems, that are at the same time causes of violations of the PCP as it stands in the above expressions, and we will sum up how they have been handled in the most recent literature.

i. Synchronicity of the data. The conversion or reversion arbitrage strategies recalled above imply synchronous trading of the options and the index. Hence, the option prices and the index value to be used in the empirical tests have to be observed at exactly the same time. This has led the empirical research to improve on the data set quality, which has become possible given the availability of high frequency prices that ensure a high level of synchronisation between the option prices and the index. For instance, Capelle-Blancard and Chaudhury(2001) use a sample where prices are required to be within one minute of each other, while Cavallo and Mammola (2000) use infra-day prices captured every fifteen minutes. The tests based on the assumption of synchronous trading are normally addressed to as ex-post tests. Capelle-Blancard and Chaudhury(2001) and Mittnik and Rieken(2000) also perform ex-ante tests to check whether some violations can be attributed to the impossibility of synchronous trading.

ii. Replication of the underlying. In order to implement an arbitrage strategy a (long or short) position in the underlying has to be taken. However, given that the underlying of index options is a basket of stocks, the implementation of the arbitrage strategy is less straightforward than for single stocks and, in principle, it could be practically unfeasible. This impossibility can stem either from short sales constraints or from the multiplicity of stocks in the index. Empirical works on European markets normally tackle only the former problem given that the index under analysis consists of a relatively small number of stocks (30 for the DAX and the Mib30 and 40 for the CAC40). As for the short sale

constraints, they are either accounted for or they do not characterise the market (as it is, for instance, in the Italian case). The alternative to the replication of the index is taking a position in the corresponding index future, but this would essentially introduce basis risk. Moreover, in most markets only a few number of stocks can account for most of the index value. For all these reasons, all the papers recalled at the beginning of this section use transaction data on the underlying index and eventually adjust it for transaction costs, as discussed next.

iii. Transaction costs. Transaction costs are basically represented by commissions on transactions and bid-ask spreads on the relevant prices[6]. They are the real critical point in the studies on the efficiency of option markets in that, most of the studies cited in this paper - both on stock and on index options – essentially conclude that arbitrage opportunities are in practice swept away by the costs involved in the arbitrage strategies. Given that both commissions and the bid-ask spreads are very specific to the market investigated, the existing papers take different assumptions about them. As for the commissions, to make the arbitrage strategy realistic, one has to consider both the commissions in the option market and the cost of replicating the index. As opposed to those analyses based on a single scenario (e.g. Mittnik and Rieken(2000)), some authors stress the multiplicity of transaction costs and prefer to work with two or more scenarios (e.g. Cavallo and Mammola(2000)) and Capelle-Blancard and Chaudhury(2001)). Each scenario essentially represents a combination of agent type (professional arbitrageurs vs. retail investors) and market liquidity (zero bid-ask spread in liquid markets vs. positive bid-ask spreads in markets displaying some sort of illiquidity). It should be noted that, while commission costs are difficult to set but in any case observable, bid-ask spreads are still observable but normally there is no data set available. This implies the need to resort to estimates based on subjective assumptions that will be discussed in the next section.

iv. Risk-free interest rate. As for the choice of the interest rate to be plugged in (2) o (3), most authors resort to interbank offer rates (Cavallo and Mammola(2000)) and Capelle-Blancard and Chaudhury(2001)) eventually adjusted for the bid-ask spread or to interbank bid and offer rates (Mittnik and Rieken(2000)). The choice of the interest rate to be used in checking arbitrage relationships can also be used to account for short selling costs. For example, Cavallo and Mammola(2000) suggest the use of a repo rate in

[6] To be more precise one should also include in the transaction costs other types of costs (e.g. clearing fees, short selling costs). However in most studies they can be considered negligible. The issue of short selling costs will be taken up again when discussing the risk-free interest rate and in the next section.

checking the short strategy because *"This allows accounting for the assumption that the cost of trading will be financed by the funds deposited with the lender of the securities"*. However it should be noted that in recent periods, interbank offer rates and repo rates do not significantly differ and therefore abstracting from this latter issue is quite acceptable.

v. Dividends. It is well known that the PCP condition has to be adjusted when the underlying stock pays a dividend during the life time of the option. If the underlying is an index, two are the possible cases. The index itself is a performance index adjusted for dividends and no modification in (2) and (3) is needed in this respect: an example is the case of the German DAX considered by Mittnik and Rieken (2000). By contrast, if the index is not adjusted for dividends they have to be estimated and considered in the arbitrage strategy. For example, Capelle-Blancard and Chaudhury(2001) assume that traders have perfect foresight of the dividends paid on the CAC40 index. Cavallo and Mammola(2000), by considering only one-month options, do not observe any dividend payment during the option life and do not tackle the problem.

The points just made can be formally included in (2) and (3) in order to make the PCP a realistic and testable relationship:

$$\left(c^{ask} + TC^{c}\right) - \left(p^{bid} - TC^{p}\right) + Xe^{-rT} - \left(g\left(I_0^{\,bid} - D\right) - TC^{s}\right) \geq 0 \qquad (4)$$

$$\left(p^{ask} + TC^{p}\right) - \left(c^{bid} - TC^{c}\right) + \left(I_0^{\,ask} - D + TC^{s}\right) - Xe^{-rT} \geq 0 \qquad (5)$$

where:

$I_0^{\,bid/ask}$ = *bid*/*ask* index price;

$c/p^{\,ask}$ = *ask* call/put price;

$c/p^{\,bid}$ = *bid* call/put price;

D = present value of the dividends paid on the index up to time T;

TC^{c} = call transaction costs;

TC^{p} = put transaction costs;

TC^{S} =index transaction costs;

g = percentage of short sales allowed.

The overall finding of the studies cited so far is that arbitrage violations of the PCP normally disappear - or at most display a limited persistence - after the inclusion of frictions. In particular, for the French market, Capelle-Blanchard and Chaudhury(2001) find that, in the period 2 January 1997 – 30 December 1999, only 1.26% (0.04%) of the short (long) arbitrage strategies are profitable for retail investors while 8% (4%) of the short (long) arbitrage strategies are profitable for institutional traders. As for the Italian market, Cavallo and Mammola(2000) find that, in the period 26 July 1996

– 18 February 1997, 2% of both the short and the long strategies are profitable for individual investors whereas 5% (6%) of the short (long) strategies are profitable for arbitrageurs. As for the German market, the methodology used by Mittnik and Rieken(2000) is not directly comparable with the one used by the other authors since, instead of testing profitability of the short (long) arbitrage strategies, they perform a regression analysis based on the PCP. They also find that arbitrage opportunities diminish dramatically when transaction costs are accounted for and in general, in line with the French study, they find that "*arbitrage restrictions, which rely on short selling of the component stocks, tend to be violated more severely than those relying on long positions in these stocks*". The authors attribute this latter finding to the existence of short selling restrictions in the German market.

In sum, findings for the French, German and Italian market are only partially consistent with the mixed findings for the North American and UK ones (e.g. Klemkosky and Resnick(1979) and Nisbet(1992) report compliance, while Evnine and Rudd(1985) and Kamara and Miller(1995) report evidence of mispricing).

3. **The data set**

To perform our analysis we use data on the underlying, the Mib30, and the MibO contracts. As for the choice of the underlying, we use the index instead of the index futures for two main reasons: i. This choice is in line with most of the reference literature; ii. In the Italian case, 8 stocks represent more that 70% of the Mib30 in the period under investigation[7] and hence portfolio replication is quite easily attainable.

In the remaining of this section we recall the main features of the MibO and we illustrate how we have constructed a dataset suitable for the PCP tests.

The MibO contract was introduced in the Italian Derivatives Market (IDEM) in November 1995, just one year after the creation of this market. The MibO is a European-style index option contract based on one of the most representative Italian indexes, the Mib30. It is quoted in index points, each worth 2.5 €, and has to respect some rules relative to the spread between the bid and ask quotations[8]. Every day six different expirations are quoted: four quarterly (March, June, September and December) and two monthly (the nearest two months). The expiration day is the third Friday of the expiration month, if the Exchange is open, the previous day of open Exchange otherwise. At expiration in the money options are automatically exercised. The exercise prices have fixed increments of 500 index points and every day at least nine different strikes for each expiration are quoted: one at the money, four in and four out of the money. The cash settlement of the options is

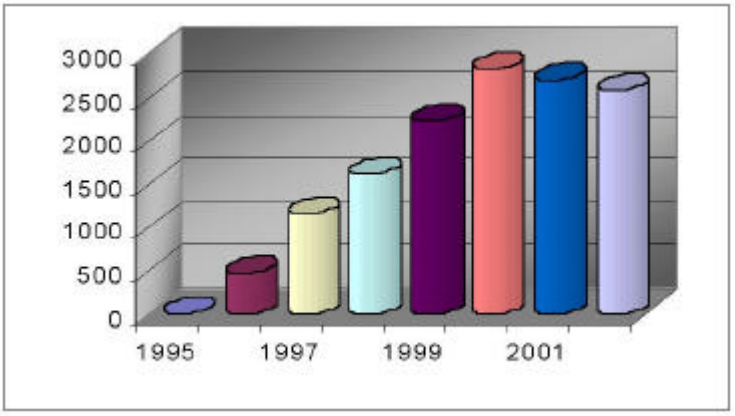
[7] More precisely, BCI, Eni, Enel, Generali, STM, Tim, Telecom and Unicredito represent 70.72% of Mib30.
[8] Further details on the contract specifications can be found in www.borsaitalia.it.

overseen by the Italian Clearing House, Cassa di Compensazione e Garanzia (CC&G), which also calculates and manages the margins. By now, no limits are provided for open interests and price changes during the negotiation time (9:15-17:40).

From its birth in 1995 up to date, the volume of MibO contracts negotiated has significantly increased, although a slight reverse in the trend is observable as from 2001 (see Figure 1).

Figure 1

MibO contracts traded every year, from 1995 to 2002.



However, the notional value of the MibO contracts exchanged every year is still very important, even bigger than that of the Italian option contracts on single stocks, Isoα (see Figures 2 and 3).

Moreover, even though the IDEM is a relatively young market, it has become the fifth derivatives market in Europe (after Liffe, DTB, Monep and Dutch Eurex), as shown in Figure 4.

Figure 2

Isoα and MibO contracts per year: volumes

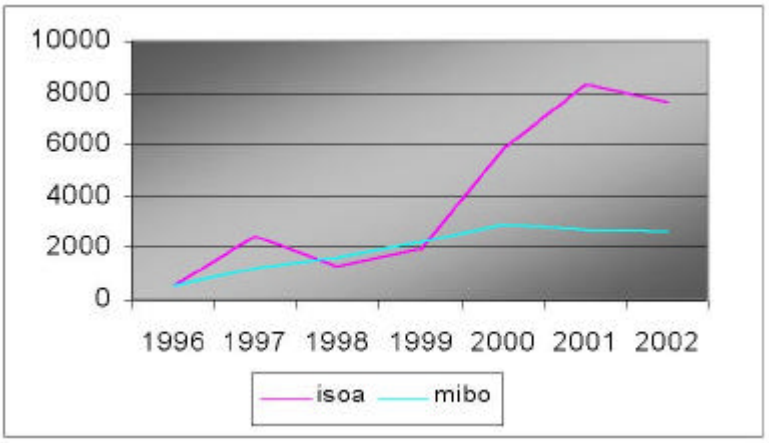


Figure 3

Isoa and MibO contracts per year: notional values

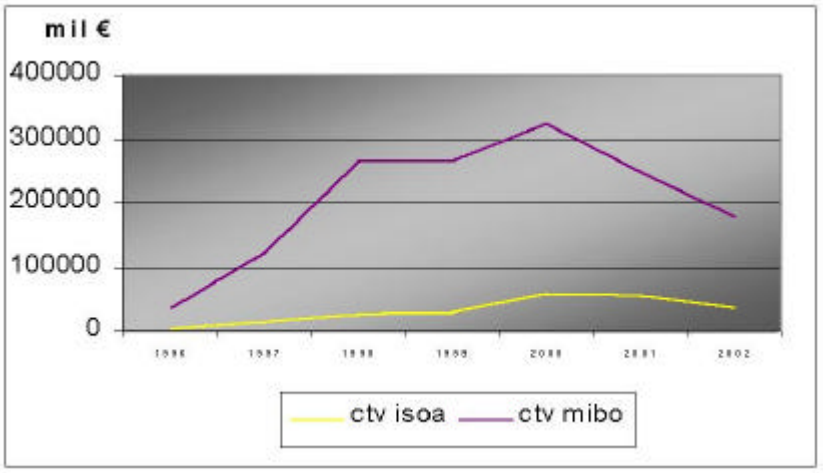
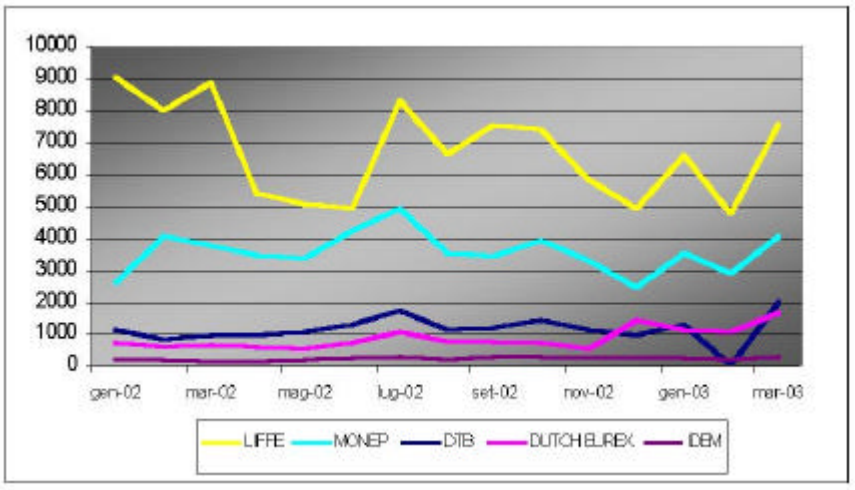


Figure 4

The five biggest index options market in Europe



Given its relevance in both the Italian and the European market, it is important to empirically test its efficiency, to assess whether and how it has evolved since its birth and to compare it with other markets, in particular the European ones.

To this end, we used intra-day high frequency data relative to the period 1 September 2002 – 31 December 2002. For each month, we have two different data sets: one including every minute Mib30 quotations and one indicating, for all the MibOs exchanged during that month, negotiation

data, trading hour, clearing hour[9], index point option price and option name, indicative of option type, expiration date and exercise price.

PCP holds only for couples of put/call options with identical maturity and strike and traded in the same instant. Therefore, in order to retain only reliable and informative data to test the PCP, we have to apply some filters to the original data set. Specifically, the data set has to satisfy the following requirements, which are discussed in the rest of this section:

1. prices synchronicity,
2. maturity and strikes matching,
3. index adjustments for dividends,
4. estimation of the transaction costs.

1. Prices synchronicity

As far as synchronicity is concerned, we follow Mittnik and Rieken (2000) and Capelle-Blancard and Chaudhury (2001) by retaining only those put/call options that were traded consequently and within 60 seconds. Then, we match each couple with the same minute Mib30 quotation[10]. In sum, we impose all prices in a given arbitrage condition to be within the same minute.

2. Maturity and strikes matching

As for the maturity and strike matching, we remove all couples of options characterised by different strike and expiration date, which implies keeping just 6.272% of the 229070 original observations.

As a proxy for the risk-free interest rate, we use the Euribor (1, 3, 6 and 12 months consistently with options maturity, source Datastream), both for comparability with other studies and because alternative choices (e.g. an IRS rate) would not affect results, given that these types of rates are not significantly different.

Moreover, note that risk free interest rates are, in the period under investigation, very similar to repo interest rates. On the basis of this latter observation we assume the same type of rate when testing both the long and the short strategy.[11]

3. Index adjustments for dividends

Since the Mib30 is not a performance-based index, we adjust it for dividends, implicitly assuming investors to know in advance future dividends, given that every year Borsa Italiana releases a

[9] The difference between trading and clearing hour, respectively the moment in which investors order the trade and the one in which CC&G closes their positions, is negligible (less than one second in 99.11% of the whole observations). Therefore, we considered only trading hours as time references.

[10] Given the discrepancy between options and index markets trading hours (9:15 – 17:40 vs 9:30 – 17:25), we attributed to those options traded before 9:30 and after 17:25 the last index prices of the previous and current day respectively.

[11] If the two types of rates were significantly different, it would be more sensible to use a repo rate when evaluating the short strategy so that the short selling cost could be implicitly accounted for (see e.g. Cavallo and Mammola (2000)).

prospect indicating the date and the amount of expected dividends of the Mib30 stocks.[12] In particular, only three stocks paid dividends over the time spanning the options maturity: Mediobanca, ordinary dividend on 18th November, Tim and Tiscali, extraordinary dividend on 16th December 2002. Given that the Mib30 is actually adjusted for extraordinary dividends (see IDEMagazine n.7, January 2003, available on Borsa Italiana web site), the only relevant dividend adjustment is the Mediobanca one. This may also explain why, in most cases, the adjustment results to be very small (the average Mib30 adjusted/Mib30 ratio came out to be 0.9868), even though for options with longer maturities the incidence is obviously bigger.

4. Estimation of transaction costs

Taking transaction costs into account is fundamental when empirically investigating PCP. Unfortunately, they are very difficult to estimate, both because there are many components to consider (commissions, trading and clearing fees, costs deriving from bid and ask prices, short-selling costs etc.) and because they all tend to vary over time, depending on the kind of strategy, on the size of transactions and on the investors type (e.g. retails vs arbitrageurs). However, in the Italian market, clearing fees are negligible (see also Cavallo and Mammola (2000)). The same is true for short-selling costs, which are also negligible given the current scenario where repo and risk-free interests rates are very low and similar. Therefore, as far as transaction costs are concerned, we decided to focus just on commission costs and the costs deriving from the bid-ask spread.

By inspection of options trade commissions on the IDEM, we noticed that the Italian option market seems remarkably diversified. Commissions depend on the type of investors as well as on the means of trade: for example, arbitrageurs usually face low commissions because of the high yearly volume of transactions they realize, even though retail investors who implement trading on line can obtain low commissions too. On the basis of this latter observation, we carry our empirical study of PCP assuming four different commissions levels, which we attribute to four different types of traders:

1. MINIMUM, equal to 1 € for option traded, which means to represent arbitrageurs who realize yearly high volume of transactions;
2. MEDIUM-LOW, equal to 10 €, which means to represent professional investors with low volume of transactions or particularly active retail investors.
3. MEDIUM-HIGH, equal to 25 €, which means to represent retail investors who trade on line
4. HIGH, equal to 40 € for option traded, which means to represent retail investors who occasionally trade options.

[12] Since we know the dividend per stock, we first compute the dividend amount for the whole Mib30 multiplying it for the number of stocks required for the replication, then we actualise it and finally we subtract it from the original Mib30 quotation. Clearly, we use these "corrected" quotations just for options traded before the dividend payment but whose maturity was after that date.

Unfortunately, bid and ask quotations are not available either for the options or for the underlying index, so we have to estimate them.

As for the options, we estimate an average bid-ask spread on the basis of a sample of bid and ask quotations (as suggested by Phillips and Smith (1980)) and we assume it constant over time, as it is common in literature (see also Capelle-Blancard and Chaudhury (2001)). To this aim, we use the bid and ask option quotations available on the Finance section of www.yahoo.com each trading day of IDEM, from 3 February to 7 March 2003[13]. The mean bid (ask) price, which resulted 0.923 (1.062) of the trading price, multiplied by the trading prices provides us with an estimated bid (ask) options quotations. Two observations are in order. First, since the real bid and ask prices are not available, we have to use estimated values for the spread, and this may inevitably introduce some bias in our results, although the values we obtain fall within the bounds imposed to the market makers by Borsa Italiana (see www.borsaitalia.it). Second, the values obtained support the asymmetry of the bid and ask spread described in recent studies (e.g. Nordèn (2002, forthcoming), Chan and Chung (1999)) and this will be relevant in interpreting our results.

As for the Mib30 bid and ask quotations, by performing an analogous estimation, the spread turned out to be negligible and hence we ignore it.[14]

4. **Empirical results for the MIBO**

In this section we present the empirical results obtained by performing ex-post tests of the PCP in the IDEM. In order to better emphasize the role of market frictions in absorbing the arbitrage opportunities, we will present our results under three different scenarios: scenario A, in which we assume a transaction cost-free market; scenario B, in which we include only the costs deriving from the option bid-ask spread and finally scenario C, in which we take into account the bid-ask costs as well as the commission costs.

Scenario A

In this scenario the profitability of the following strategies is tested:

Long $c + Xe^{-r\tau} - p - (I - D)$

Short $p + (I - D) - c - Xe^{-r\tau}$

The results obtained in this scenario are reported in Tables 1 and 2. Four comments are here in order. First, we have reported only results for positive profits since the two strategies are perfectly symmetrical and the number of profitable long strategies is identical to the number of short strategies that have negative profits.

[13] Even though this period does not match with the one under investigation, we can assume that the average options bid-ask spread has not remarkably changed, since we assume it constant.

[14] Details and data on the estimated Mib30 bid-ask spread are available upon request.

Second, Table 1 shows that the short strategy is always more profitable than the long one. In fact, on average over the four months, the short (long) strategy is profitable in 57.84% (42.16%) of the cases. These values are very close to the ones reported by Capelle-Blancard and Chaudhury (2001) and by Ackert and Tian (2001), who in a similar scenario reported a frequency of short (long) strategies profitability equal to 58% (42%) and to 52% (38%), respectively for the French and the US option markets. The bigger profitability of the short strategy denotes a tendency to overvaluate put options with respect to call.

Third, in this scenario the frequency of PCP violations is substantial, as it was to be expected. Obviously, this is not a sufficient condition to conclude that the Italian option market is not efficient in pricing the MibO contracts, as we have not taken any transaction cost into account.

Table 1 : Frequency of PCP violations in scenario A, by month [a].

No commissions - No *bid*/*ask*		
Month	**Strategy**	
	Long	*Short*
September	1030 **(41.96%)**	1425 **(58.04%)**
October	737 **(48.87%)**	771 **(51.13%)**
November	520 **(37.55%)**	865 **(62.45%)**
December	742 **(40.41%)**	1094 **(59.59%)**
Whole sample	3029 **(42.16%)**	4155 **(57.84%)**

a = The table reports the number and the percentage (in parenthesis) of PCP violations registered in each month analysed. We do not find cases of perfectly null profit.

Finally, even though the violations of PCP are very frequent in this scenario, in nearly all cases the size of the violations is not very remarkable. Table 2 shows that the average long profit is 60.38 €, while the average short profit amounts to 62.39 €, so that the short strategy is slightly more profitable than long one not only as far as frequency is concerned, but also as far as the amount is concerned. Moreover, only very few cases over the whole sample display profits bigger than 200 € and, as we will show later, these cases of exceptional returns are usually due to particular instances of MibO contracts, such as very long maturity or particular trading hour (e.g. the case of profit bigger than 2400 € is realized trading the only options with one year maturity).

Table 2: Descriptive statistics for long and short strategies profits, scenario A [a].

LONG						SHORT				
Included observations: 3029						Included observations: 4155				
	Mean	Max	Min.	Obs.	%	Mean	Max	Min.	Obs.	%
[0, 200)	49.13	198.08	0.012	2909	96.04%	56.72	198.93	0.011	4061	97.74%
[200, 400)	253.96	399.57	200.29	91	3.00%	256.96	398.35	200.06	78	1.88%
[400, 600)	457.03	570.08	403.34	23	0.76%	510.31	597.91	406.54	12	0.29%
[600, 800)	648.50	690.96	627.27	3	0.1%	689.44	745.59	638.66	4	0.09%
[800, 1000)	904.74	979.33	830.15	2	0.06%	-	-	-	-	-
[2400, 2600)	2598.67	2598.67	2598.67	1	0.03%	-	-	-	-	-
All	60.38	2598.67	0.012	3029	100.00%	62.39	745.59	0.011	4155	100.00%

a = This table reports for each strategy (long and short) and each profit range: the mean, the maximum, the minimum, the number of observations and the percentage of the profit range over the whole sample. Monetary amounts are in €.

Scenario B

Scenario B includes the costs associated with the bid-ask spread. Specifically, given that the bid-ask spread on the underlying turned out to be negligible, we only accounted for the bid-ask spread on the option prices, which also in literature is referred to as the most important among the implicit transaction costs (e.g. Demsetz (1968), Phillips and Smith (1980) and Stoll (1989)). Therefore, in scenario B the profitability of the following strategies is tested:

Long $c^{bid} + Xe^{-r\tau} - p^{ask} - (I - D)$

Short $p^{bid} + (I - D) - c^{ask} - Xe^{-r\tau}$

The results relative to this scenario are reported in Tables 3 and 4.

Since we lose the symmetric nature of the two strategies, Table 3 reports the results both for the long and the short strategy. This feature is due to the asymmetric bid-ask spread included in this scenario which makes the two strategies asymmetric.

This table shows that the only inclusion of the costs deriving from the option bid-ask spread significantly diminishes the arbitrage opportunities in the IDEM, which drop from 42.16% (57.84%) to 2.43% (1.63%) for the long (short) strategy over the whole sample.

Moreover, in contrast with scenario A, the higher and more frequent profitability of the short strategy is no longer true. This is likely to be a consequence of the asymmetric nature of the bid-ask

spread introduced in this scenario. More precisely, the option transaction prices we use are on average closer to the ask than to the bid quotations. This, together with put prices being on average bigger than call prices[15], may make long strategies more profitable than the short ones. However, the difference is not so remarkable, except for October.

Table 3: Frequency of PCP violations in scenario B, by month [a].

No commissions - Including *bid*/*ask*			
	Profits	**Strategy**	
		Long	*Short*
September	Negative	2415 (98.37%)	2410 (98.17%)
	Positive	40 **(1.63%)**	45 **(1.83%)**
	Total	2455 (100.00%)	2455 (100.00%)
October	Negative	1459 (96.75%)	1498 (99.34%)
	Positive	49 **(3.25%)**	10 **(0.66%)**
	Total	1508 (100.00%)	1508 (100.00%)
November	Negative	1365 (98.56%)	1368 (98.77%)
	Positive	20 **(1.44%)**	17 **(1.23%)**
	Total	1385 (100.00%)	1385 (100.00%)
December	Negative	1771 (96.46%)	1791 (2.45%)
	Positive	65 **(3.54%)**	45 **(2.45%)**
	Total	1836 (100.00%)	1836 (100.00%)
Whole sample	Negative	7010 (97.57%)	7067 (98.37%)
	Positive	174 **(2.43%)**	117 **(1.63%)**
	Total	7184 (100.00%)	7184 (100.00%)

a =The table reports the number and the percentage (in parenthesis) of PCP violations registered each month.

Table 4 shows that the level of the violations in this scenario is not very dissimilar to the one detected in the previous one, given that in most cases the arbitrage profits are less than 200 € for

[15] We observe that put option prices are generally bigger than call prices. This is probably due to the fact that in our sample we have more frequently in the money put options (20.98%), rather than in the money call (10.48%).

both the long and short strategy. However, the average profit is higher: 100.35 € for long strategies and 89.71 € for the short ones. The profits earned by investors in this scenario are on average bigger than the ones recorded in the previous scenario, probably because the small ones have been swept away by the bid-ask spread.

Table 4: Descriptive statistics for long and short strategies profits, scenario B [a].

LONG						SHORT				
Included observations: 174						Included observations: 117				
	Mean	Max	Min.	Obs.	%	Mean	Max	Min.	Obs.	%
[0, 200)	58.67	196.16	0.09	151	86.78%	52.10	183.11	1.30	101	86.33%
[200, 400)	263.75	353.15	203.78	19	10.92%	260.67	364.49	205.92	12	10.26%
[400, 600)	443.62	460.09	410.70	3	1.73%	483.57	503.40	455.85	3	2.56%
[600, 800)	-	-	-	-	-	655.57	655.57	655.57	1	0.85%
[2200, 2400)	2260.13	2260.13	2260.13	1	0.57%	-	-	-	-	-
All	100.35	2260.13	0.09	174	100.00%	89.71	655.57	1.30	117	100.00%

a = This table reports for each strategy (long and short) and each profit range: the mean, the maximum, the minimum, the number of observations and the percentage of the profit range over the whole sample. Monetary amounts are in €.

Scenario C

Scenario C includes the bid - ask spread costs as well as the commission costs, set at four different levels (as discussed in the previous section). Thus the profitability of the following strategies is tested :

Long $c^{bid} + Xe^{-r\tau} - p^{ask} - (I - D) - TC$

Short $p^{bid} + (I - D) - c^{ask} - Xe^{-r\tau} - TC$

Given that this is the most realistic scenario, the results obtained can be considered a good approximation of some actual arbitrage opportunities offered to investors by the IDEM. Similarly to previous scenarios, we will present both the frequency and the level of the PCP violations and, additionally, we will perform some analyses in order to single out the causes of these violations.

Table 5 reports the frequency of PCP violations recorded in this last scenario. First it can be noted that the inclusion of commission costs has further reduced the frequency of arbitrage opportunities, but not as much as the inclusion of the bid-ask spread. This confirms the weight of the option bid-ask spread and therefore the importance of including it when testing PCP violations.

Clearly, the arbitrage opportunities decrease as the commission costs tend to increase. Most interestingly, table 5 shows that in this scenario there is no longer a systematic tendency of long strategies to be more often profitable than short ones. So, in contrast with the results reported for US option markets (e.g. Klemkosky and Resnick (1979)) and other European markets (e.g. Capelle-Blancard e Chaudhury (2001) and Mittnik and Rieken (2000)), we do not observe a systematic higher profitability of the short strategy. This result, which was reported also by Cavallo and Mammola (2000), can be attributed to the absence of institutional and actual short selling restrictions in the Italian Market.

Table 5 : Frequency of PCP violations in scenario C, by month and commission levels [a].

Strategy	**Month**	**Commission levels**			
		Minimum	*Medium low*	*Medium high*	*Maximum*
Long	*September*	29 **(1.18%)**	21 **(0.86%)**	14 **(0.57%)**	3 **(0.12%)**
	October	44 **(2.92%)**	37 **(2.45%)**	32 **(2.12%)**	30 **(1.99%)**
	November	14 **(1.01%)**	9 **(0.65%)**	9 **(0.65%)**	2 **(0.14%)**
	December	48 **(2.61%)**	34 **(1.85%)**	23 **(1.25%)**	19 **(1.03%)**
	Whole sample	135 **(1.88%)**	101 **(1.14%)**	78 **(1.08%)**	54 **(0.75%)**
Short	*September*	37 **(1.51%)**	27 **(1.10%)**	24 **(0.98%)**	12 **(0.49%)**
	October	7 **(0.46%)**	4 **(0.27%)**	4 **(0.27%)**	4 **(0.27%)**
	November	13 **(0.94%)**	11 **(0.79%)**	7 **(0.51%)**	5 **(0.36%)**
	December	29 **(1.58%)**	20 **(1.09%)**	15 **(0.82%)**	13 **(0.71%)**
	Whole sample	86 **(1.20%)**	62 **(0.86%)**	50 **(0.70%)**	34 **(0.47%)**

a = The table reports the number and the percentage (in parenthesis) of long and short portfolios that get positive profits taking advantage of the PCP violations. We distinguished by month (rows) and by commission level (columns).

As far as the amount of these violations is concerned Table 6 supports two observations. First, both strategies display profits that range from very negligible values (around half a Euro) to very high ones (e.g. more than 2000 € attaining a long strategy), even though 90% of positive profits by means of both strategies are below 250 €. Second, the average profits are not negligible both for arbitrageurs and for retail investors, ranging from 97.41 € (105.98 €) for arbitrageurs who implement a short (long) strategy to 122.02 € (142.09 €) if the strategies are implemented by retail investors. This apparently strange result can be explained as follows: as the commission costs

increase, the low-profit cases disappear, only the high-profit ones persist and the average profits rise. Therefore we can conclude that, in the very few cases of PCP violations, it is still possible to implement really profitable arbitrage strategies independently of the investor's type.

Table 6 : Descriptive statistics of profits realized, scenario C [a].

Strategy	Commission levels	**Descriptive statistics**			
		Minimum	*Maximum*	*Mean*	*Total observations*
Long	*Minimum*	0.59	2239.38	105.98	135
	Medium low	3.93	2221.38	119.07	101
	Medium high	1.39	2191.38	130.54	73
	Maximum	0.19	2161.38	142.09	54
Short	*Minimum*	0.96	634.82	97.41	86
	Medium low	2.11	616.82	111.62	62
	Medium high	4.76	586.82	107.80	50
	Maximum	0.56	556.82	122.02	34

a = This table reports for each strategy (long and short) and each commission level: the minimum, the maximum and the mean profit realized and the number of observed profitable portfolios over the whole sample. Monetary amounts are in €.

However, it has to be stressed that the percentage of PCP violations in this latter and more realistic scenario is very low. It ranges from a maximum of 2.92% (October, long strategy, minimum level of commission costs) to a minimum of 0.12% (September, long strategy, maximum level of commission costs) and on average, only the 1.05% of portfolios over the whole sample gained positive profits. These latter results point at maintaining that, in the period under analysis, the PCP substantially holds in the Italian index option market.

However, the values obtained are affected by the presence of some cases of abnormal profit, most of which are likely to depend on very particular features of the contract. Specifically, we noted that bigger arbitrage opportunities are reached when options are either out of the money, or they have very short or very long maturity (5 days vs. more than three months) or when the synchronicity with the underlying is not attained.

Therefore, after removing from the data set these "extreme" cases, we have replicated the analysis.[16] The results obtained are not very dissimilar to those reported above. In fact the frequency of PCP violations has not substantially changed and it is still very low: around 1.5% for long strategies and 0.5% for short ones, while the profit level has naturally reduced for both long and short strategies.

At this stage a direct comparison of our results with the previous study for the MibO by Cavallo and Mammola(2000) is in order. Before comparing figures, it has to be stressed that the authors consider only two types of traders – arbitrageurs and individual investors - which, as for the level of commissions, are comparable with two of the four that we have considered, i.e. arbitrageurs and professional investors. Overall, in the most realistic scenario where both commission costs and bid–ask spread are accounted for, Cavallo and Mammola (2000) observe, in the period July 1996 – February 1997, a higher frequency of PCP violations. More precisely, profitable short (long) strategies amount to 5% (6%) for Cavallo and Mammola's arbitrageurs vs. 1.20% (1.88%) for our arbitrageurs; and 2%(2%) for Cavallo and Mammola's individual investors vs. 0.86%(1.14%) for our professional investors. A comparison of Cavallo and Mammola's result for individual investors with our results for trading on line or occasional retail investors would highlight an even bigger difference between the frequencies of violations in their study with respect to ours.

Given that the PCP is more likely to be fulfilled by at the money options with short maturities, the frequency of PCP violations observed by Cavallo and Mammola (2000) should have been lower than the one reported in this study, since they base their analysis only on at the money options with one month maturity. By contrast, although we use options with any moneyness and maturities ranging from one month to one year, we observed a lower frequency of profitable strategies. This suggests that arbitrage opportunities have decreased both for arbitrageurs and retail investors from the period analysed by Cavallo and Mammola, which represent the infancy of the market, to the period investigated in this study, where the market has possibly reached its maturity.

The more favourable results that we obtain with respect to the previous study can be explained, on one hand, by the increase in the market volumes which, from 1997 to 2002, was substantial and was possibly enhanced by the shift to Euro in 1999 (see Figure 1), on the other, by the availability of higher frequency data that allows the degree of synchronicity required by the tests.

To sum up, our analysis supports the validity of the PCP on the Italian index option market thus pointing at a substantial and increased pricing efficiency of the very same market.

[16] The cases omitted and details on the results are available upon request.

5. Conclusions

In the present paper we have tested the efficiency of the Italian index option market in the period 1 September 2002 – 31 December 2002 by checking the validity of the most famous no-arbitrage relationship in option markets: the Put-Call Parity. By means of a high frequency dataset, we have conducted our analysis in essentially three steps: in the first step we tested the PCP in the total absence of frictions, in the second one we have included the bid-ask spread into the analysis and, finally, we have included commission costs too. The inclusion of frictions has raised many estimation problems given that neither bid-ask spread quotations are available nor commissions costs can be uniquely defined.

The main findings can be summed up as follows.

Arbitrage opportunities are swept away by frictions: more precisely, in the presence of a bid-ask spread on the option prices, the average percentage of profitable short (long) arbitrage strategies drops from 57.84% (42.16%) to 1.63% (2.43%). This latter datum further drops when commissions are accounted for. Given the disparate level of commissions in Italian the market, we have conducted our analysis under four different assumptions about the level of commissions, ranging from a minimum to a maximum, which identify four different type of traders: arbitrageurs, professional investors, retail investors trading on line and occasional retails. Depending on the trader's type, the percentage of PCP violations ranges form 1.20% (1.88%) for short (long) arbitrageurs to a 0.47% (0.75%) for short (long) occasional retails.

Thus our results are in line with previous studies on other European markets as far as the role of frictions is concerned. However, by contrast with Capelle-Blancard and Chaudhury(2001) and Mittnik and Rieken(2000), our results do not support a systematic higher frequency of arbitrage violations for portfolios involving short-selling of the underlying. This latter result, which stems form the actual absence of short selling restrictions on the Italian market, is in line with the previous study by Cavallo and Mammola(2000). However, with respect to our results, the authors observe in the period July 1996 – February 1997 a higher frequency of PCP violations although they use only at the money options with one-month maturity.

By a comparative inspection of the two studies, we can conclude that, for all traders types, arbitrage opportunities in the Italian index option market have decreased from its introduction to the present.

The more favourable results that we obtain with respect to the previous study can be explained, on one hand, by the increase in the market volumes which from 1997 to 2002 was substantial and possibly enhanced by the shift to Euro in 1999 (see Figure 1), on the other, by the availability of higher frequency data that allows the degree of synchronicity required by the tests performed.

Finally, we have checked the robustness of our results by removing from our analysis some abnormal profits due to the very particular nature of the option (e.g. out of the money options, very long or very short maturities). The results remain unaffected in terms of frequency, whereas the profit levels naturally drop.

Overall, we can conclude that, in the period under analysis, the MibO market was efficient in that the frequency of arbitrage opportunities is low for arbitrageurs and much lower for occasional retails. However, in the very few cases of PCP violations, it is possible to implement profitable arbitrage strategies independently of the investor's type.

The efficiency analysis performed in this paper can be extended in many directions: first, the tests performed in this work are ex-post tests and therefore cannot highlight the role of non synchronous trading in the arbitrage violations; secondly the PCP is a cross market relationship and other strategies such as various types of spreads can be used to check the internal option market efficiency. These issues will be investigated in a different paper.

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